# The Most Valuable Eigenvector

Hung-yi Lee

# Search


http://incomebully.com/does-pr-pagerank-still-matter/

Google 台灣大學

全部 地圖 圖片 新聞 影片 更多 搜尋工具

約有 5,160,000 項結果 (搜尋時間：1.01 秒)

國立臺灣大學
www.ntu.edu.tw/
包含學校簡介、系所介紹、校園資訊。成立於1928年，前身為臺北帝國大學。1945年更名為臺灣大學。
您已造訪這個網頁 3 次。上次造訪日期：2016/5/1

ntu.edu.tw 內容的搜尋結果

招生資訊
碩士班招生 - 學士班轉學考 - 碩士班甄試入學 - ...

學術單位
文學院 - 工學院 - 理學院 - 生物資源暨農學院 - ...

資訊網路與多媒體研究所
碩士班 - 本所成員 - 碩士班修業規定 - 課程介紹 - ...

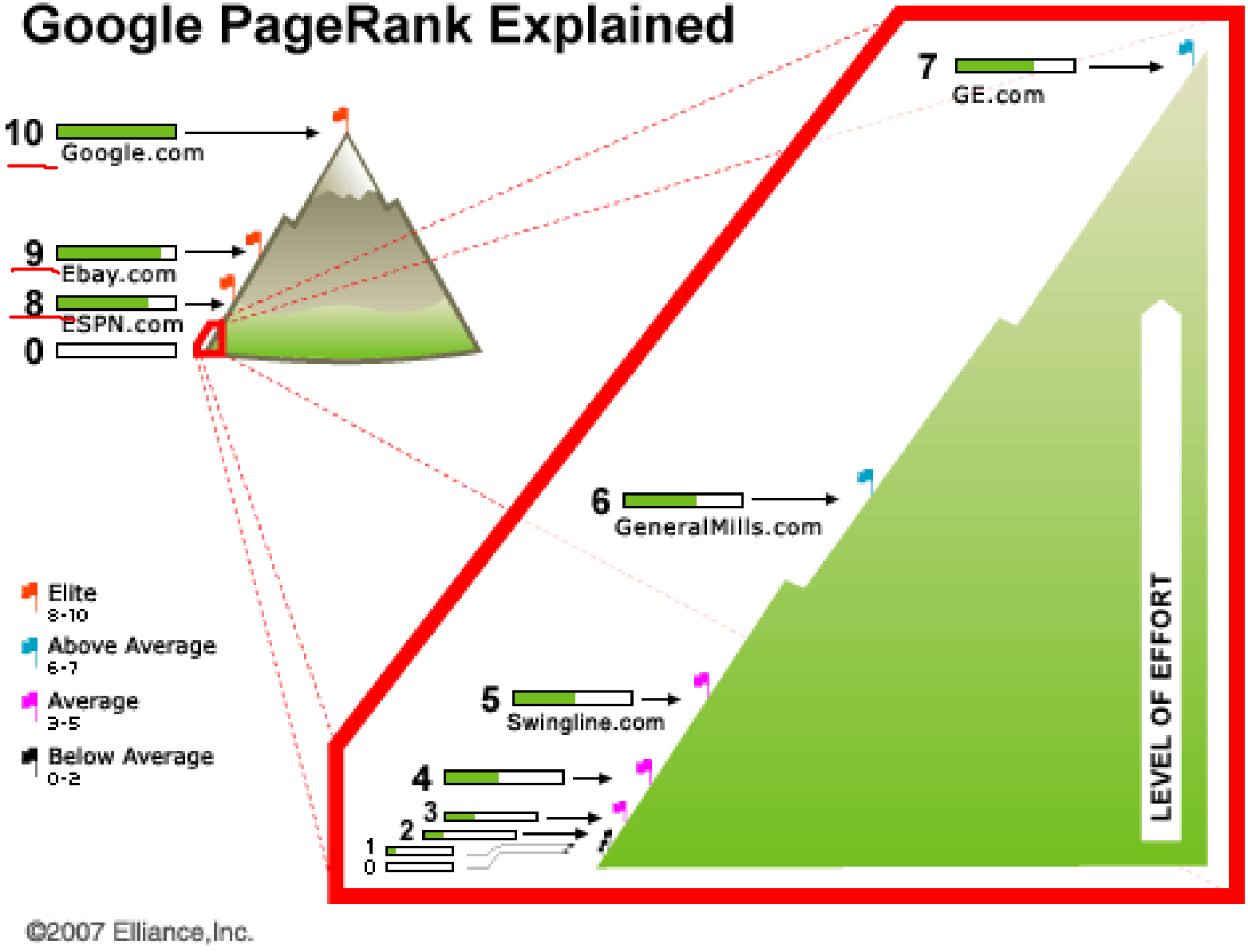
圖書館
館藏資源 - 電子資源 - 資料庫 - 開放時間 - 學生 - ...

