信號與系統

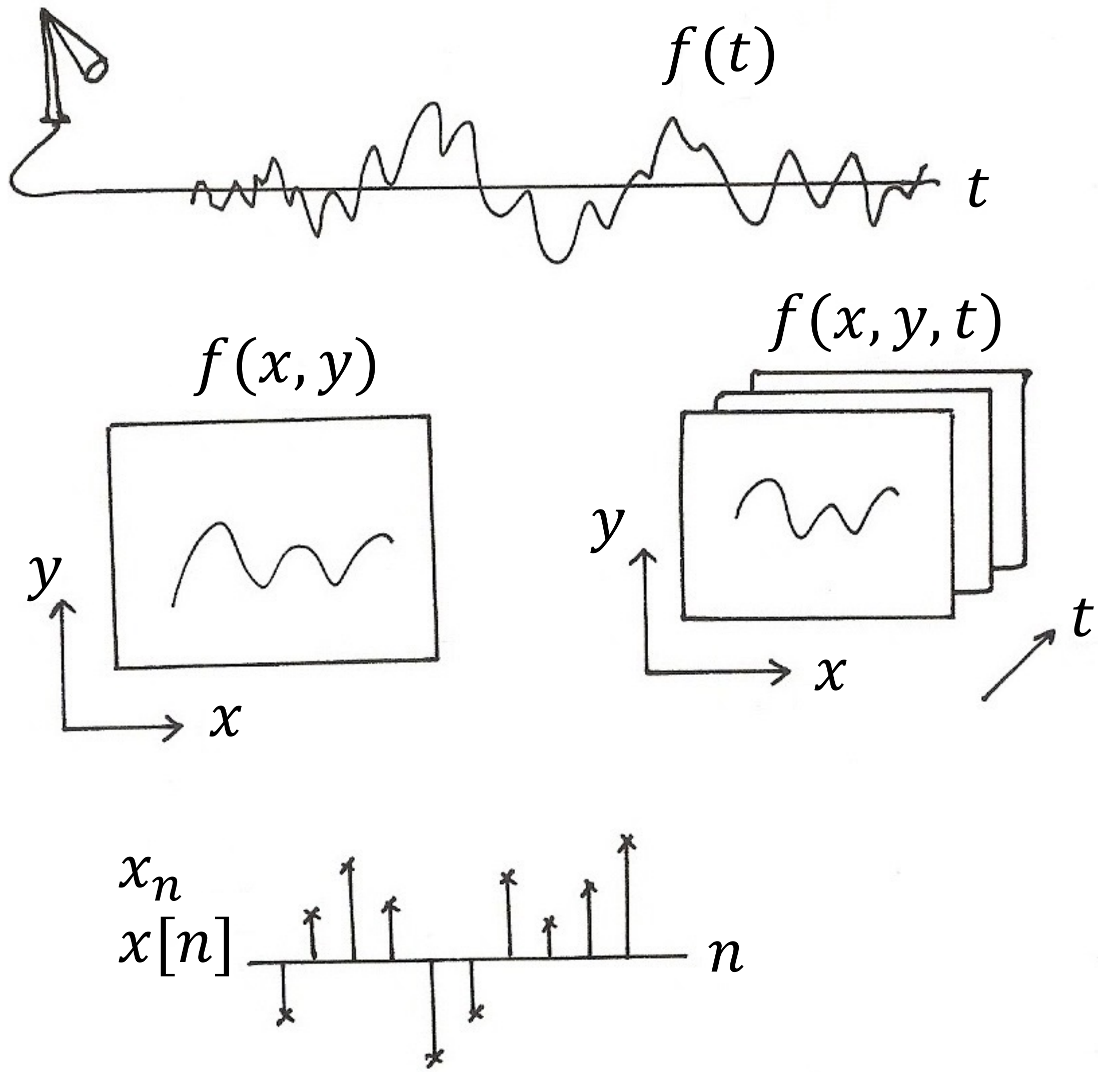
Signals & Systems

李琳山

A Signal

- A signal is a function of one or more variables, which conveys information on the nature of some physical phenomena.
- Examples
  - f (t) : a voice signal, a music signal
  - f (x , y) : an image signal, a picture
  - f (x , y , t) : a video signal
  - x_n : a sequence of data (n: integer)
  - b_n : a bit stream (b:1 or 0)
  - continuous-time, discrete-time
  - analog, digital
- Human Perceptible/Machine Processed

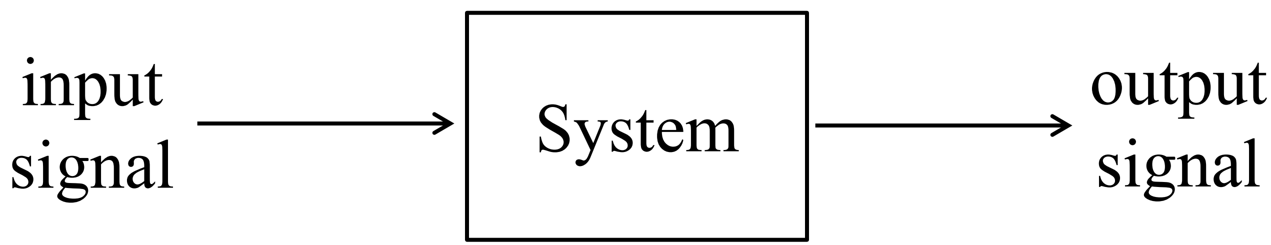
A Signal



b_n: 1 1 0 1 0 1 0 0 1 0 1 0 0 1 …

A System

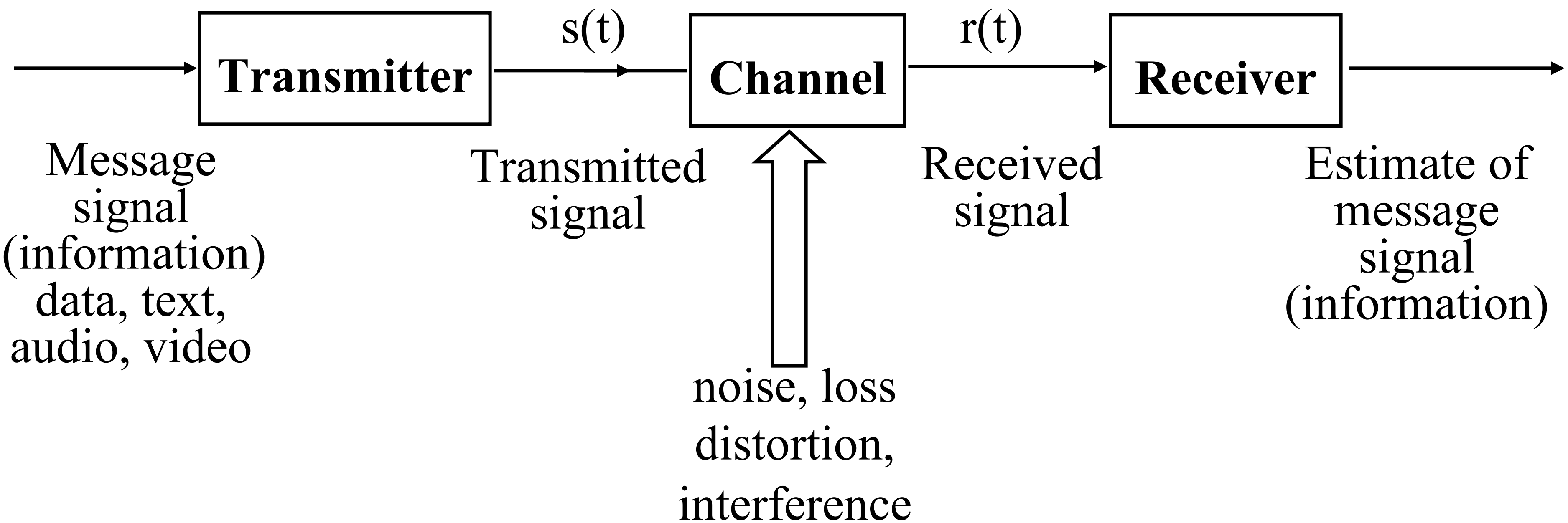
- An entity that manipulates one or more signals to accomplish some function, including yielding some new signals.



- Examples
  - an electric circuit
  - a telephone handset
  - a PC software receiving pictures from Internet
  - a TV set
  - a computer with some software handling some data

