

13.0 Speaker Variabilities: Adaption and Recognition

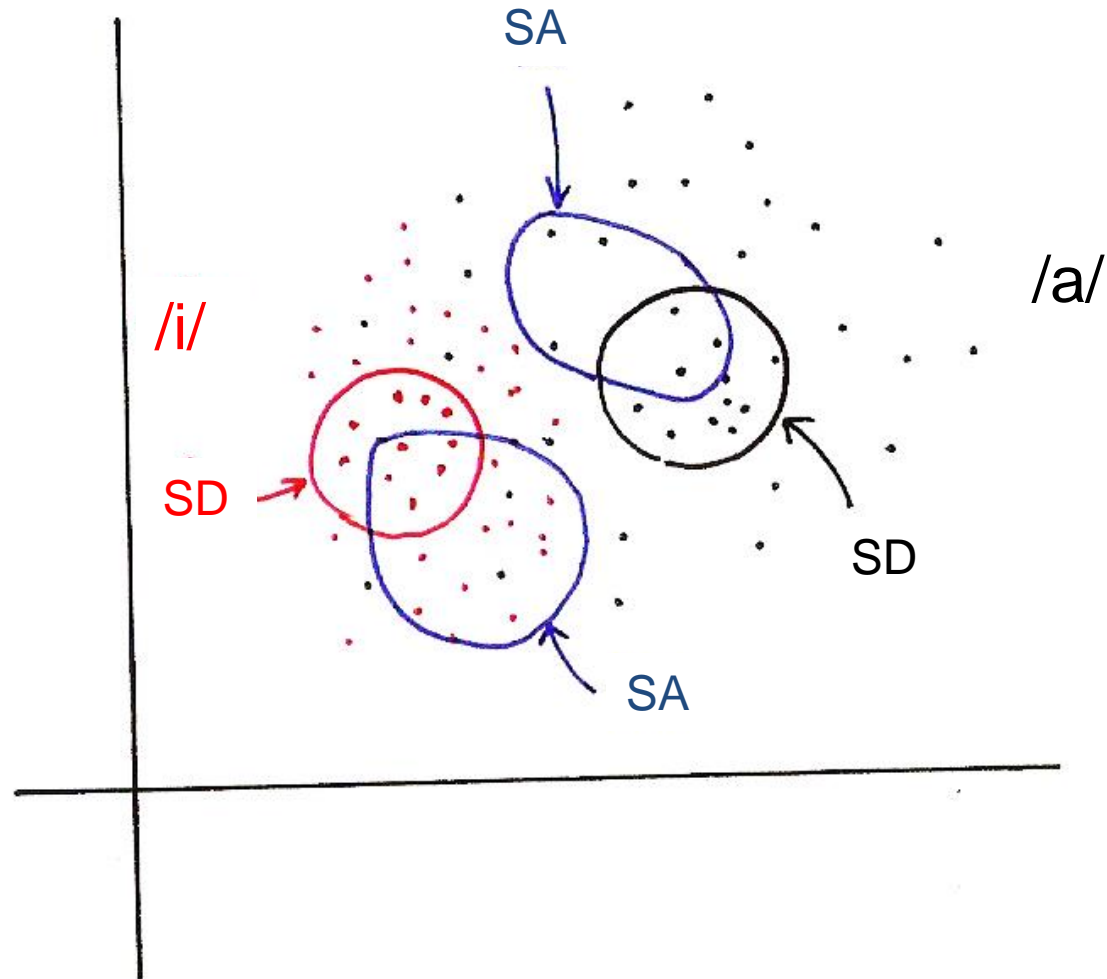
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Speaker Dependent/Independent/Adaptation

- **Speaker Dependent (SD)**
 - trained with and used for 1 speaker only, requiring huge quantity of training data, best accuracy
 - practically infeasible
- **Multi-speaker**
 - trained for a (small) group of speakers
- **Speaker Independent (SI)**
 - trained from large number of speakers, each speaker with limited quantity of data
 - good for all speakers, but with relatively lower accuracy
- **Speaker Adaptation (SA)**
 - started with speaker independent models, adapted to a specific user with limited quantity of data (adaptation data)
 - technically achievable and practically feasible
- **Supervised/Unsupervised Adaptation**
 - supervised: text (transcription) of the adaptation data is known
 - unsupervised: text (transcription) of the adaptation data is unknown, based on recognition results with speaker-independent models, may be performed iteratively
- **Batch/Incremental/On-line Adaptation**
 - batch: based on a whole set of adaptation data
 - incremental/on-line: adapted step-by-step with iterative re-estimation of models
e.g. first adapted based on first 3 utterances, then adapted based on next 3 utterances or first 6 utterances,...

Speaker Dependent/Independent/Adaptation



MAP (Maximum A Posteriori) Adaptation

- **Given Speaker-independent Model set $\Lambda = \{\lambda_i = (A_i, B_i, \pi_i), i=1, 2, \dots, M\}$ and A set of Adaptation Data $\bar{O} = (o_1, o_2, \dots, o_t, \dots, o_T)$ for A Specific Speaker**

$$\Lambda^* = \arg \max_{\Lambda} \text{Prob}[\Lambda | \bar{O}] = \arg \max_{\Lambda} \frac{\text{Prob}[\bar{O} | \Lambda] \text{Prob}[\Lambda]}{\text{Prob}[\bar{O}]} = \arg \max_{\Lambda} \text{Prob}[\bar{O} | \Lambda] \text{Prob}[\Lambda]$$

- **With Some Assumptions on the Prior Knowledge Prob $[\Lambda]$ and some Derivation (EM Theory)**

- example adaptation formula

$$\mu_{jk}^* = \frac{\tau_{jk} \mu_{jk} + \sum_{t=1}^T [\gamma_t(j, k) o_t]}{\tau_{jk} + \sum_{t=1}^T \gamma_t(j, k)}$$

μ_{jk} : mean of the k - th Gaussian in the j - th state for a certain λ_i

μ_{jk}^* : adapted value of μ_{jk}

$$\gamma_t(j, k) = \left[\frac{\alpha_t(j) \beta_t(j)}{\sum_{j=1}^N \alpha_t(j) \beta_t(j)} \right] \left[\frac{c_{jk} N(o_t; \mu_{jk}, U_{jk})}{\sum_{m=1}^L c_{jm} N(o_t; \mu_{jm}, U_{jm})} \right]$$

↑

$$\gamma_t(j) = P(q_t = j | \bar{O}, \lambda_i)$$

τ_{jk} : a parameter having to do with the prior knowledge about μ_{jk}

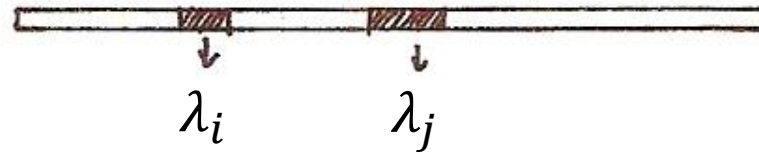
may have to do with number of samples used to train μ_{jk}