myNTU臺大人入口網
... 計中帳號登入！ SSO1.3. 登入 · ※ 預防帳號遭盜用，請定期修改 ...

國立臺灣大學學士班轉學考試
招生名額及科目 一般生(不招收陸生)陸生(限在臺就讀). 預定日程 一 ...

國立臺灣大學- 維基百科，自由的百科全书
https://zh.wikipedia.org/zh-tw/國立臺灣大學
國立臺灣大學，簡稱臺灣大學、臺大，乃臺灣最早的現代綜合大學，前身是於1928年創立的臺北帝國大學，籌設之初定位為只辦醫學和農學的實業大學，伊澤多喜男力排 ...

國立臺灣大學National Taiwan University - Facebook
www.facebook.com › Places › Taipei, Taiwan › Landmark
★★☆☆☆ 評分：1.8 - 11,822 票
國立臺灣大學National Taiwan University, Taipei, Taiwan. 37715 likes · 1580 talking about this · 175196 were here. 國立臺灣大學粉絲專頁.




http://www.hobo-web.co.uk/google-pr-update/

# PageRank

- Information of 2008

**Rank：8**
Google台灣首頁：www.google.com.tw
Youtube台灣首頁：http://tw.youtube.com
台灣大學網站首頁：www.ntu.edu.tw

**Rank：7**
痞客邦首頁：www.pixnet.net
104人力銀行：www.104.com.tw
無名小站首頁：www.wretch.cc

**Rank：6**
博客來網站：www.books.com.tw
聯合新聞網首頁：http://udn.com
天下雜誌網站首頁：www.cw.com.tw

**Rank：5**
推推王網站：http://funp.com
愛情公寓網站：www.i-part.com.tw
KKman網站首頁：
www.kkman.com.tw

**Rank：4**
工頭堅部落格：http://worker.bluecircus.net
白木怡言部落格：www.yubou.tw
RO仙境傳說網站：http://ro.gameflier.com

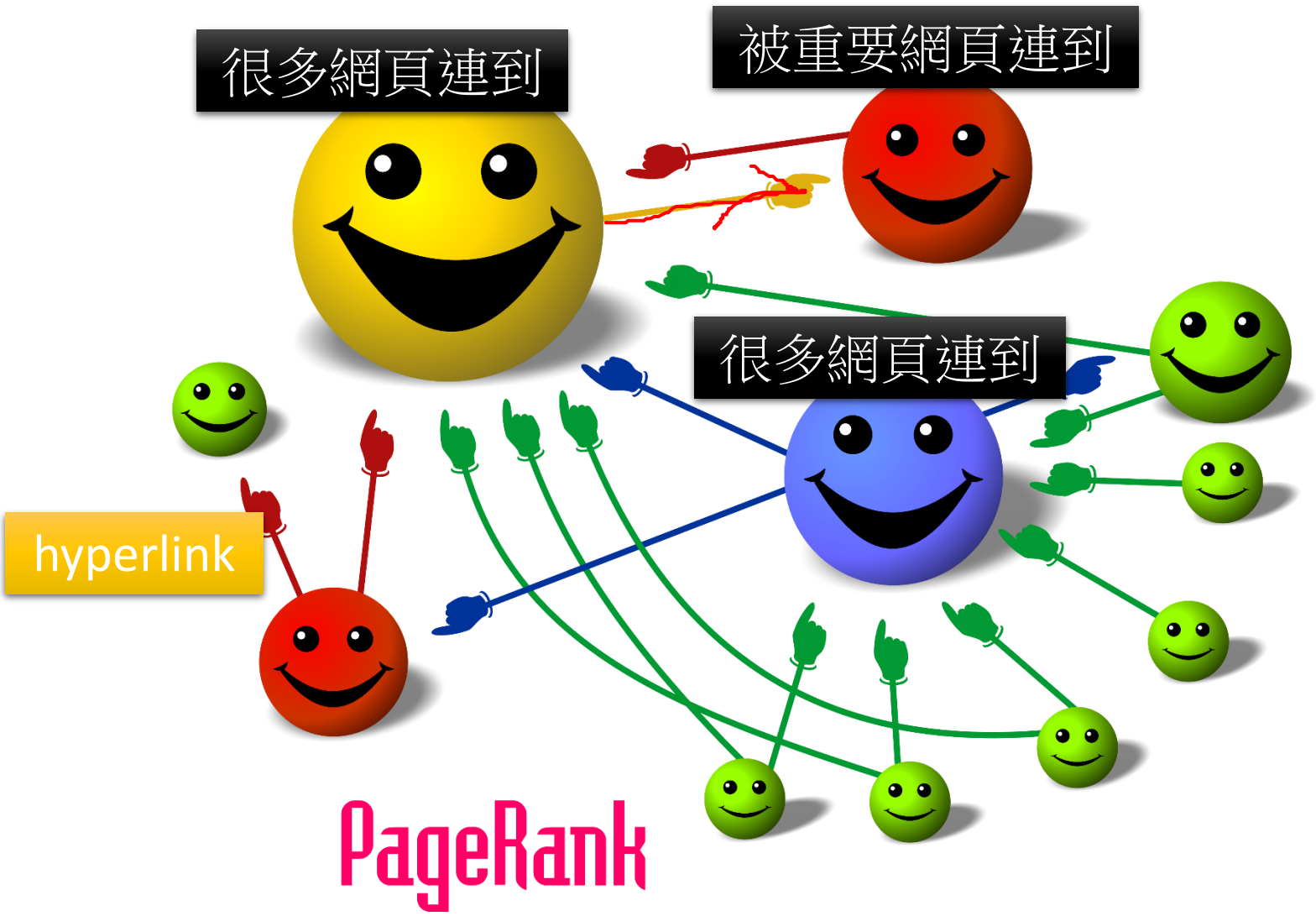
Source: http://terrylogin.blogspot.com/2008/09/pagerank.html

# PageRank

# PageRank

- *Webpages with a higher PageRank are more likely to appear at the top of Google search results.*
- *PageRank relies on the uniquely democratic nature of the web by using its vast link structure as an indicator of an individual page's value.*
- ***Google interprets a link from page A to page B as a vote, by page A, for page B.***