Typical Examples of Signals/Systems Concerned

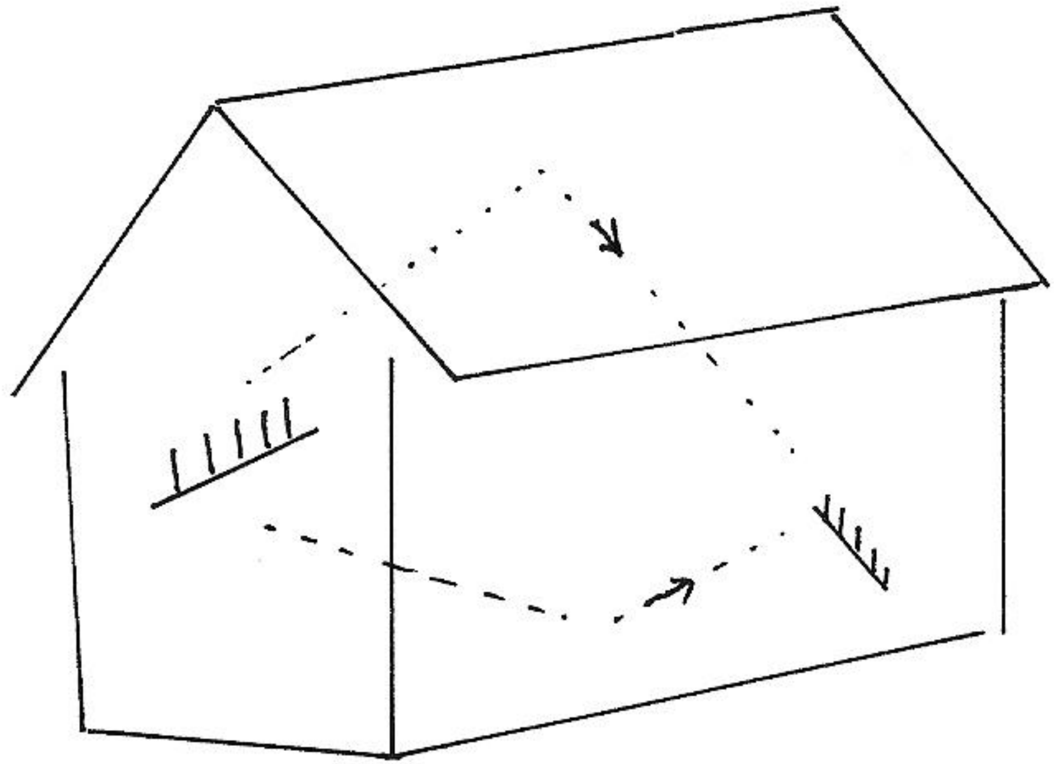
- Communication Systems



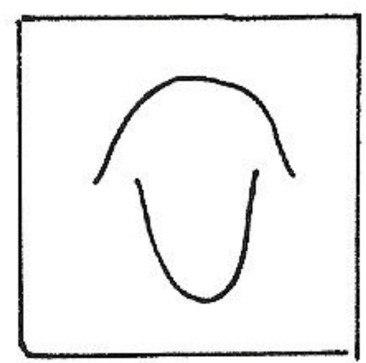
Typical Examples of Signals/Systems Concerned

- Computers
- Signal Processing Systems
  - software systems processing the signal by computation/ memory
  - examples : audio enhancement systems, picture processing systems, video compression systems, voice recognition/ synthesis systems, array signal processors, equalizers, etc.

Audio Enhancement

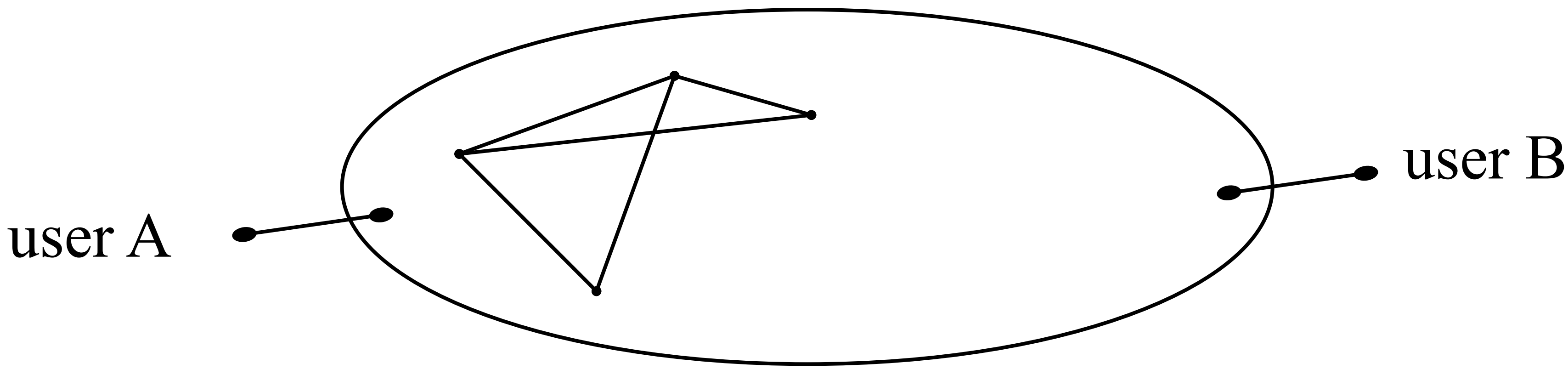


Picture Processing



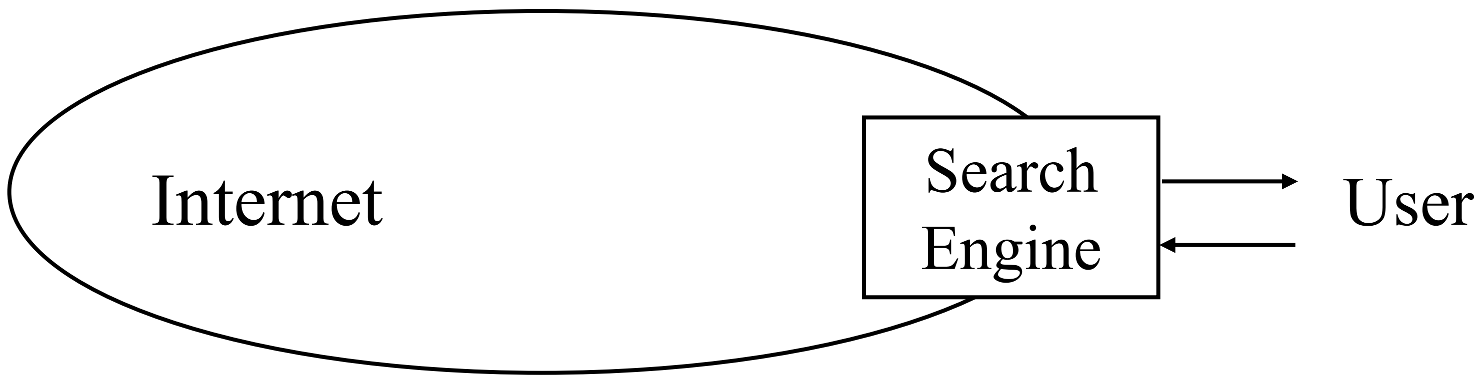
Typical Examples of Signals/Systems Concerned

- Networks



Typical Examples of Signals/Systems Concerned

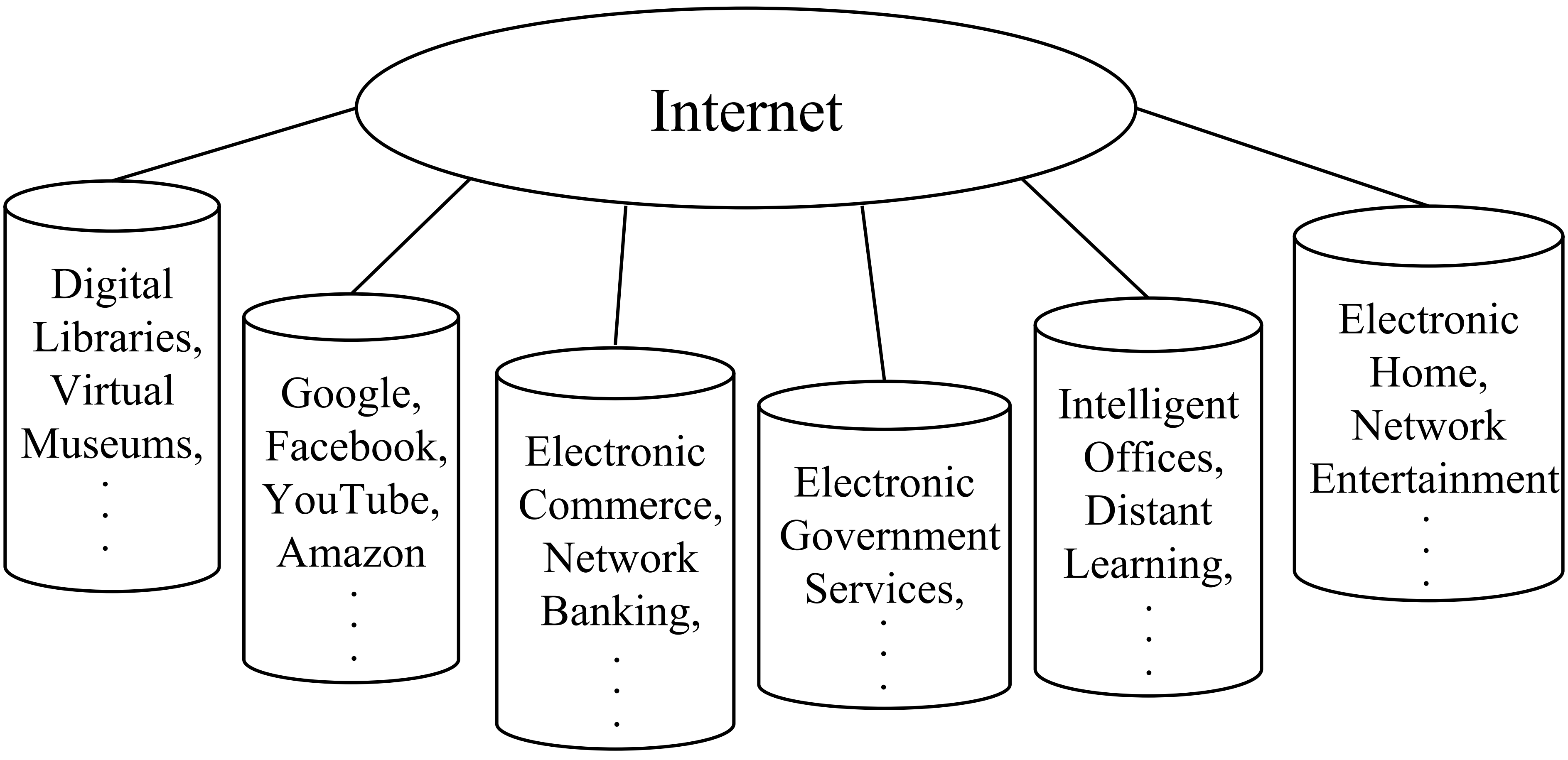
- Information Retrieval Systems



- Internet

- Other Information Systems
  - examples : remote sensing systems, biomedical signal processing systems, etc.