- a weighted sum shifting μ_{jk} towards those directions of o_t (in j-th state and k-th Gaussian)
larger τ_{jk} implies less shift

- **Only Those Models with Adaptation Data will be Modified, Unseen Models remain Unchanged — MAP Principle**

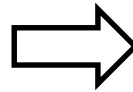
- good with larger quantity of adaptation data
- poor performance with limited quantity of adaptation data

MAP Adaptation

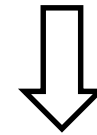


$$\frac{a\vec{v} + b\vec{o}}{a + b}$$

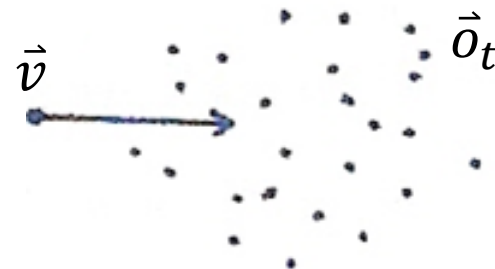
$$= \lambda\vec{v} + (1 - \lambda)\vec{o}$$



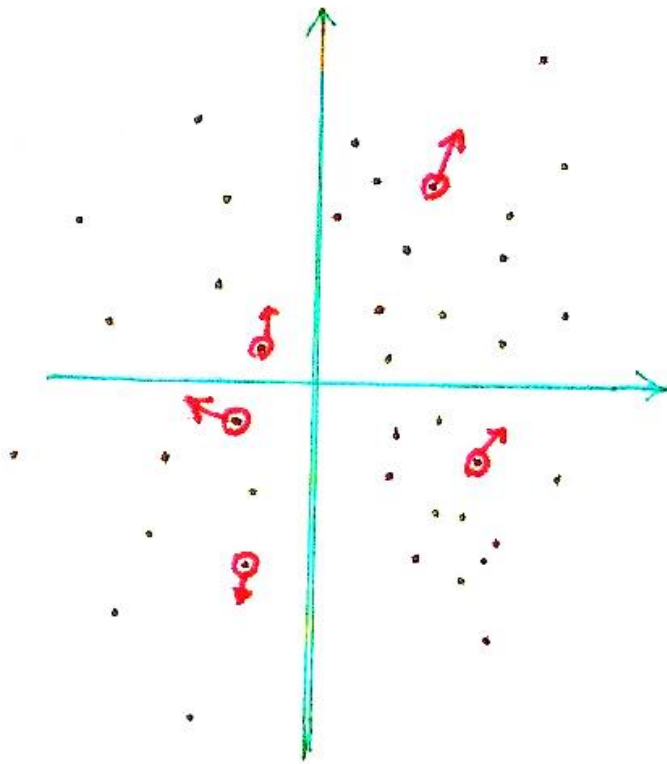
$$\frac{a\vec{v} + (\sum_i b_i)\vec{o}}{a + (\sum_i b_i)}$$



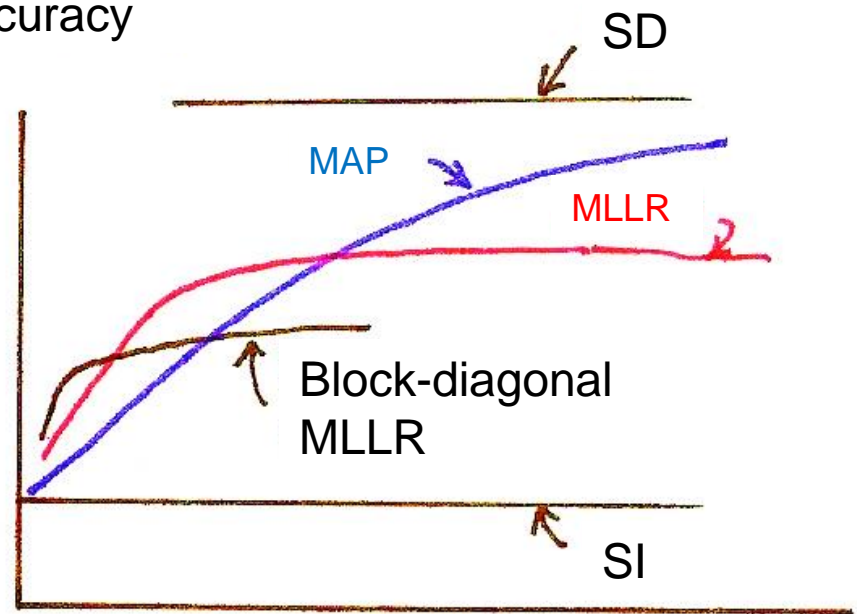
$$\frac{a\vec{v} + (\sum_t b_t \vec{o}_t)}{a + (\sum_t b_t)}$$



MAP Adaptation



Accuracy



Adaptation Data

Maximum Likelihood Linear Regression (MLLR)

- **Divide the Gaussians (or Models) into Classes C_1, C_2, \dots, C_L , and Define Transformation-based Adaptation for each Class**

$$\mu_{jk}^* = A \mu_{jk} + b, \quad \mu_{jk} : \text{mean of the } k\text{-th Gaussian in the } j\text{-th state}$$

- linear regression with parameters A, b estimated by maximum likelihood criterion

$$[A_i, b_i] = \arg \max_{A, b} \text{Prob}[\bar{O} | \Lambda, A_i, b_i] \text{ for a class } C_i$$

A_i, b_i estimated by EM algorithm

- All Gaussians in the same class updated with the same A_i, b_i : parameter sharing, adaptation data sharing
- unseen Gaussians (or models) can be adapted as well
- A_i can be full matrices, or reduced to diagonal or block-diagonal to have less parameters to be estimated
- faster adaptation with much less adaptation data needed, but saturated at lower accuracy with more adaptation data due to the less precise modeling

