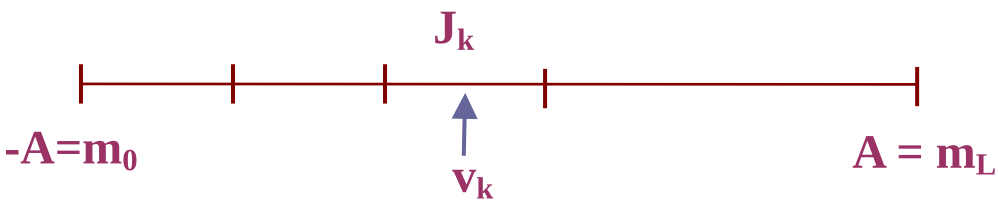
2.4 Vector Quantization (VQ)

Scalar Quantization



$$S = \bigcup_{k=1}^{L} J_k \,, \; V = \{ v_1 , v_2 , \ldots, v_L \}$$

$$Q : S \to V$$

$$Q(x[n]) = v_k \quad \text{if} \quad x[n] \in J_k$$

$$L = 2^R$$

Each v_k represented by a R-bit pattern

- Quantization characteristics (codebook)

$\{ J_1 , J_2 , \ldots, J_L \}$ and $\{ v_1 , v_2 , \ldots, v_L \}$

designed considering at least

1. error sensitivity
2. probability distribution of $x[n]$

2-dim Vector Quantization (VQ)

Example:

$\overline{x}_{\mathbf{n}} = (x[n] , x[n+1])$

$S = \{\overline{x}_{\mathbf{n}} = (x[n] , x[n+1]) ; |x[n]| < A , |x[n+1]|<A\}$

- **VQ**

- S divided into L 2-dim regions $J_{\mathbf{1}} , J_{\mathbf{2}} , \ldots, J_{\mathbf{k}} ,\ldots J_{\mathbf{L}}$

$$S = \bigcup_{k=1}^{L} J_{\mathbf{k}}$$

each with a representative vector $\overline{v}_{\mathbf{k}} \in J_{\mathbf{k}}$

$V= \{ \overline{v}_{\mathbf{1}} , \overline{v}_{\mathbf{2}} , \ldots, \overline{v}_{\mathbf{L}} \}$

- $Q : S \rightarrow V$

$Q(\overline{x}_{\mathbf{n}})= \overline{v}_{\mathbf{k}}$ if $\overline{x}_{\mathbf{n}} \in J_{\mathbf{k}}$

$L = 2^{\mathbf{R}}$

each $\overline{v}_{\mathbf{k}}$ represented by a R-bit pattern

- Considerations
  1. error sensitivity may depend on x[n], x[n+1] jointly
  2. distribution of x[n] , x[n+1] may be correlated statistically
  3. more flexible choice of $J_{\mathbf{k}}$

- Quantization Characteristics (codebook)

$\{ J_{\mathbf{1}} , J_{\mathbf{2}} , \ldots, J_{\mathbf{L}} \}$ and $\{ \overline{v}_{\mathbf{1}} , \overline{v}_{\mathbf{2}} , \ldots, \overline{v}_{\mathbf{L}} \}$

N-dim Vector Quantization

$$\bar{x} = (x_1, x_2, \ldots, x_N)$$

$$S = \{\bar{x} = (x_1, x_2, \ldots, x_N), \mid x_k \mid < A, k = 1,2,\ldots N\}$$

$$S = \bigcup_{k=1}^{L} J_k, \; V = \{\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_L\}$$

$$Q : S \rightarrow V$$

$$Q(\bar{x}) = \bar{v}_k \quad \text{if} \quad \bar{x} \in J_k$$

$L = 2^R$, each $\bar{v}_k$ represented by an R-bit pattern

Codebook Trained by a Large Training Set

- **Define distance measure between two vectors $\bar{x}$, $\bar{y}$**

$d(\bar{x}, \bar{y}) : S \times S \rightarrow R^+$ (non-negative real numbers)

- desired properties

$$d(\bar{x}, \bar{y}) \geq 0$$

$$d(\bar{x}, \bar{x}) = 0$$

$$d(\bar{x}, \bar{y}) = d(\bar{y}, \bar{x})$$

$$d(\bar{x}, \bar{y}) + d(\bar{y}, \bar{z}) \geq d(\bar{x}, \bar{z})$$

examples :

$$d(\bar{x}, \bar{y}) = \sum_k (x_i - y_i)^2$$

$$d(\bar{x}, \bar{y}) = \sum_k \mid x_i - y_i \mid$$

Codebook Trained by a Large Training Set

- **Iterative training**

(1)
Fixed $\{ \bar{v}_1, \bar{v}_2, \ldots, \bar{v}_L \}$
find best set of
$\{ J_1, J_2, \ldots, J_L \}$

(2)
Fixed $\{ J_1, J_2, \ldots, J_L \}$
find best set of
$\{ \bar{v}_1, \bar{v}_2, \ldots, \bar{v}_L \}$

(1) $J_k = \{ \bar{x} \mid d(\bar{x}, \bar{v}_k) < d(\bar{x}, \bar{v}_j), \ j \neq k \}$

$$\rightarrow D = \sum_{\text{all } \bar{x}} d(\bar{x}, Q(\bar{x})) = \min$$

nearest neighbor condition

(2) For each k

$$\bar{v}_k = \frac{1}{M} \sum_{\bar{x} \in J_k} \bar{x}$$

$$\rightarrow D_k = \sum_{\bar{x} \in J_k} d(\bar{x}, \bar{v}_k) = \min$$

centroid condition

(3) Convergence condition

$$D = \sum_{k=1}^{L} D_k$$

after each iteration D is reduced, but $D \geq 0$

$$| D^{(m+1)} - D^{(m)} | < \epsilon, \quad m : \text{iteration}$$

Lloyd-Max Algorithm

Applications of VQ

- Any set of parameters with somehow related properties can be grouped into a vector for VQ
- Number of bits to be transmitted can be reduced

Ref: 10.1, 10.2, Gersho and Gray, "Vector Quantization and Signal Compression", Kluwer, 1992