Internet



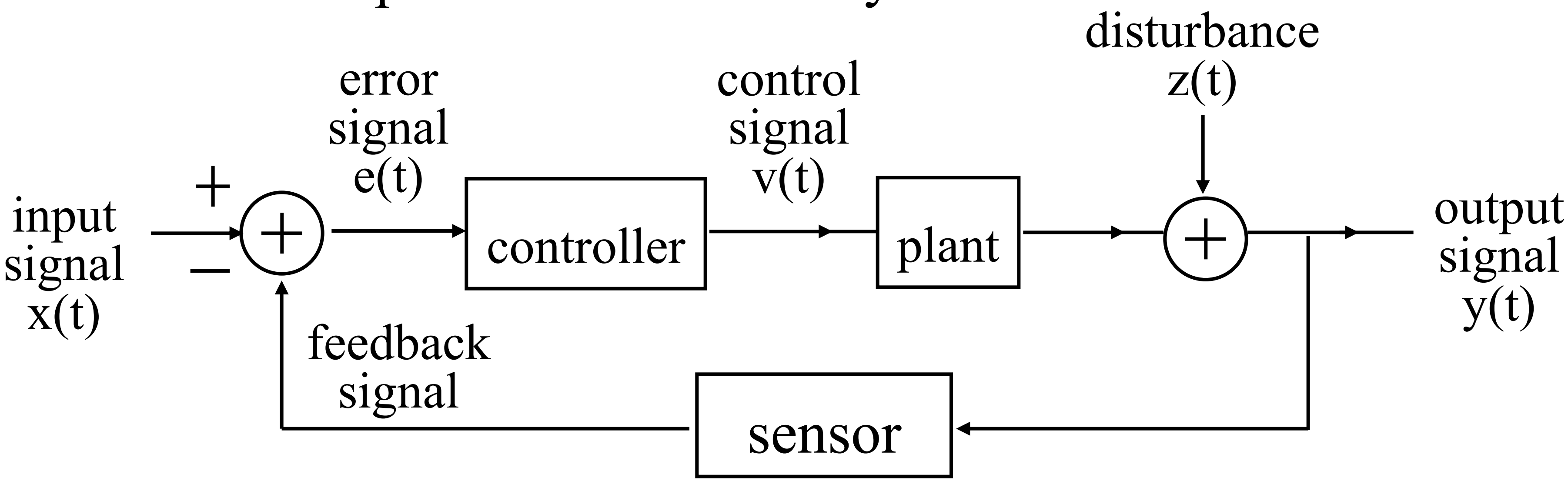
Internet
Digital Libraries, Virtual Museums,
. . .
Google, Facebook, YouTube, Amazon
. . .
Electronic Commerce, Network Banking,
. . .
Electronic Government Services,
. . .
Intelligent Offices, Distant Learning,
. . .
Electronic Home, Network Entertainment
. . .

Internet

- Network Technology Connects Everywhere Globally
- Huge Volume of Information Disseminated across the Globe in Microseconds
- Multi-media, Multi-lingual, Multi-functionality
- Cross-cultures, Cross-domains, Cross-regions
- Integrating All Knowledge Systems and Information related Activities Globally

Typical Examples of Signals/Systems Concerned

- Control Systems
  - close-loop/feedback control systems



  - example: aircraft landing systems, satellite stabilization systems, robot arm control systems, etc.

Typical Examples of Signals/Systems Concerned

- Other Systems
  - manufacturing systems, computer-aided-design systems, mechanical systems, chemical process systems, etc.

Scope of The Course

- Those Signals/Systems Operated by Electricity, in Particular by Software and Computers, with Extensive Computation and Memory, for Information and Control Primarily

- Analytical Framework to Handle Such Signals/Systems

- Mathematical Description/Representation of Such Signals/Systems

Scope of The Course

- Language and Tools to Solve Problems with Such Signals/Systems

- Closely Related to: Communications, Signal Processing, Computers, Networks, Control, Biomedical Engineering, Circuits, Chips, EM Waves, etc.

- A Fundamental Course for E.E.

Text/Reference Books and Lecture Notes

- Textbook:
  - Oppenheim & Willsky, “Signals & Systems”, 2nd Ed. 1997
  - Prentice-Hall, 新月
- Reference:
  - S. Haykin & B.Van Veen, “Signals & Systems”, 1999
  - John Willey & Sons, 歐亞
- Lecture Notes:
  - Available on web before the day of class

Course Outline

1. Fundamentals
2. Linear Time-invariant Systems
3. Fourier Series & Fourier Transform
4. Discrete Fourier Transform (DFT)
5. Time/Frequency Characterization of Signals/Systems
6. Sampling & Sampling Theorem
7. Communication Systems
8. Laplace Transform
9. Z-Transform
10. Linear Feedback Systems
11. Some Application Examples

History of the Area

- Independently Developed by People Working on Different Problems in Different Areas
- Fast Development after Computers Become Available and Powerful
- Re-organized into an Integrated Framework

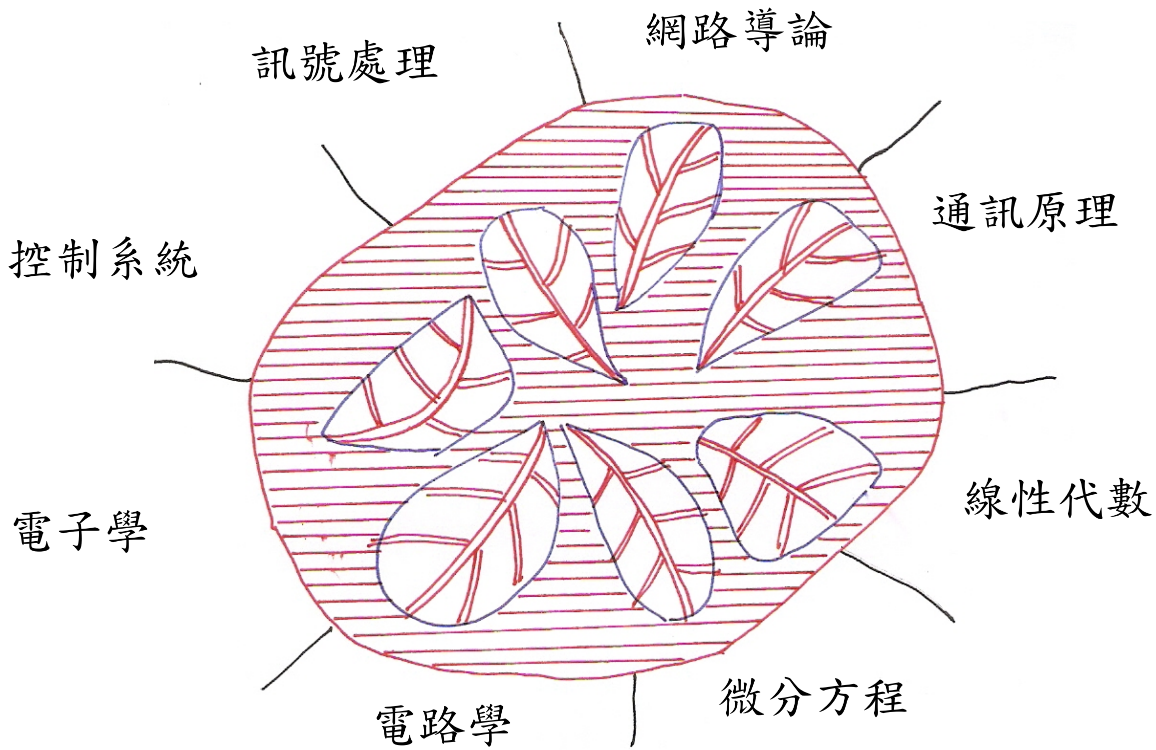
Background Required

- 2nd semester of 2nd year of EE
- Mathematics
- Pre-requisite : No

Grading

• Midterm	35%
• Final	35%
• MATLAB Problems	20%
• Homeworks	10%

學習要領



- 每週準時上課認真聽講，不遲到缺席
- 每週自行閱讀課本，跟上上課進度
- 課本中上課未能提到之處，自行仔細研讀(含例題、習題)

1.0 Fundamentals

1.1 Signals

Continuous/Discrete-time Signals

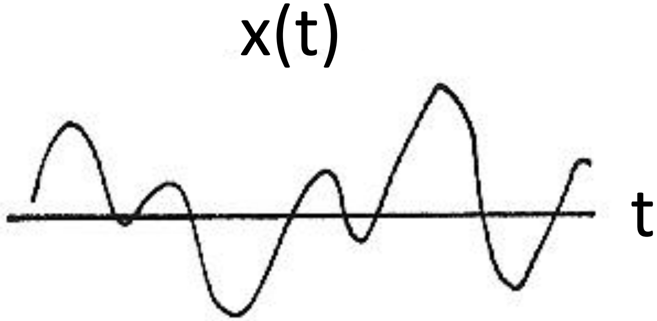
$x(t),\ x[n]$

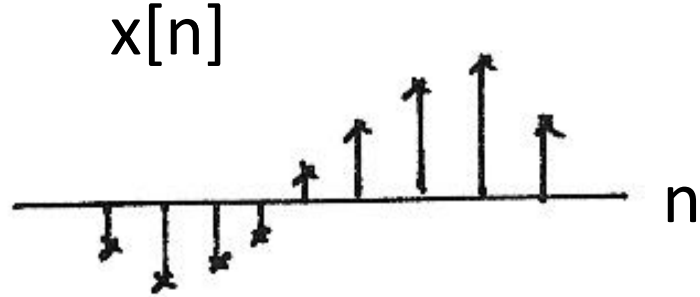
Signal Energy/Power

$$E = \int_{t_1}^{t_2} |\, x(t) \,|^2 \, dt \ , \quad E = \sum_{n=n_1}^{n_2} |\, x[n] \,|^2$$