- **Clustering the Gaussians (or Models) into L Classes**

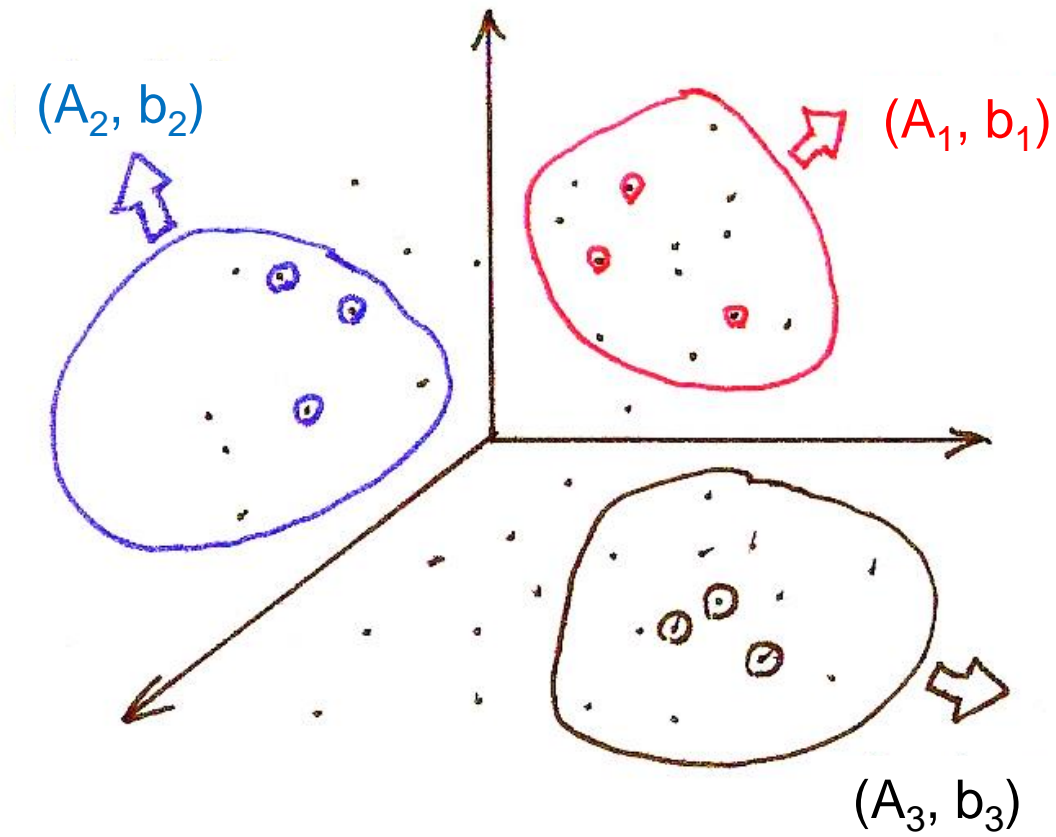
- too many classes requires more adaptation data, too less classes becomes less accurate
- basic principle: Gaussian (or models) with similar properties and “just enough” data form a class
- data-driven (e.g. by Gaussian distances) primarily, knowledge driven helpful

- **Tree-structured Classes**

- the node including minimum number of Gaussians (or models) but with adequate adaptation data is a class
- dynamically adjusting the classes as more adaptation data are observed