# Importance



# PageRank

## The Anatomy of a Large-Scale Hypertextual Web Search Engine

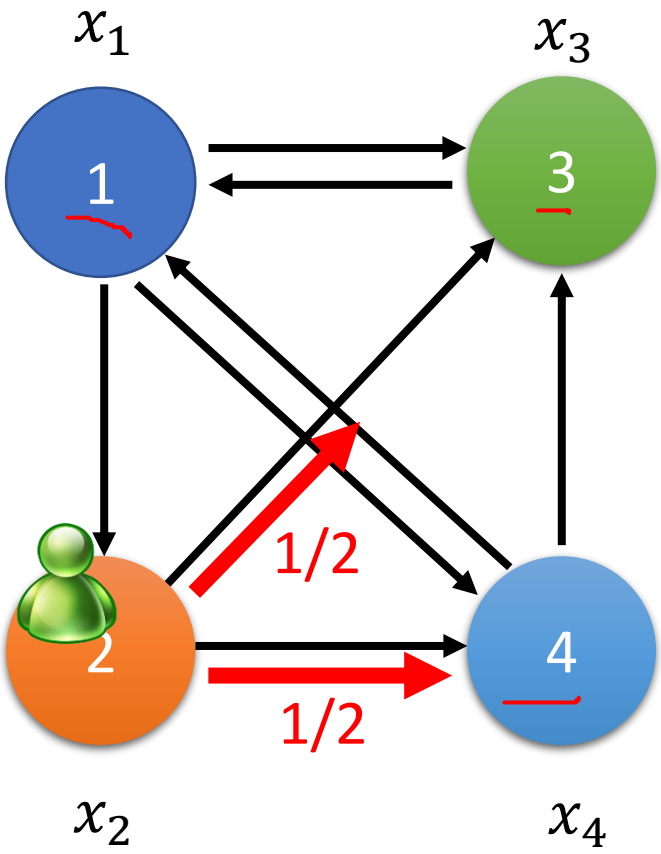
Sergey Brin and Lawrence Page

*Computer Science Department,*
*Stanford University, Stanford, CA 94305, USA*
sergey@cs.stanford.edu and page@cs.stanford.edu

**Abstract**

In this paper, we present Google, a prototype of a large-scale search engine which makes heavy use of the structure present in hypertext. Google is designed to crawl and index the Web efficiently and produce much more satisfying search results than existing systems. The prototype with a full text and hyperlink database of at least 24 million pages is available at http://google.stanford.edu/
To engineer a search engine is a challenging task. Search engines index tens to hundreds of millions of web pages involving a comparable number of distinct terms. They answer tens of