$$P = E/(t_2 - t_1) \quad , \quad P = E/(n_2 - n_1 + 1)$$

Continuous/Discrete-time





Transformation of A Signal

- Time Shift

$$x(t) \to x(t - t_0) \quad , \quad x[n] \to x[n - n_0]$$

- Time Reversal

$$x(t) \to x(-t) \quad , \quad x[n] \to x[-n]$$

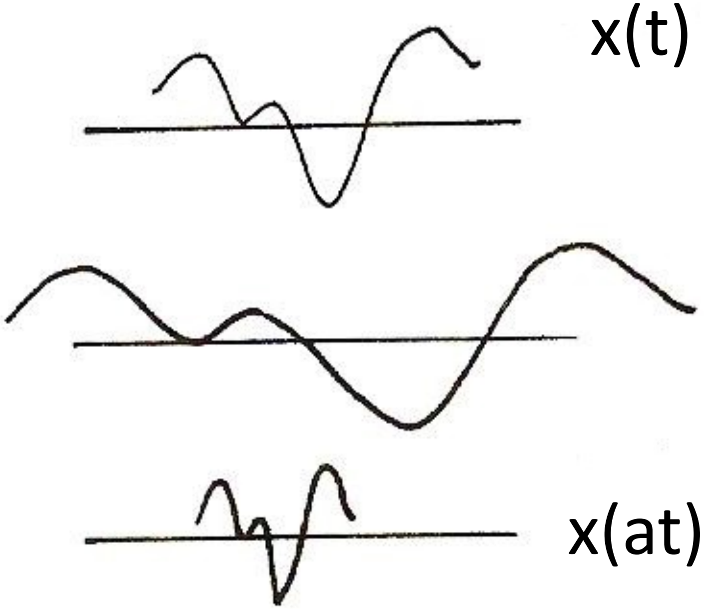
- Time Scaling

$$x(t) \to x(at) \quad , \quad x[n] \to ?$$

- Combination

$$x(t) \to x(at + b) \quad , \quad x[n] \to ?$$

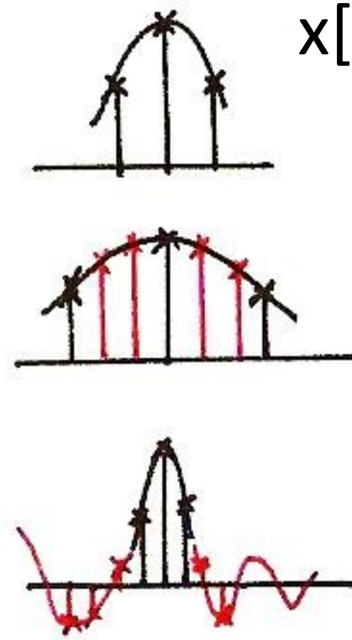
Time Scaling



x(t)

x(at), a<1

x(at), a>1



x[n]

?

?

Periodic Signal

$$x(t) = x(t+T) \quad , \quad T : period$$

$$x(t) = x(t+mT) \quad , \quad m : \text{integer}$$

T_0 : Fundamental period : the smallest positive value of T

aperiodic : NOT periodic

$$x[n] = x[n+N] = x[n+mN] \quad , \quad N_0$$

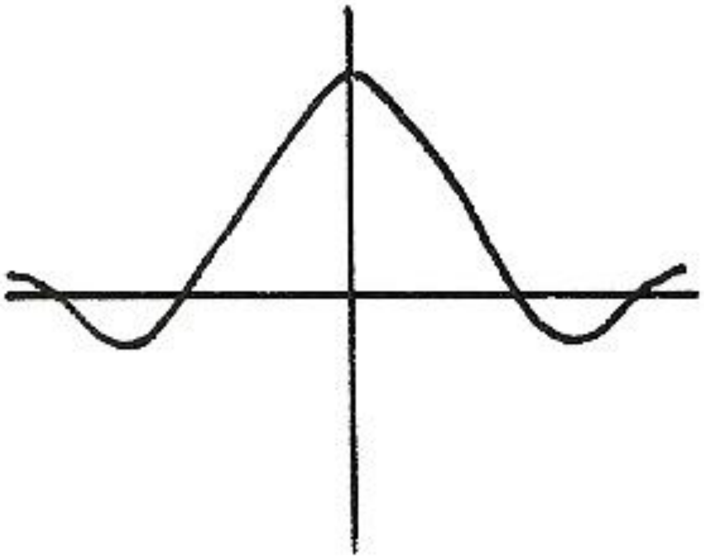
Even/Odd Signals

- Even $x(-t) = x(t)$, $x[-n] = x[n]$
- Odd $x(-t) = -x(t)$, $x[-n] = -x[n]$
- Any signal can be discomposed into a sum of an even and an odd

$$x_1(t) = \frac{1}{2}[x(t) + x(-t)], \quad x_2(t) = \frac{1}{2}[x(t) - x(-t)]$$

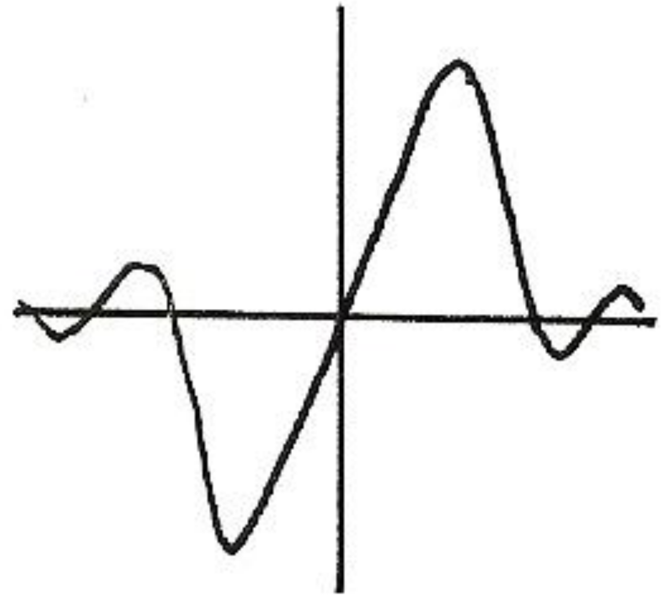
Even/Odd

Even



$x(-t)=x(t)$

Odd



$x(-t)=-x(t)$

Exponential/Sinusoidal Signals

- Basic Building Blocks from which one can construct many different signals and define frameworks for analyzing many different signals efficiently

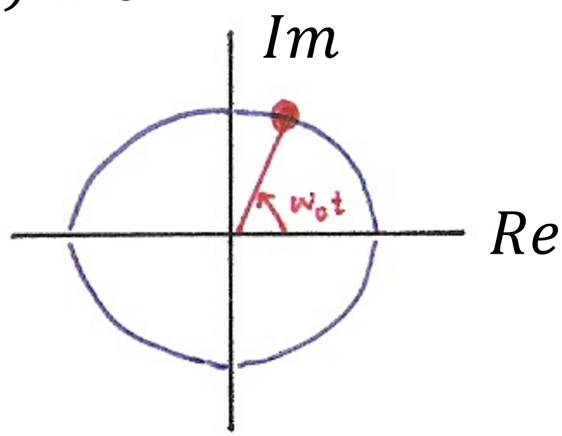
$$x(t) = e^{j\omega_0 t}, \quad \text{fundamental period} \quad T_0 = \frac{2\pi}{|\omega_0|}$$

$$\text{fundamental frequency} \quad \omega_0 = \frac{2\pi}{T_0}$$

$$\omega_0 : rad / \sec$$

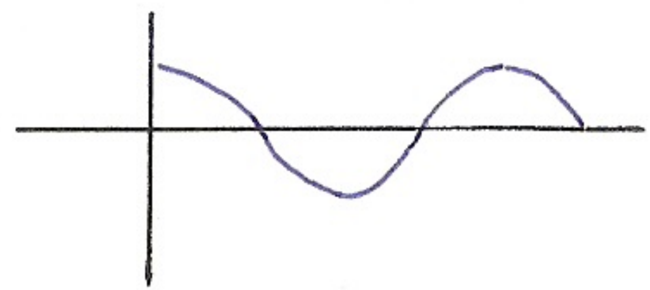
Exponential/Sinusoidal Signals

$x(t) = e^{j\omega_0 t}$

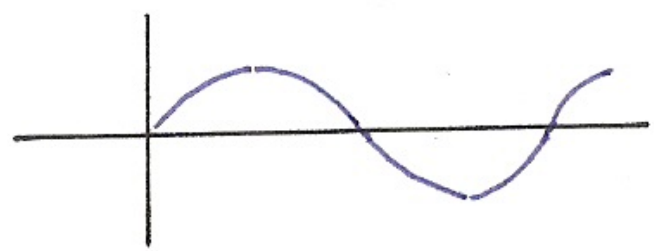


$e^{jx} = \cos x + j \sin x$

$Re\{e^{j\omega_0 t}\} = \cos \omega_0 t$



$Im\{e^{j\omega_0 t}\} = \sin \omega_0 t$

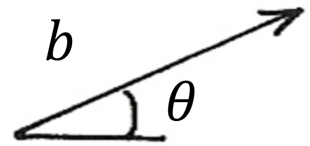


Vector Space

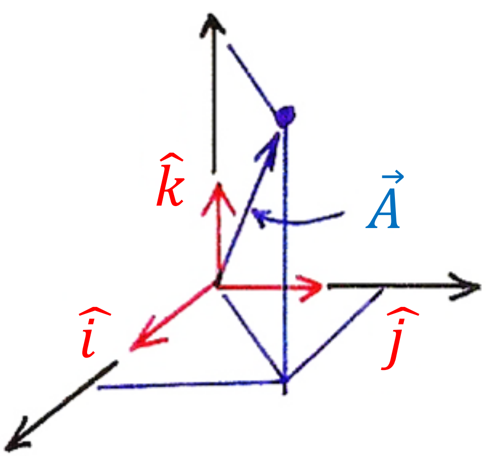
$$V = \{v \mid \cdots\}$$

$$av$$

$$v_1 + v_2$$



3-dim Vector Space



$$(\vec{A}) \cdot \hat{j} = (a\hat{i} + b\hat{j} + c\hat{k}) \cdot \hat{j}$$ (合成)

$$b = \vec{A} \cdot \hat{j}$$ (分析)