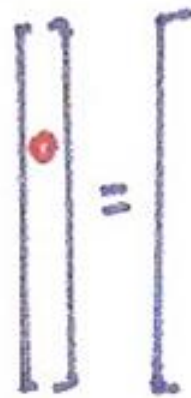
- **Feature-based MLLR (fMLLR)**

MLLR

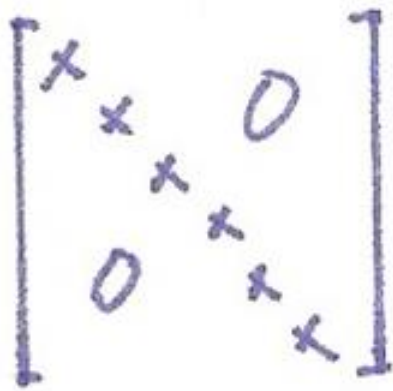


MLLR

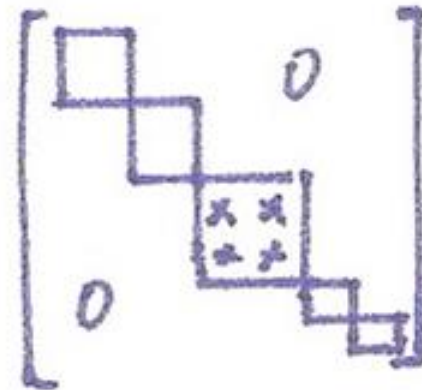
$$\vec{y} = A\vec{x}$$



Full

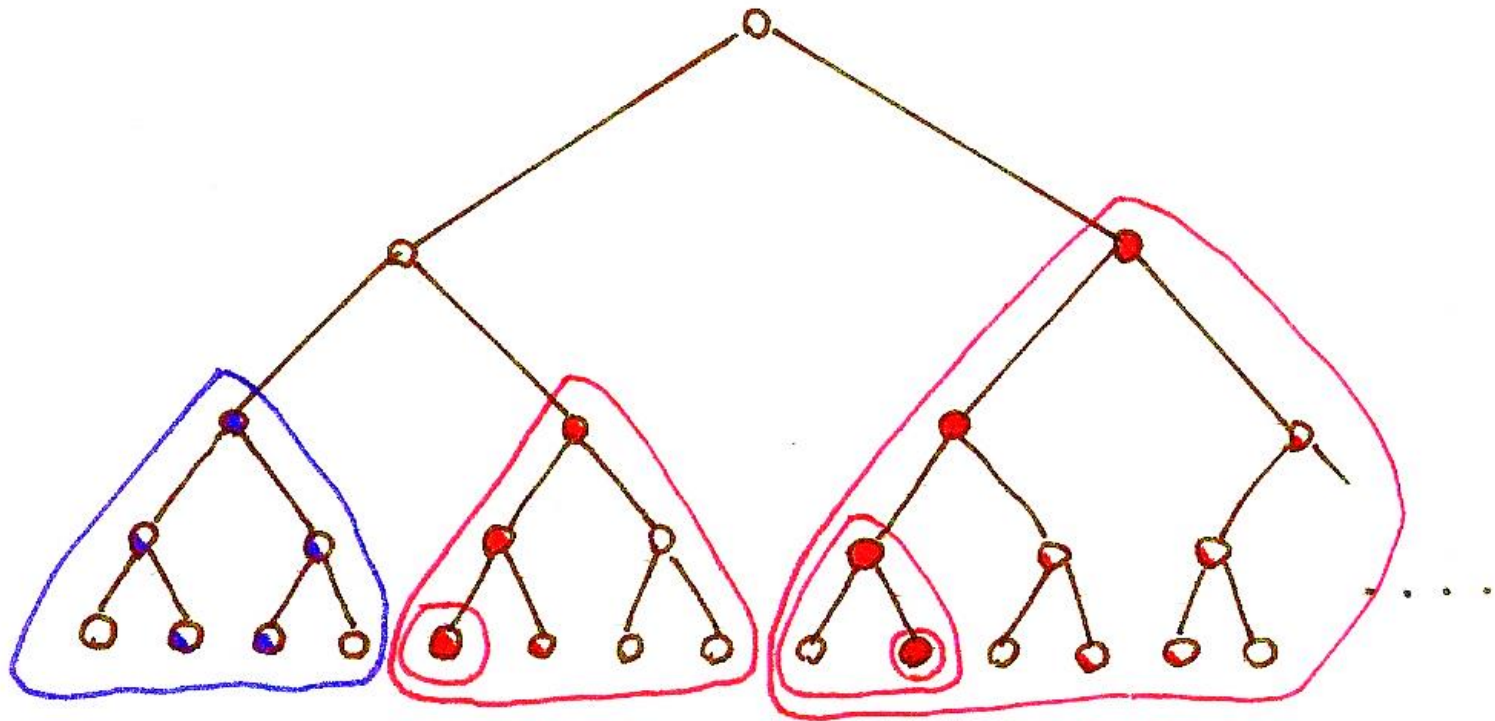


Diagonal



Block-diagonal

MLLR



Principal Component Analysis (PCA)

• Problem Definition:

- for a zero mean random vector \mathbf{x} with dimensionality N , $\mathbf{x} \in \mathbb{R}^N$, $E(\mathbf{x})=0$, iteratively find a set of k ($k \leq N$) orthonormal basis vectors $\{\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_k\}$ so that
 - (1) $\text{var}(\mathbf{e}_1^T \mathbf{x}) = \max$ (x has maximum variance when projected on \mathbf{e}_1)
 - (2) $\text{var}(\mathbf{e}_i^T \mathbf{x}) = \max$, subject to $\mathbf{e}_i \perp \mathbf{e}_{i-1} \perp \dots \perp \mathbf{e}_1$, $2 \leq i \leq k$
(x has next maximum variance when projected on \mathbf{e}_2 , etc.)

• Solution: $\{\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_k\}$ are the eigenvectors of the covariance matrix Σ for \mathbf{x} corresponding to the largest k eigenvalues

- new random vector $\mathbf{y} \in \mathbb{R}^k$: the projection of \mathbf{x} onto the subspace spanned by $\mathbf{A} = [\mathbf{e}_1 \ \mathbf{e}_2 \ \dots \ \mathbf{e}_k]$, $\mathbf{y} = \mathbf{A}^T \mathbf{x}$
- a subspace with dimensionality $k \leq N$ such that when projected onto this subspace, \mathbf{y} is “closest” to \mathbf{x} in terms of its “randomness” for a given k
- $\text{var}(\mathbf{e}_i^T \mathbf{x})$ is the eigenvalue associated with \mathbf{e}_i

• Proof

- $\text{var}(\mathbf{e}_1^T \mathbf{x}) = \mathbf{e}_1^T E(\mathbf{x} \mathbf{x}^T) \mathbf{e}_1 = \mathbf{e}_1^T \Sigma \mathbf{e}_1 = \max$, subject to $|\mathbf{e}_1|^2 = 1$
- using Lagrange multiplier

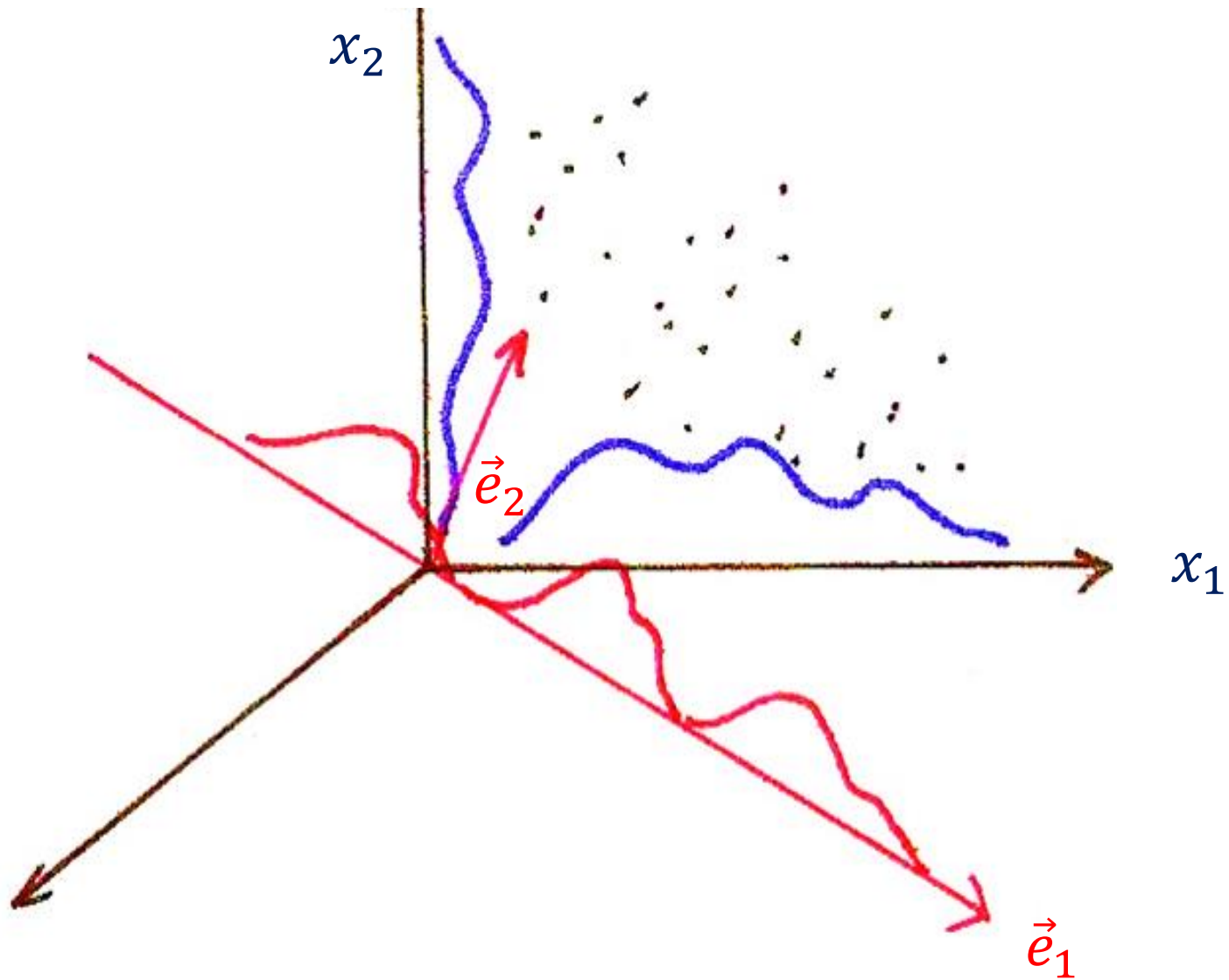
$$J(\mathbf{e}_1) = \mathbf{e}_1^T E(\mathbf{x} \mathbf{x}^T) \mathbf{e}_1 - \lambda(|\mathbf{e}_1|^2 - 1), \quad \frac{\partial J(\mathbf{e}_1)}{\partial \mathbf{e}_1} = 0$$

$$\Rightarrow E(\mathbf{x} \mathbf{x}^T) \mathbf{e}_1 = \lambda_1 \mathbf{e}_1, \quad \text{var}(\mathbf{e}_1^T \mathbf{x}) = \lambda_1 = \max$$