# Importance - Formulas




$$x_1 = x_3 + \frac{1}{2}x_4$$

$$x_2 = \frac{1}{3}x_1$$

$$x_3 = \frac{1}{3}x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_4$$

$$x_4 = \frac{1}{3}x_1 + \frac{1}{2}x_2$$

Consider a random surfer

# Importance - Formulas

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$Ax = x$$

The solution x is in the eigenspace of eigenvalue $\lambda = 1$

$$x_1 = x_3 + \frac{1}{2}x_4$$

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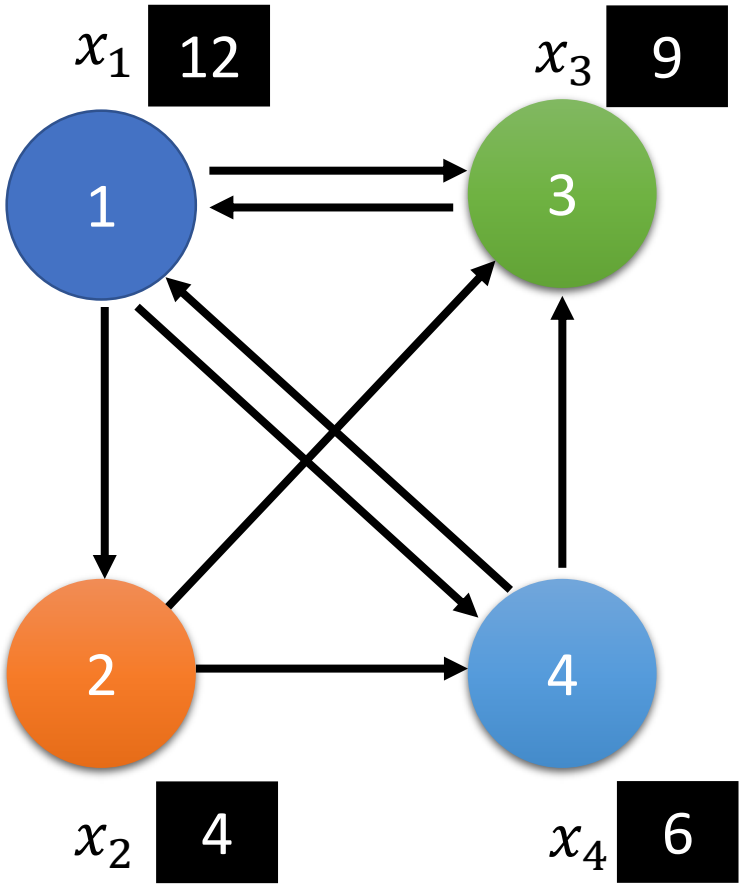