N-dim Vector Space

$$\vec{A} = \sum_{k=1}^{N} a_k \hat{v}_k$$ (合成)

$$a_j = \vec{A} \cdot \widehat{v}_j$$ (分析)

$$\widehat{v}_i \cdot \widehat{v}_j = \delta_{ij}$$

Signal Analysis

$$x(t) = \sum_k a_k \, x_k(t)$$

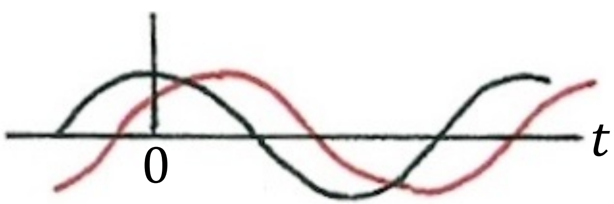
$t = n$ $\qquad$ $t = n$

$$x_k(t) = Re\{e^{j\omega_k t}\} = \cos \omega_k t$$

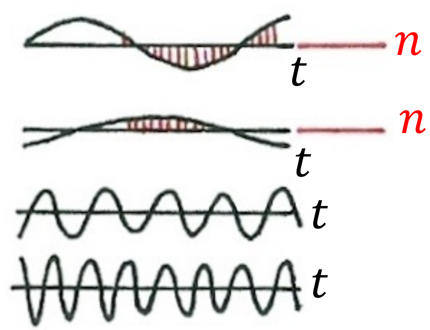
$$a_k = A_k e^{j\phi_k}$$

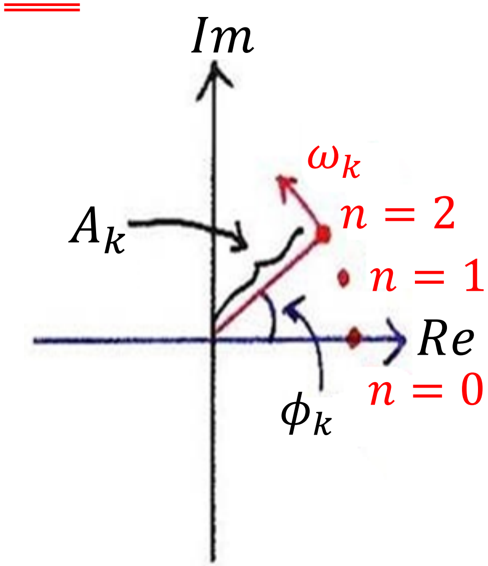
$t = n$

$$Re\{(A_k e^{j\phi_k})(e^{j\omega_k t})\} = A_k \cos(\omega_k t + \phi_k)$$



$$\omega_k = k\omega_0$$





Exponential/Sinusoidal Signals

- Harmonically related signal sets

$$\{\phi_k(t) = e^{jk\omega_0 t}, k = 0, \pm 1, \pm 2, \ldots.\}$$

fundamental period $T_k = \dfrac{2\pi}{|k\omega_0|}$

fundamental frequency $|k\omega_0|$

all with common period $T_1 = \dfrac{2\pi}{|\omega_0|}$

Exponential/Sinusoidal Signals

- Sinusoidal signal

$$x(t) = A\cos(\omega_0 t + \phi) = \mathrm{Re}\{Ae^{j(\omega_0 t + \phi)}\}$$

- General format

$$x(t) = Ce^{at} = |C|e^{j\theta} \cdot e^{(r + j\omega_0)t} = |C|e^{rt} \cdot e^{j(\omega_0 t + \theta)}$$

- Discrete-Time

$$x[n] = e^{j\omega_0 n}, \omega_0 : rad$$

$$x[n] = A\cos(\omega_0 n + \phi)$$

$$x[n] = Ce^{\beta n}$$

Exponential/Sinusoidal Signals

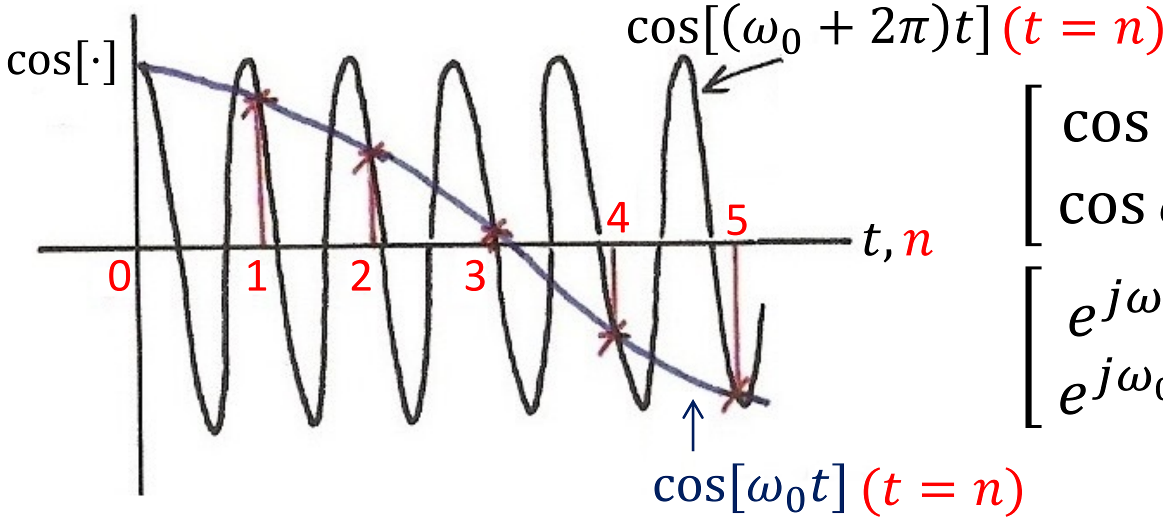
- Important Differences Between Continuous-time and Discrete-time Exponential/Sinusoidal Signals
  - For discrete-time, signals with frequencies ω_0 and $\omega_0 + m \cdot 2\pi$ are identical. This is Not true for continuous-time.

$$e^{j(\omega_0 + m\cdot 2\pi)n} = e^{j\omega_0 n}$$

$$e^{j(\omega_0 + \mathrm{X})t} = e^{j(\omega_0 + \mathrm{X})t}$$

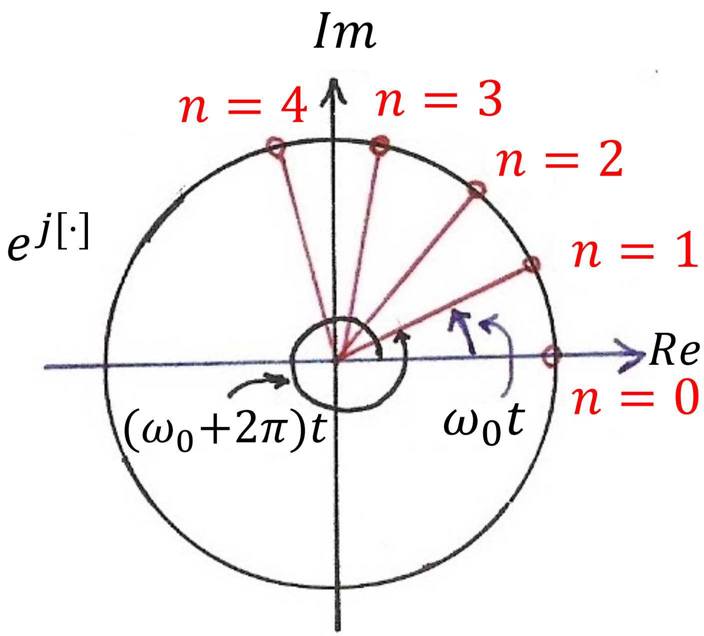
see : Fig.1.27, p.27 of text

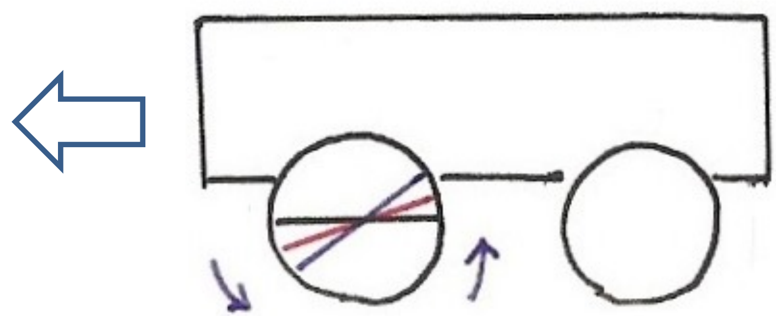
Continuous/Discrete Sinusoidals



$$\begin{cases} \cos \omega_0 t \neq \cos(\omega_0 + 2\pi)t \\ \cos \omega_0 n = \cos(\omega_0 + 2\pi)n \end{cases}$$

$$\begin{cases} e^{j\omega_0 t} \neq e^{j(\omega_0 + 2\pi)t} \\ e^{j\omega_0 n} = e^{j(\omega_0 + 2\pi)n} \end{cases}$$