- similar for \mathbf{e}_2 with an extra constraint $\mathbf{e}_2^T \mathbf{e}_1 = 0$, etc.

$$\begin{array}{ccc} Av & = & u \\ Av & = & \lambda v \\ \uparrow & & \uparrow \\ \text{eigenvector} & & \text{eigenvalue} \end{array}$$

PCA



$$\begin{aligned} \begin{bmatrix} \cdots & e_{1k}^T & \cdots \end{bmatrix} \begin{bmatrix} \vdots \\ x_k \\ \vdots \end{bmatrix} &= \vec{e}_1 \cdot \vec{x} \\ &= \underbrace{|\vec{e}_1|}_{\substack{\parallel \\ 1}} |\vec{x}| \cos \theta \end{aligned}$$

$$\vec{y} = A^T \vec{x} = \begin{bmatrix} \vec{e}_1^T \\ \vec{e}_2^T \\ \vdots \\ \vec{e}_k^T \end{bmatrix} \vec{x}$$

Basic Problem 3 (P.35 of 4.0)

$$\bar{U} = \begin{bmatrix} & & m \\ & & \vdots \\ & & \vdots \\ \ell & \cdots & \bar{u}_{lm} & \cdots \\ & & \vdots \\ & & \vdots \end{bmatrix} = E \left(\begin{bmatrix} x_1 - \bar{x}_1 \\ x_2 - \bar{x}_2 \\ \vdots \\ \vdots \end{bmatrix} [x_1 - \bar{x}_1, x_2 - \bar{x}_2, \cdots] \right)$$

$$\bar{u}_{lm} = E[(x_l - \bar{x}_l)(x_m - \bar{x}_m)]$$

PCA

The diagram illustrates the PCA decomposition of a covariance matrix Σ . It shows a matrix of eigenvectors $\begin{bmatrix} \vec{e}_1 & \vec{e}_2 & \dots & \vec{e}_k & \dots & \vec{e}_N \end{bmatrix}$ multiplied by a diagonal matrix of eigenvalues $\begin{bmatrix} \lambda_1 & & & & & \\ & \lambda_2 & & & & \\ & & \ddots & & & \\ & & & \lambda_k & & \\ & & & & \ddots & \\ & & & & & \lambda_N \end{bmatrix}$, which is approximately equal to the covariance matrix Σ . The covariance matrix is also shown as $\Sigma = E[x x^T] = E[(x - \bar{x})(x - \bar{x})^T]$. The eigenvectors are shown as columns of a matrix, and the eigenvalues are shown as elements of a diagonal matrix. The covariance matrix is shown as a matrix of inner products $\begin{bmatrix} \vec{e}_1^T & \vec{e}_2^T & \dots & \vec{e}_k^T & \vdots & \vec{e}_N^T \end{bmatrix}$.

$$\begin{bmatrix} \vec{e}_1 & \vec{e}_2 & \dots & \vec{e}_k & \dots & \vec{e}_N \end{bmatrix} \begin{bmatrix} \lambda_1 & & & & & \\ & \lambda_2 & & & & \\ & & \ddots & & & \\ & & & \lambda_k & & \\ & & & & \ddots & \\ & & & & & \lambda_N \end{bmatrix} \approx \Sigma$$

$$\Sigma = E[x x^T]$$

$$\Sigma = E[(x - \bar{x})(x - \bar{x})^T]$$

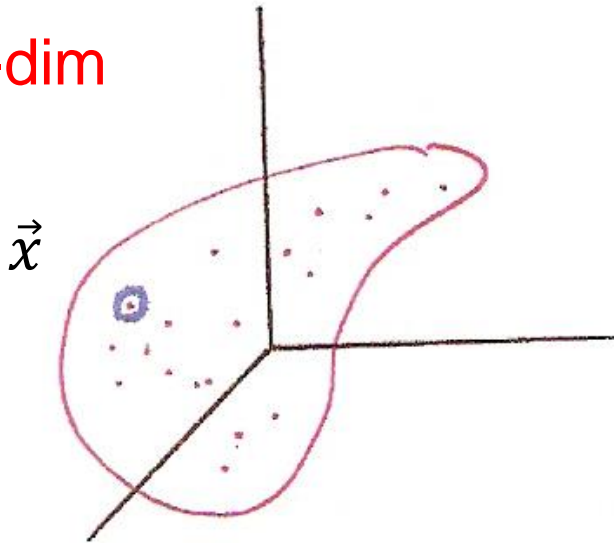
$$\sum \underbrace{\vec{e}_i}_{\text{eigenvector}} = \underbrace{\lambda_i}_{\text{eigenvalue}} \underbrace{\vec{e}_i}_{\text{eigenvector}}$$

$$\begin{aligned} Av &= u \\ Av &= \lambda v \\ \uparrow \quad \quad \uparrow \\ \text{eigenvector} \quad & \text{eigenvalue} \end{aligned}$$