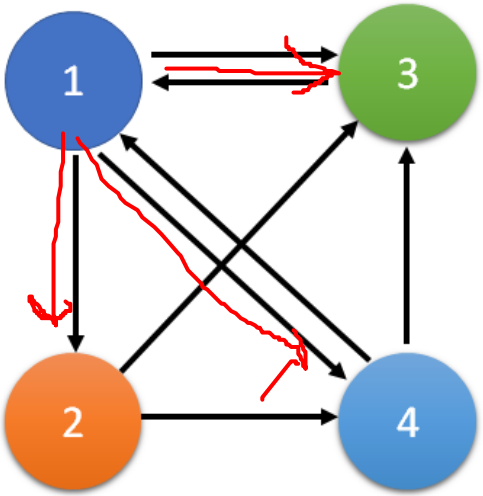
$$Ax = x$$

The solution x is in the eigenspace of eigenvalue $\lambda = 1$

$$Span\{[12 \quad 4 \quad 9 \quad 6]^T\}$$



# Eigenvalue = 1




$$Ax = x$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{bmatrix}$$

Column-stochastic Matrix

Column-stochastic matrix always have eigenvalue $\lambda = 1$ Proof

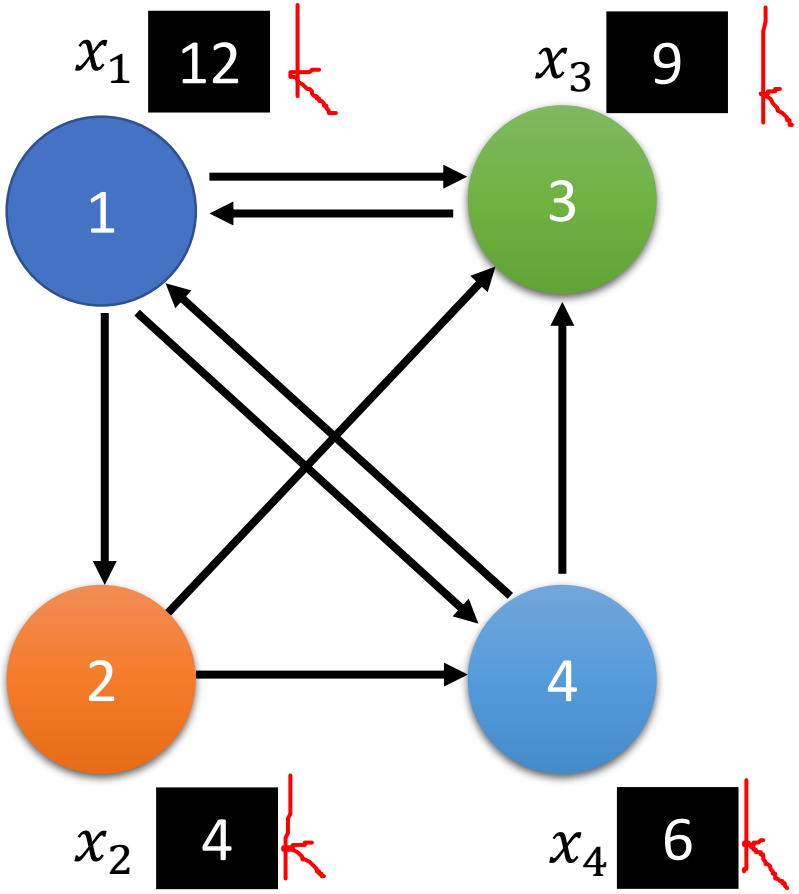
How about the Dangling nodes (只入不出)?