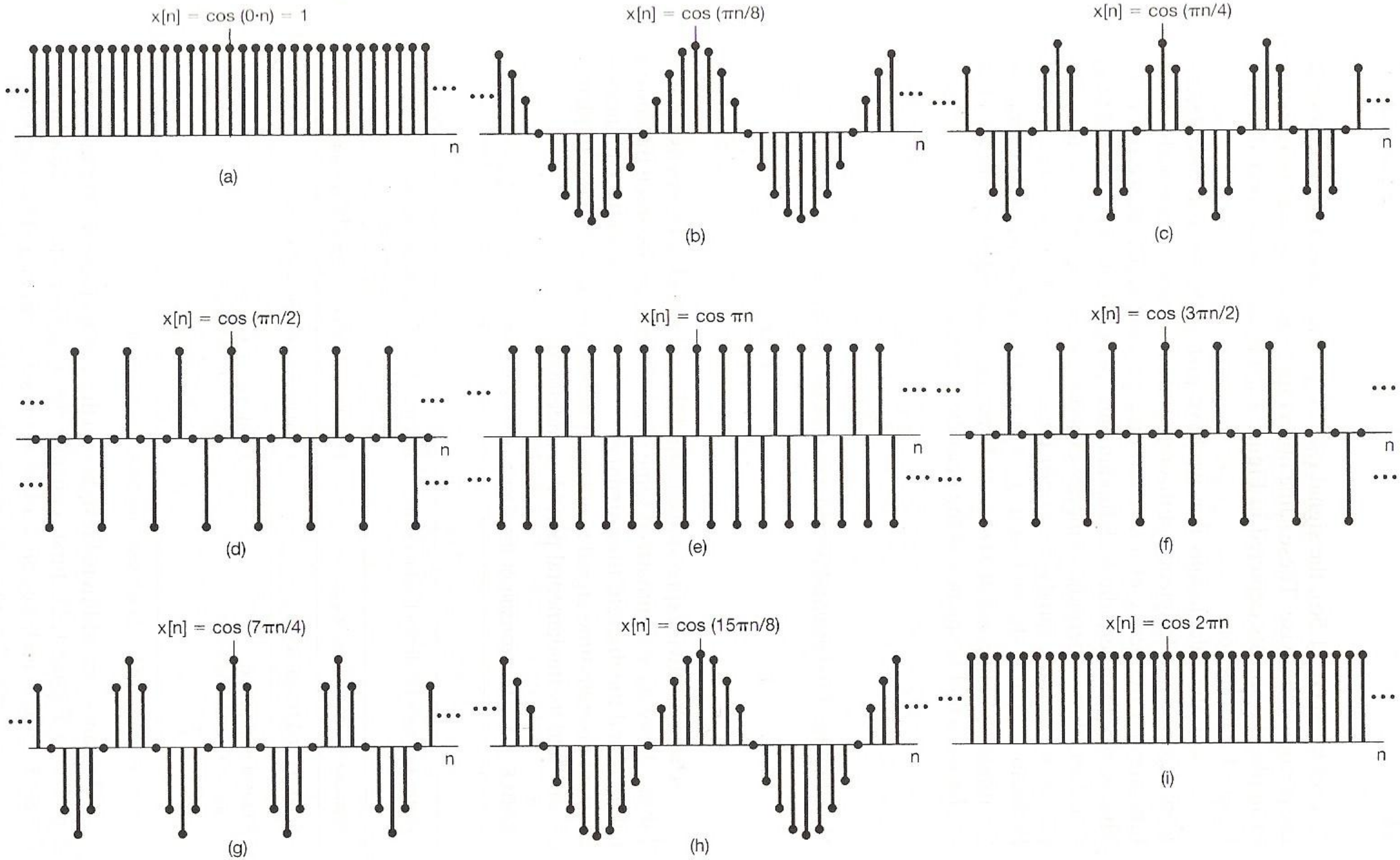


Figure 1.27 Discrete-time sinusoidal sequences for several different frequencies.

Fig. 1.27

$$e^{j\omega_0 n} \neq e^{j(2\pi-\omega_0)n}$$

but

$$Re\left\{e^{j\omega_0 n}\right\} = Re\left\{e^{-j\omega_0 n}\right\}$$

$$= Re\left\{e^{j(2\pi-\omega_0)n}\right\}$$

Note:

$$Im\left\{e^{j\omega_0 n}\right\} \neq Im\left\{e^{j(2\pi-\omega_0)n}\right\}$$

Exponential/Sinusoidal Signals

- Important Differences Between Continuous-time and Discrete-time Exponential/Sinusoidal Signals
  - For discrete-time, ω_0 is usually defined only for $[-\pi, \pi]$ or $[0, 2\pi]$. For continuous-time, ω_0 is defined for $(-\infty, \infty)$
  - For discrete-time, the signal is periodic only when

$$\omega_0 N = 2\pi m, \quad \omega_0 = \left(\frac{2\pi}{N}\right)m = 2\pi\left(\frac{m}{N}\right)$$

see : Fig.1.25, p.24 of text

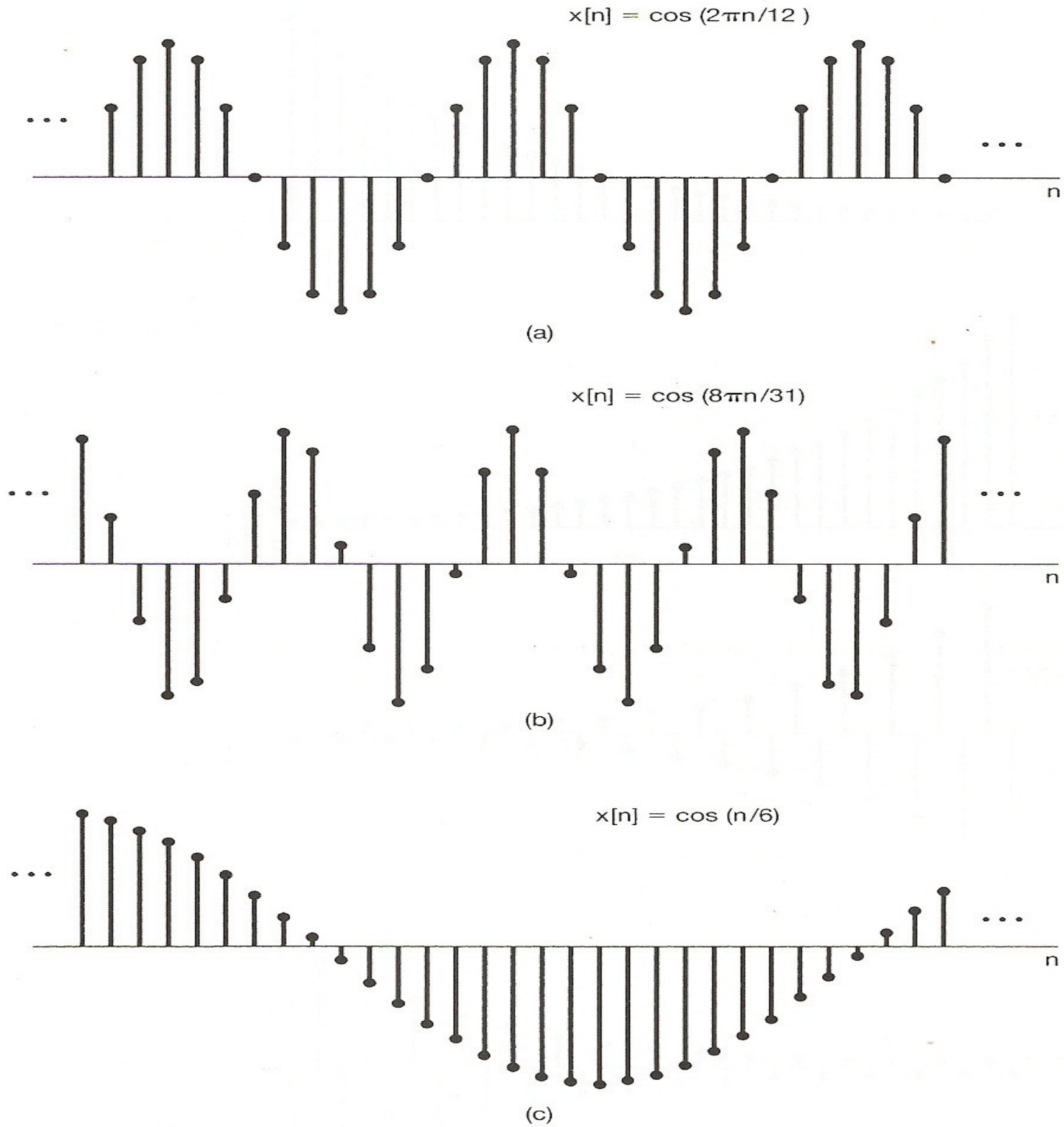


Figure 1.25 Discrete-time sinusoidal signals.