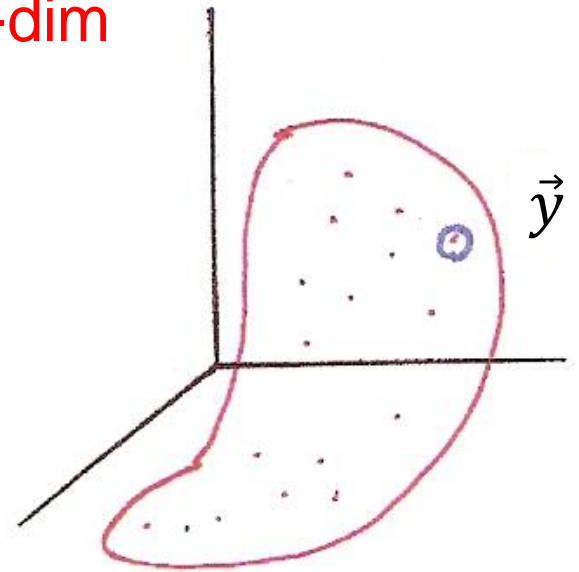
PCA

$$\vec{y} = A^T \vec{x}$$

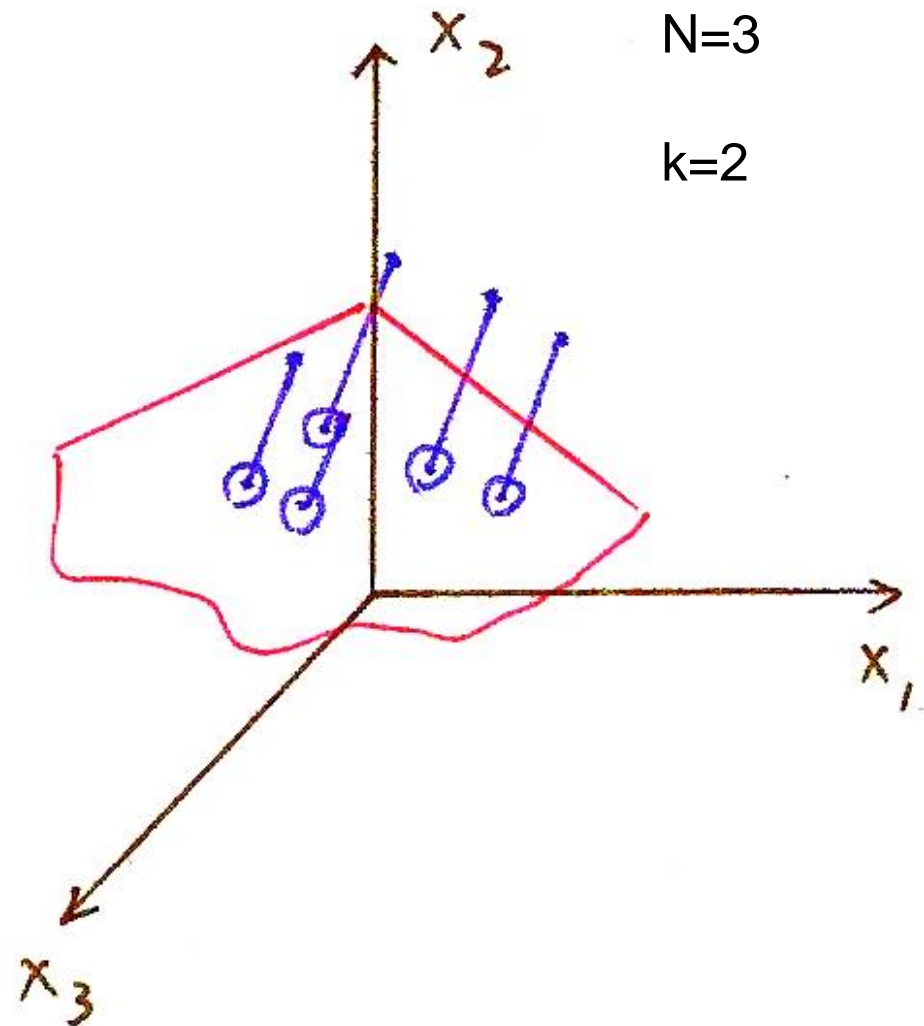
N-dim



k-dim



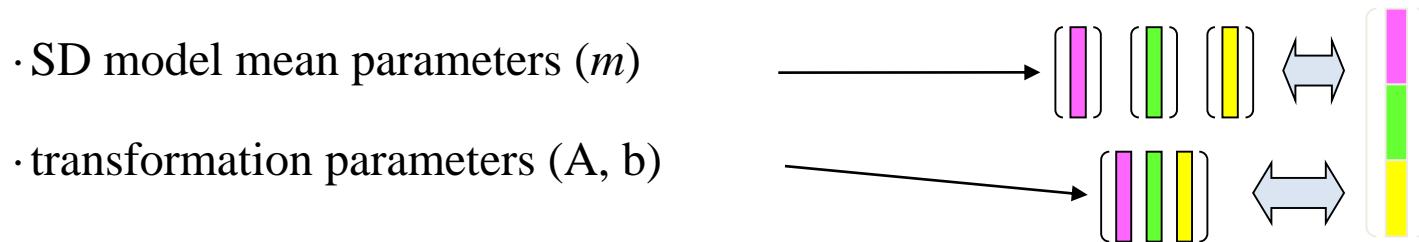
PCA



Eigenvoice

- **A Supervector \mathbf{x} constructed by concatenating all relevant parameters for the speaker specific model of a training speaker**

- concatenating the mean vectors of Gaussians in the speaker-dependent phone models
- concatenating the columns of A , b in MLLR approach
- \mathbf{x} has dimensionality N ($N = 5,000 \times 3 \times 8 \times 40 = 4,800,000$ for example)



- **A total of L ($L = 1,000$ for example) training speakers gives L supervectors $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_L$**
 - $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \dots, \mathbf{x}_L$ are samples of the random vector \mathbf{x}
 - each training speaker is a point (or vector) in the space of dimensionality N

- **Principal Component Analysis (PCA)**

- $\mathbf{x}' = \mathbf{x} - E(\mathbf{x})$, $\Sigma = E(\mathbf{x}' \mathbf{x}'^T)$,
 $\Sigma \approx [e_1, e_2, \dots, e_k][\lambda_i][e_1, e_2, \dots, e_k]^T$, $[\lambda_i]$: diagonal with λ_i as elements

$\{e_1, e_2, \dots, e_k\}$: eigenvectors with maximum eigenvalues $\lambda_1 > \lambda_2 > \dots > \lambda_k$
 k is chosen such that $\lambda_j, j > k$ is small enough ($k=250$ or 50 for example)

Eigenvoice

- **Principal Component Analysis (PCA)**

- $\mathbf{x}' = \mathbf{x} - E(\mathbf{x})$, $\Sigma = E(\mathbf{x}' \mathbf{x}'^T)$,

- $\Sigma \approx [e_1, e_2, \dots, e_K][\lambda_1, \lambda_2, \dots, \lambda_K]^T$, $[\lambda_i]$: diagonal with λ_i as elements

- $\{e_1, e_2, \dots, e_K\}$: eigenvectors with maximum eigenvalues $\lambda_1 > \lambda_2 > \dots > \lambda_K$

- k is chosen such that $\lambda_j, j > k$ is small enough ($k=50$ for example)

- **Eigenvoice Space: spanned by $\{e_1, e_2, \dots, e_K\}$**

- each point (or vector) in this space represents a whole set of tri-phone model parameters