# Unique Ranking?

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{bmatrix}$$

Having eigenvalue $\lambda = 1$

The dimension of the subspace is 1




constraint

Unique Ranking

Unique Score

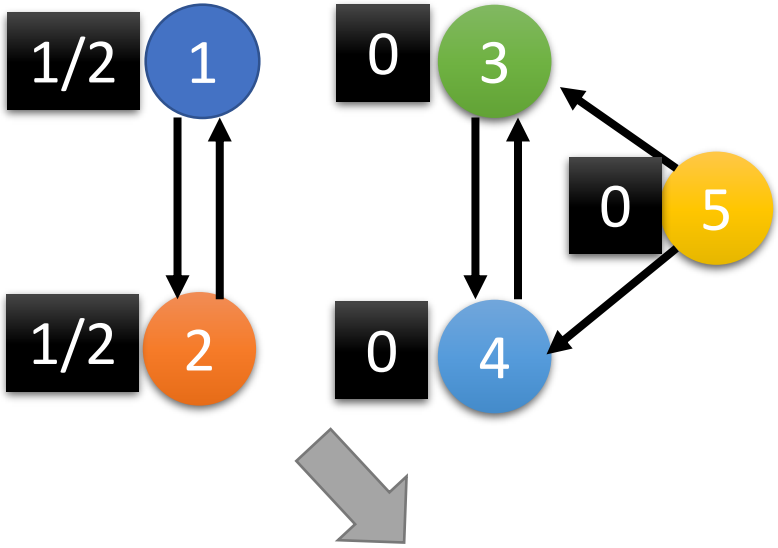
# Unique Ranking?

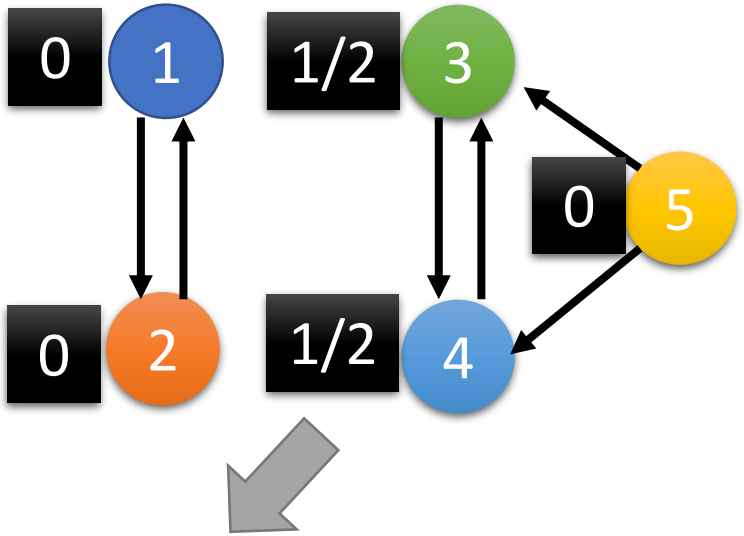
$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

How about dimension > 1

Dim for $\lambda = 1$ is 2

Basis:




Any linear combination is in the eigenspace

Not Unique Ranking

# Unique Ranking?

Can it be non-uniform?