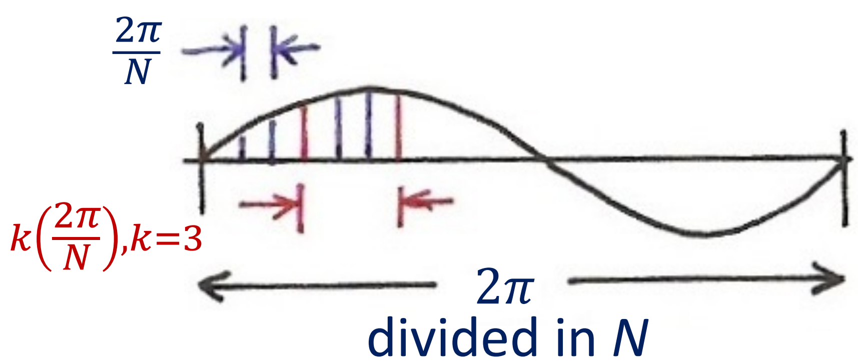
Harmonically Related Signal Sets

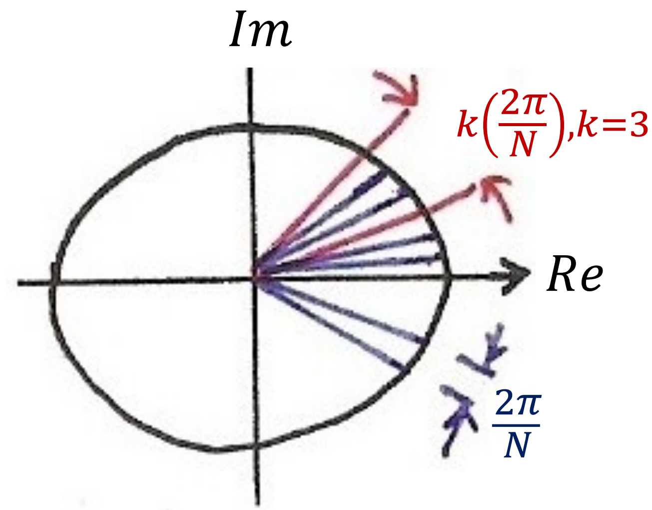
For being periodic

real period in cycles

$$\omega_k \cdot \underline{\underline{N}} = 2\pi \cdot \overline{\overline{k}} \quad \Rightarrow \quad \omega_k = k\left(\frac{2\pi}{N}\right)$$

real period in *n*





$$\phi_k[n] = e^{jk\left(\frac{2\pi}{N}\right)n} \quad \Rightarrow \quad \phi_{k+N}[n] = e^{j(k+N)\left(\frac{2\pi}{N}\right)n} = e^{jk\left(\frac{2\pi}{N}\right)n} = \phi_k[n]$$

Exponential/Sinusoidal Signals

- Harmonically related discrete-time signal sets

$$\{\phi_k[n] = e^{jk(\frac{2\pi}{N})n} \quad , \quad \text{k} = 0, \pm 1, \pm 2, \ldots\ldots\ldots\}$$

all with common period N

$$\phi_{k+N}[n] = \phi_k[n]$$

This is different from continuous case. Only *N* distinct signals in this set.

Unit Impulse and Unit Step Functions

- Continuous-time

  $\delta(t) \quad , \quad u(t)$

  – First Derivative

  $$\delta(t) = \frac{du(t)}{dt}$$

  see: Fig1.33, Fig1.34, P,33 of text

  – Running Integral

  $$u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$$

  – Sampling property

  $$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$

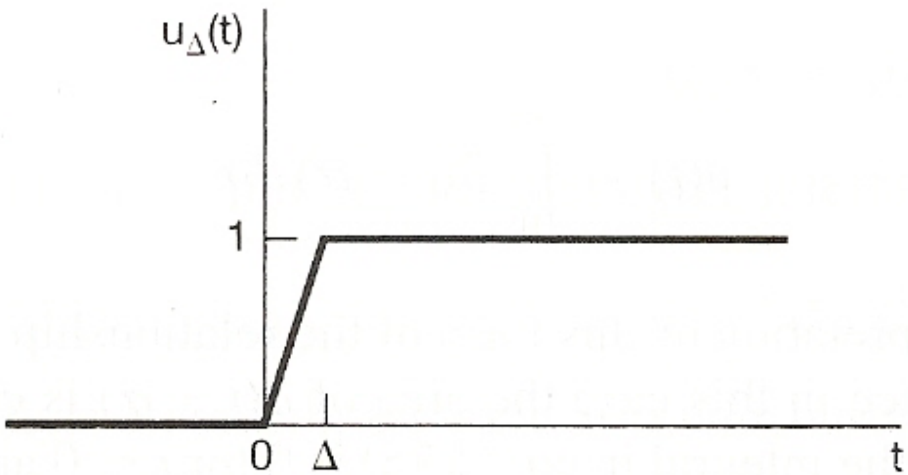


Figure 1.33 Continuous approximation to the unit step, $u_\Delta(t)$.

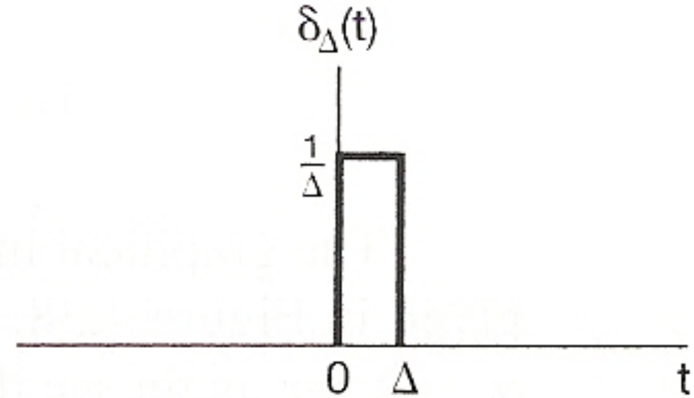


Figure 1.34 Derivative of $u_\Delta(t)$.

Unit Impulse and Unit Step Functions

- Discrete-time

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases} \qquad u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

  - First difference

$$\delta[n] = u[n] - u[n-1] \qquad (\lim_{\Delta \to 0}[\frac{x(t) - x(t-\Delta)}{\Delta}])$$

  - Running Sum

$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

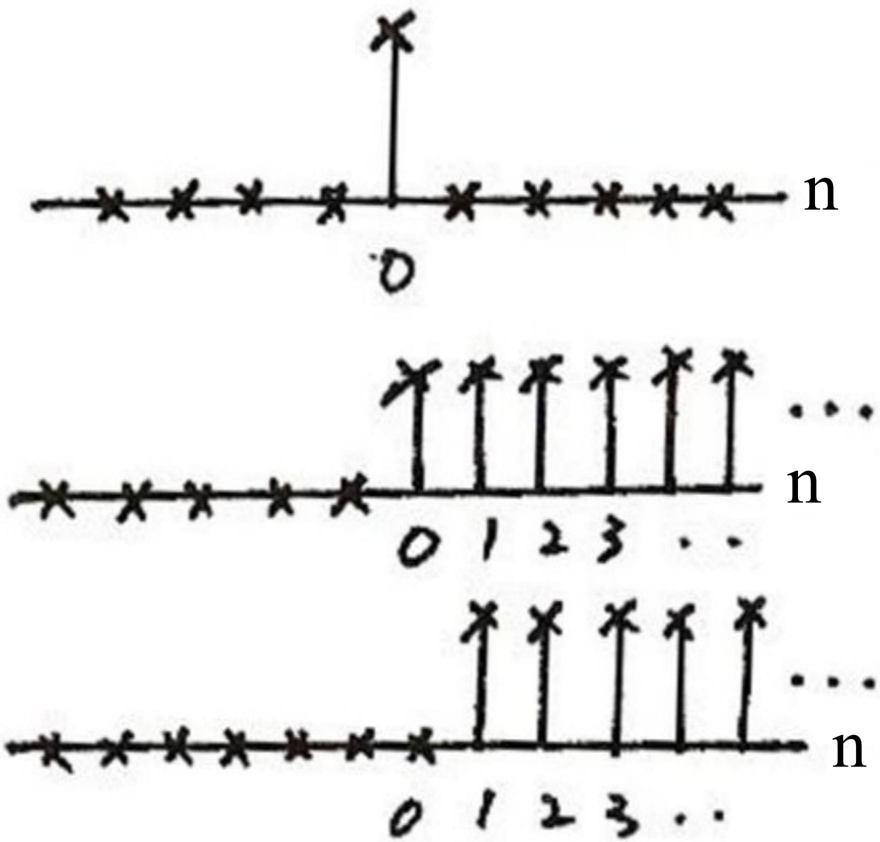
$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

  - Sampling property

$$x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]$$

Unit Impulse & Unit Step

- Discrete-time



$\delta\,[n]$

$u\,[n]$

$u\,[n-1]$

$$\delta\,[n] = u[n] - u[n-1]$$

Vector Space Representation of Discrete-time Signals

- n-dim

$$\vec{a} = (a_1, a_2, \cdots a_n)$$

$$\vec{b} = (b_1, b_2, \cdots b_n)$$

$$\widehat{v_1} = (1, 0, 0, \cdots 0)$$

$$\widehat{v_2} = (0,1, 0, \cdots\ 0)$$

$$\vdots$$

$$\vec{a} \cdot \vec{b} = \sum_i a_i\, b_i$$

$\Rightarrow$

$$\vec{x} = (x_1, x_2, \cdots x_n)$$

$$\vec{x} = \sum_i x_i\, \widehat{v_i}$$ 合成

$$x_j = \vec{x} \cdot \widehat{v_j}$$ 分析

Vector Space Representation of Discrete-time Signals

- n extended to $\pm\infty$

$$\vec{x} = (\cdots, x_{-2}, x_{-1}, x_0, x_1, x_2, x_3, \ \cdots)$$

$$\widehat{v_0} = (\cdots, \quad 0, \quad 0, \ 1, \ 0, \ 0, \ 0, \cdots) = \delta[\,n\,]$$

$$\widehat{v_1} = (\cdots, \quad 0, \quad 0, \ 0, \ 1, \ 0, \ 0, \cdots) = \delta[\,n - 1\,]$$

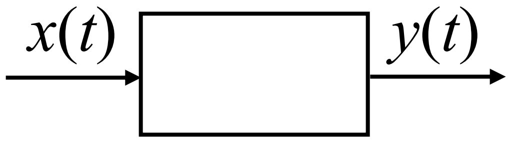
$$\widehat{v_k} = (\cdots, \quad 0, \quad 0, \ 0, \cdots, \ 0, \ 1, \cdots) = \delta[\,n - k\,]$$