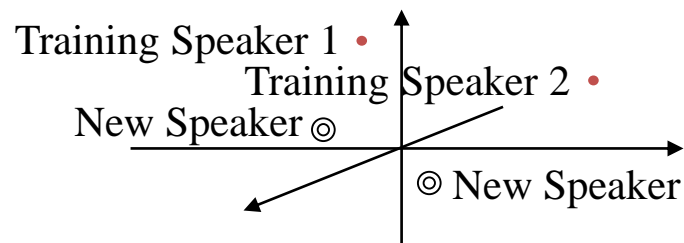
- $\{e_1, e_2, \dots, e_K\}$ represents the most important characteristics of speakers extracted from huge quantity of training data by large number of training speakers

- each new speaker as a point (or vector)

- in this space, $y = \sum_{i=1}^k a_i e_i$

- a_i estimated by maximum likelihood principle (EM algorithm)

- $$\bar{a}^* = \arg \max_{\bar{a}} \text{Prob}[\bar{O} \mid \sum_{i=1}^k a_i e_i]$$



- **Features and Limitations**

- only a small number of parameters $a_1 \dots a_K$ is needed to specify the characteristics of a new speaker

- rapid adaptation requiring only very limited quantity of training data

- performance saturated at lower accuracy (because too few free parameters)

- high computation/memory/training data requirements

Speaker Adaptive Training (SAT) and Cluster Adaptive Training (CAT)

• Speaker Adaptive Training (SAT)

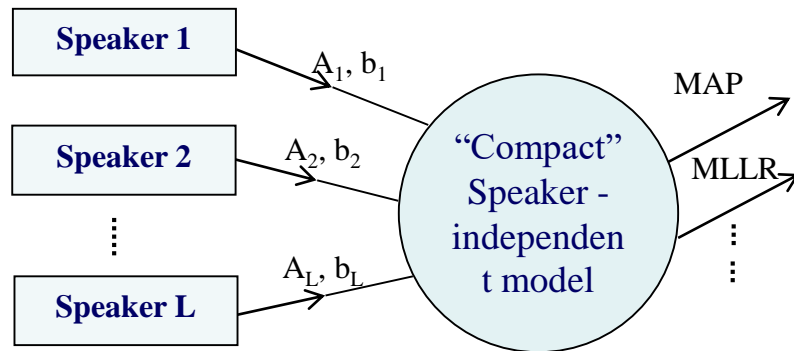
- trying to decompose the phonetic variation and speaker variation
- removing the speaker variation among training speakers as much as possible
- obtaining a “compact” speaker-independent model for further adaptation
- $y = Ax + b$ in MLLR can be used in removing the speaker variation

• Clustering Adaptive Training (CAT)

- dividing training speakers into R clusters by speaker clustering techniques
- obtaining mean models for all clusters (may include a mean-bias for the “compact” model in SAT)
- models for a new speaker is interpolated from the mean vectors

• Speaker Adaptive Training (SAT)

Training Speakers

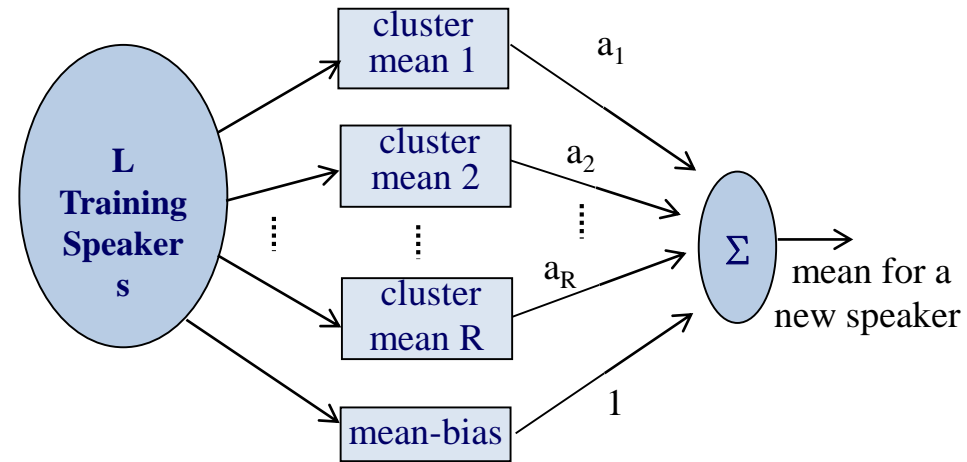


$$\text{Original SI : } \Lambda^* = \arg \max_{\Lambda} \text{Prob}(\bar{o}_{1,2,\dots,L} | \Lambda)$$

$$\text{SAT : } [\Lambda_c^*, (A, b)_{1,\dots,L}^*] = \arg \max_{\Lambda_c, (A, b)_{1,\dots,L}} \text{Prob}(\bar{o}_{1,2,\dots,L} | \Lambda_c, (A, b)_{1,\dots,L})$$

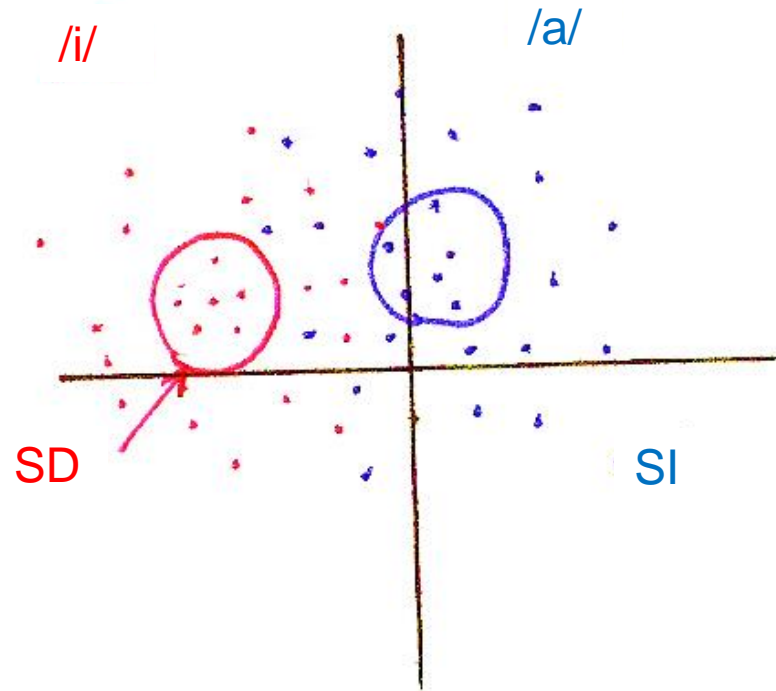
EM algorithm used

• Cluster Adaptive Training (CAT)



$$m^* = \sum_{i=1}^R a_i m_i + m_b, \quad m_i: \text{cluster mean } i, \quad m_b: \text{mean-bias}$$

a_i estimated with maximum likelihood criterion



$$\vec{y} = A\vec{x} + \vec{b}$$

$$\vec{x} = A^{-1} \vec{y} - A^{-1} \vec{b}$$

Speaker Recognition/Verification

- **To recognize the speakers rather than the content of the speech**
 - phonetic variation/speaker variation
 - speaker identification: to identify the speaker from a group of speakers
 - speaker verification: to verify if the speaker is as claimed

