4.0 Digital Transmission (II) — Modulation and Systems

4.1 Basic Digital Modulation Schemes

- **Modulation :**
  modifying some parameters of a carrier signal (at carrier frequency f_c) according to the data message, while the carrier signal can be transmitted in some desired frequency band

- **Basic Forms**

$$p(t) = \begin{cases} s_1(t), & [\,0, T\,] \;, \quad \text{for “1”} \\ s_0(t), & [\,0, T\,] \;, \quad \text{for “0”} \end{cases}$$

T : bit duration

$$s(t) = \sum_k p(t-kT)$$

- **3 Basic Schemes**
  - Amplitude Shift Keying (ASK)

$$s_1(t) = A\,cos(2\pi f_c t) \;, \quad [\,0, T\,]$$
$$s_0(t) = \quad 0 \qquad\qquad , \; [\,0, T\,]$$

  - Frequency Shift Keying (FSK)

$$s_1(t) = A\,cos(2\pi f_{c1} t) \;, \quad [\,0, T\,]$$
$$s_0(t) = A\,cos(2\pi f_{c2} t) \;, \quad [\,0, T\,]$$

See Fig. 6.25 , p. 382 of Haykin

  - Phase Shift Keying (PSK)

$$s_1(t) = A\,cos(2\pi f_c t) \;, \quad [\,0, T\,]$$
$$s_0(t) = A\,cos(2\pi f_c t+\pi) = -A\,cos(2\pi f_c t) \;, \; [\,0, T\,]$$

For each bit pulse having integer number of cycles

$$f_c = n \bullet r = n\,/\,T$$

n = interger

$$r = \frac{1}{T} = \text{bit rate}$$

Binary PSK (BPSK)

- **Vector Space Analysis**

  - 1- dim space is adequate

    $\phi(t) = (\frac{2}{T})^{1/2} \cos(2\pi f_c t)$, [0, T]

    $\overline{s_1} = (E_b)^{1/2}$

    $\overline{s_0} = -(E_b)^{1/2}$

    See Fig. 6.3 , p. 350 of Haykin

    $Pe = Q([2(\frac{E_b}{N_0})]^{1/2})$

  - A minimum energy signal set, efficient in using E_b / N_0

  - Antipodal signal set

    $\overline{s_0} = -\overline{s_1}$

- **Spectral Behavior**

$R_s(\tau) = E[s(t)s(t+\tau)]$

$S_s(f) = F\{R_s(\tau)\}$

$= \frac{1}{4} A^2 T[\,sinc^2(\frac{f+f_c}{r}) + sinc^2(\frac{f-f_c}{r})]$

$sinc(x) = \frac{\sin(\pi x)}{\pi x}$

$B \propto r$, $\eta = \frac{r}{B}$ bandwidth efficiency

Differential PSK (DPSK)

- difficult to distinguish at the receiver which phase represents 1 or 0
- data represented by "relative phase" instead of "absolute phase"

- **BPSK**

$$s_1(t) = A\,cos(2\pi f_c t)\ , \quad [\,0, T\,]$$

$$s_0(t) = A\,cos(2\pi f_c t+\pi) = -\,s_0(t)\ , \quad [\,0, T\,]$$

data represented by "absolute phase"

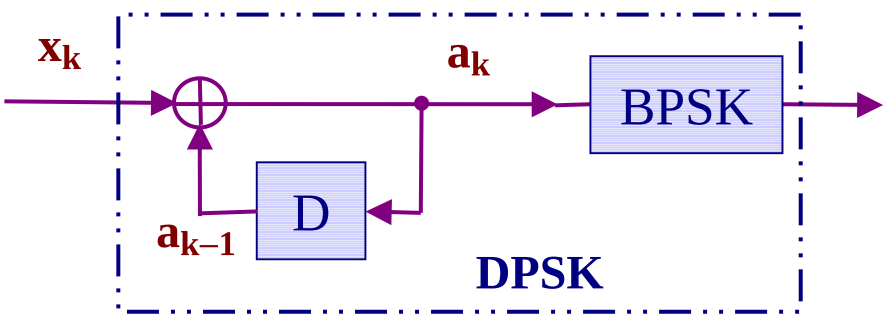
- **DPSK**

$$s(t) = A\,cos(2\pi f_c t+\phi_k)\ , \quad [\,kT\,, (k+1)T\,]$$

$$\phi_k = \phi_{k-1} + \pi \quad , \quad x_k = 1$$

$$\phi_k = \phi_{k-1} \quad , \quad x_k = 0$$

data represented by "relative phase"



$$a_k = a_{k-1} \oplus x_k$$

Quadriphase-shift Keying (QPSK)

- **Signal Sets**

$$\bar{s}_i(t) = A\,cos[2\pi f_c t + (2i-1)\frac{\pi}{4}]\ , \quad [\,0,\ T_s = 2T\,]$$

$$i = 1, 2, 3, 4 \quad \text{for} \quad 00, 01, 11, 10$$

2 bits/symbol

$$s_i(t) = A\,cos[(2i-1)\frac{\pi}{4}]cos(2\pi f_c t)$$
$$-A sin[(2i-1)\frac{\pi}{4}]sin(2\pi f_c t)$$
$$= A/(2^{1/2})[\pm cos(2\pi f_c t) \pm sin(2\pi f_c t)], [0, T_s=2T]$$

- It can be shown

$$\phi_1(t) = (\frac{2}{2T})^{1/2}\,cos(2\pi f_c t)\ , \quad [\,0,\ 2T\,]$$

$$\phi_2(t) = (\frac{2}{2T})^{1/2}\,sin(2\pi f_c t)\ , \quad [\,0,\ 2T\,]$$

are orthonormal basis

$$\bar{s}_i = (\pm\frac{A}{2}(2T)^{1/2}, \pm\frac{A}{2}(2T)^{1/2})\ , \quad i = 1, 2, 3, 4$$

4 signal vectors on $(\phi_1(t), \phi_2(t))$ plane, N=2, M=4

See Fig. 6.6 , p. 355 of Haykin

Quadriphase-shift Keying (QPSK)

- **Receiver Structure 2 (N = 2 < 4 = M)**

  $\overline{x} = (x_1 , x_2)$

  $x_j = \int_0^T x(t)\phi_j(t)dt$

  $= [x(t)] \cdot [\phi_j(t)]$

  - $[\pm a_1\phi_1(t) \pm a_2\phi_2(t) + n(t)] \cdot [\phi_1(t)] = \pm a_1 + n_1$
  - orthogonal signal component deleted
  - just like a BPSK in each dimension

- **Exactly like 2 BPSK signals on orthogonal dimensions**
  - same P_e
  - same $S_s(f)$ but with half bandwidth or doubled bandwidth efficiency

M-ary PSK

$M = 2^n$ symbols, n bits/symbol

$s_i(t) = A\cos[2\pi f_c t + \frac{2\pi}{M}(i-1)]$, $i=1,2,\ldots M$, $[0,T_s=nT]$

$s_i(t)$ can be similarly expanded in 2-dim plane of $(\phi_1(t), \phi_2(t))$

n = 3	8 PSK
n = 4	16PSK
n = 5	32PSK

See Fig. 6.15 , p. 366 of Haykin

- degraded error rate performance but improved bandwidth efficiency

Ref : 6.1, 6.2, 6.3 of Haykin