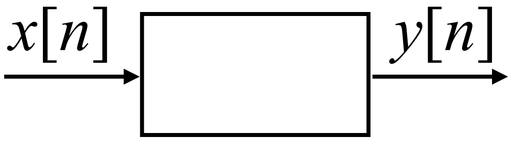
$$\{\delta[n-k], \ k\text{: inter}\} \Longrightarrow \vec{x} = \sum_i x_i \widehat{v}_i \quad \text{合成}$$

$$x_j = \vec{x} \cdot \widehat{v}_j \quad \text{分析}$$

1.2 Systems

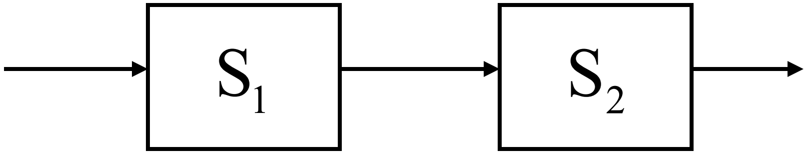
- Continuous/Discrete-time Systems



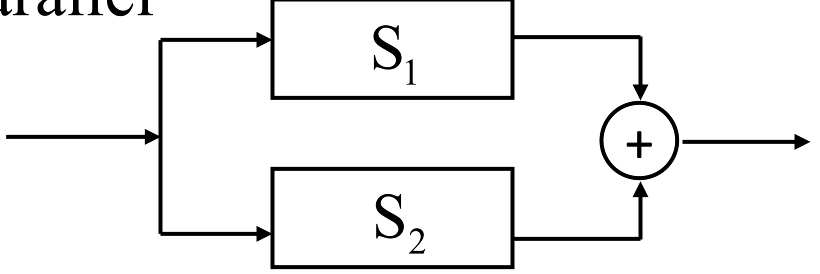


- Interconnections of Systems

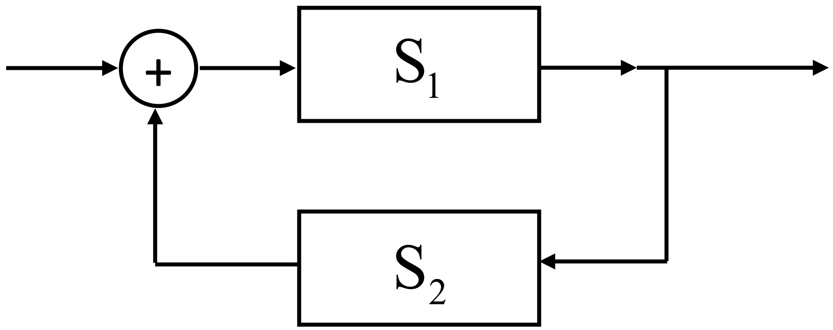
  – Series



  – Parallel



- Interconnections of Systems
  - Feedback

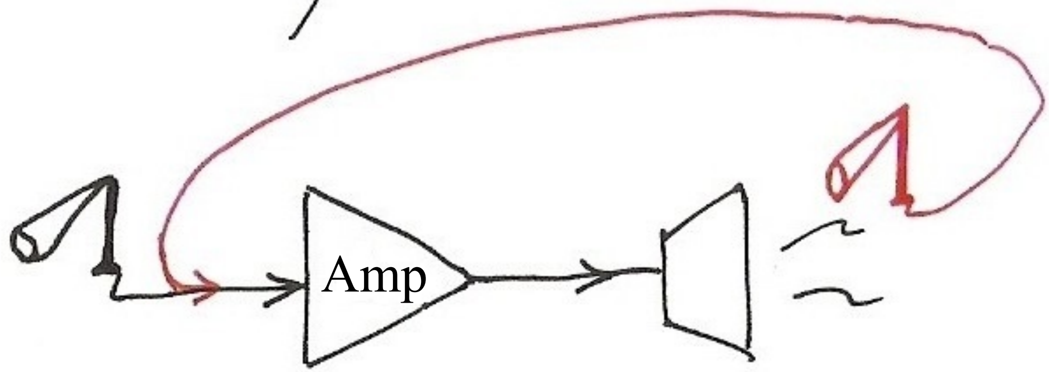


  - Combinations

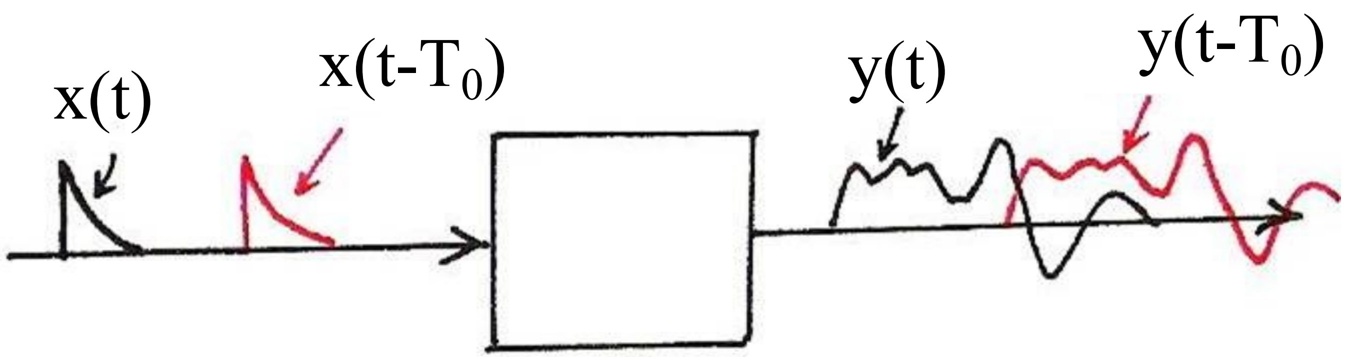
- Stability
  - stable : bounded inputs lead to bounded outputs
- Time Invariance
  - time invariant : behavior and characteristic of the system are fixed over time

Stability

Examples of unstable systems



Time Invariance



- Linearity
  - linear : superposition property

$$x_k[n] \rightarrow y_k[n]$$

$$\sum_k a_k x_k[n] \rightarrow \sum_k a_k y_k[n]$$

  - scaling or homogeneity property

$$x[n] \rightarrow y[n]$$

$$ax[n] \rightarrow ay[n]$$

  - additive property

$$x_i[n] \rightarrow y_i[n]$$

$$x_1[n] + x_2[n] \rightarrow y_1[n] + y_2[n]$$

- Memoryless/With Memory
  - Memoryless : output at a given time depends only on the input at the same time

    eg. $$y[n] = (ax[n] - x^2[n])^2$$

  - With Memory

    eg. $$y[n] = \sum_{k=-\infty}^{n} x[k]$$

- Invertibility

  – invertible : distinct inputs lead to distinct outputs, i.e. an inverse system exits

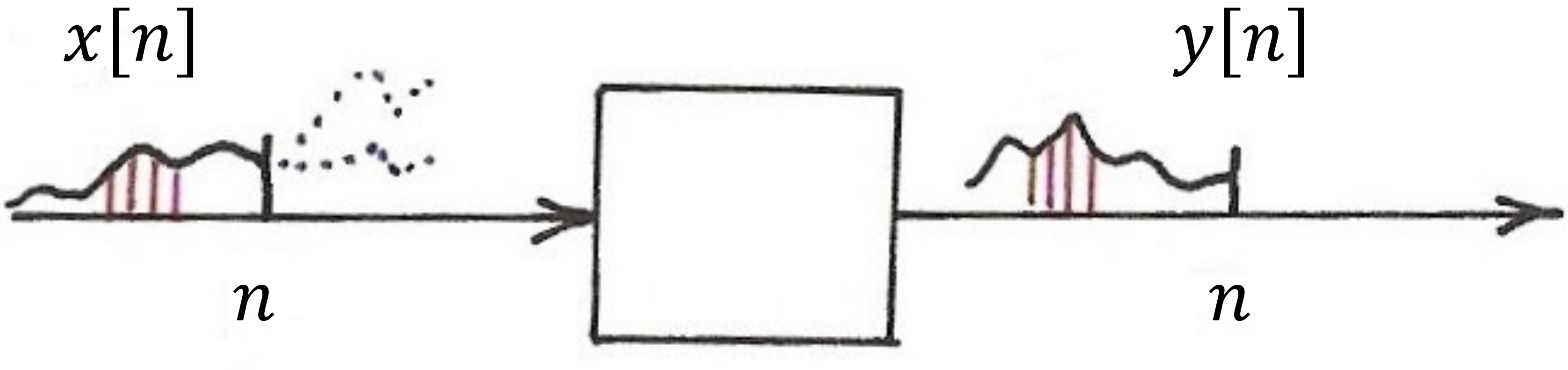
  eg.
  $$y[n] = \sum_{k=-\infty}^{n} x[k]$$
  $$z[n] = y[n] - y[n-1]$$

- Causality

  – causal : output at any time depends on input at the same time and in the past

  eg.
  $$y[n] = \sum_{k=-\infty}^{n} x[k]$$

Causality



$$y[n] = \sum_{k=-\infty}^{n+m} x[k]$$

Examples

- Example 1.12, p.47 of text

$$y[n] = x[-n]$$ “NOT” causal

$$y(t) = x(t)\cos(t+1)$$ causal

- Example 1.13, p.49 of text

$$y(t) = tx(t)$$ unstable

$$y(t) = e^{x(t)}$$ stable

Examples

- Example 1.20, p.55 of text

$$y[n] = 2x[n] + 3$$ "NOT" linear

  - zero input leads to zero output for linear systems
  - incrementally linear: difference between the responses to any two inputs is a linear function of the difference between the two inputs

Problem 1.35, p.64 of text

- $x[n] = e^{jk\left(\frac{2\pi}{N}\right)n}$ fundamental period=?

$$k\left(\frac{2\pi}{N}\right)N_0 = 2\pi m \quad , \quad N_0 = \frac{N}{k/m} = \frac{N}{a} \le N$$

  - a has to divide N for N_0 being an integer and $N_0 \le N$
  - a has to divide k for m being an integer

$$a = gcd(k, N), \quad N_0 = N/gcd(k, N)$$

example: N=12, k=3, N_0 =4, m=1

N=12, k=9, N_0 =4, m=3

- Selected problems for chap1: 4, 9, 14, 16, 18, 19, 27, 30, 35, 37, 47