- **Gaussian Mixture Model (GMM)**

$$\lambda_i = \{(w_j, \mu_j, \Sigma_j), j=1,2,\dots,M\} \text{ for speaker } i$$

$$\text{for } \bar{O} = o_1 o_2 \dots o_t \dots o_T, \quad b_i(o_t) = \sum_{j=1}^M w_j N(o_t; \mu_j, \Sigma_j)$$

- maximum likelihood principle

$$i^* = \arg \max_i \text{Prob}(\bar{O} | \lambda_i)$$

- **Feature Parameters**

- those carrying speaker characteristics preferred
- MFCC
- MLLR coefficients A_i, b_i , eigenvoice coefficients a_i , CAT coefficients a_i

- **Speaker Verification**

- text dependent: higher accuracy but easily broken
- text independent
- likelihood ratio test

$$\rho(\bar{O}; \lambda_i) = \frac{p(\bar{O} | \lambda_i)}{p(\bar{O} | \bar{\lambda}_i)} > th$$

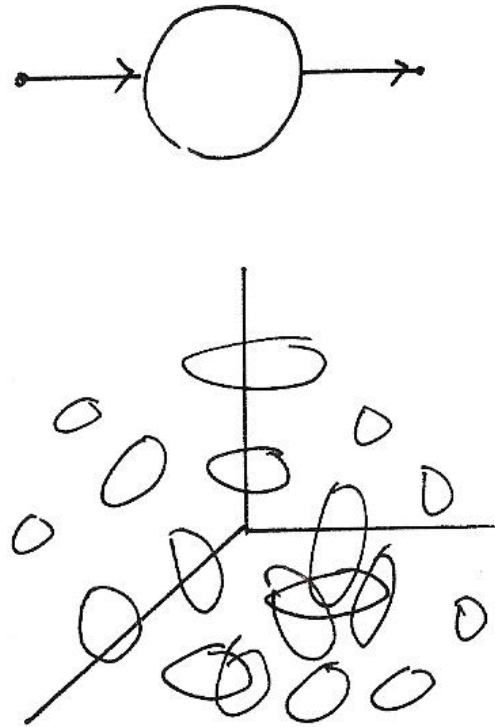
$\bar{\lambda}_i$: background model or anti - model for speaker i , trained by other speakers, competing speakers, or speaker - independent model

th : threshold adjusted by balancing missing/false alarm rates and ROC curve

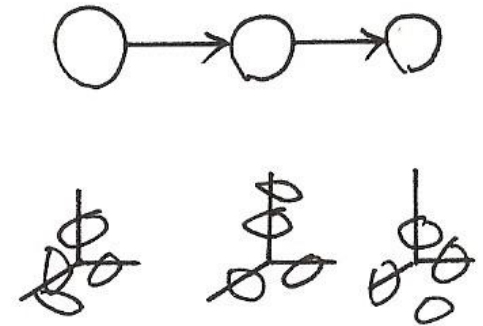
- speech recognition based verification

Speaker Recognition

Gaussian Mixture Model (GMM)



HMM



Likelihood Ratio Test

- **Detection Theory— Hypothesis Testing/Likelihood Ratio Test**

- 2 Hypotheses: H_0, H_1 with prior probabilities: $P(H_0), P(H_1)$
observation: X with probabilistic law: $P(X | H_0), P(X | H_1)$

- MAP principle

choose H_0 if $P(H_0 | X) > P(H_1 | X)$

choose H_1 if $P(H_1 | X) > P(H_0 | X)$

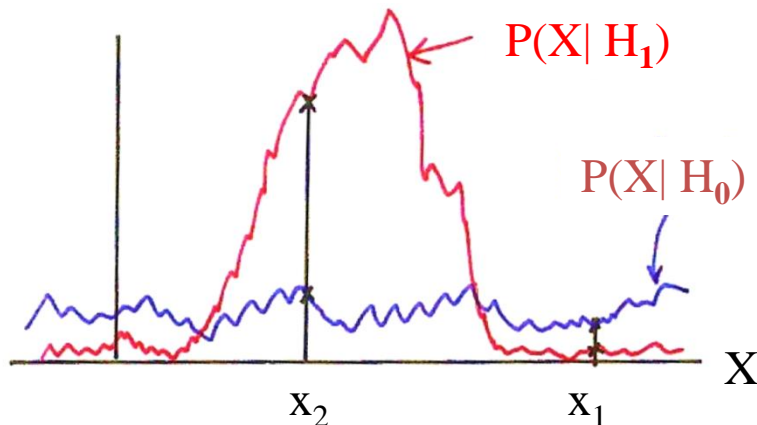
$$\Rightarrow \frac{P(H_0 | X)}{P(H_1 | X)} \underset{H_1}{\overset{H_0}{\gtrless}} 1$$

- Likelihood Ratio Test

$$P(H_i | X) = P(X | H_i) P(H_i) / P(X), i=0,1$$

$$\Rightarrow \frac{P(X | H_0)}{P(X | H_1)} \underset{H_1}{\overset{H_0}{\gtrless}} \frac{P(H_1)}{P(H_0)} = Th$$

likelihood ratio-Likelihood Ratio Test



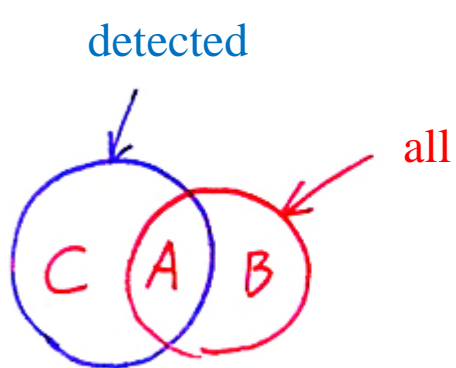
- Type I error: missing (false rejection)

Type II error: false alarm (false detection)

false alarm rate, false rejection rate, detection rate, recall rate, precision rate

Th: a threshold value adjusted by balancing among different performance rates ²⁴

Receiver Operating Characteristics (ROC) Curve



$$\text{Missing rate} = \frac{B}{A+B} = 1 - \frac{A}{A+B} \quad \text{recall}$$
$$\text{False Alarm rate} = \frac{C}{A+C} = 1 - \frac{A}{A+C} \quad \text{precision}$$