All entries are 1/n

$$\mathbf{M} = (1 - m)\mathbf{A} + m\mathbf{S}$$

0.15

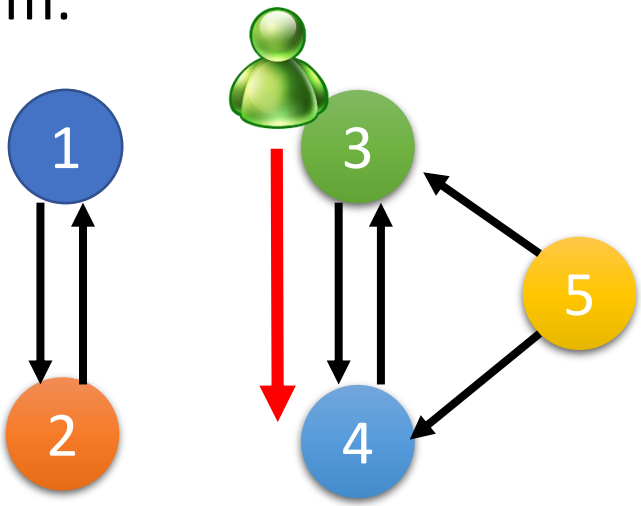
$M\chi = \chi$

Follow the link

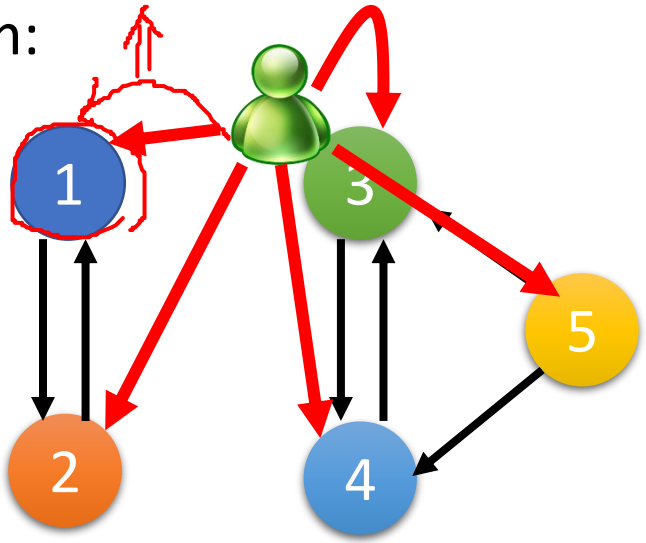
random

There are two ways to surf the web

Prob 1 – m:




Prob m:



# Unique Ranking?

$$\mathbf{M} = (1 - m)\mathbf{A} + m\mathbf{S}$$

- Unique ranking
- For M, the dim of the eigenvalue $\lambda = 1$ is 1

M is Column-stochastic matrix and "positive"

Proof → Dim = 1

Hint: For M, the eigenvectors for eigenvalue $\lambda = 1$ are all "positive" or "negative"

# Power method


Find $x^*$, such that $x^* = Mx^*$, $\|x^*\|_1 = 1$

Start from $x_0$, $\|x_0\|_1 = 1$

$$x_1 = Mx_0$$

$$x_2 = Mx_1$$

$$\vdots$$

$$x_k = Mx_{k-1}$$

If $k \to \infty$

$$x_k = x^*$$

Proof

# Actually ……

- **The Last Toolbar Pagerank Update was December 2013**
- Google declared thereafter: “P*ageRank is something that we haven’t updated for over a year now, and we’re probably not going to be updating it again going forward, at least the Toolbar version.*“

# Reference

- THE $25,000,000,000 EIGENVECTOR: THE LINEAR ALGEBRA BEHIND GOOGLE
  - http://userpages.umbc.edu/~kogan/teaching/m430/GooglePageRank.pdf
- A SURVEY OF EIGENVECTOR METHODS FOR WEB INFORMATION RETRIEVAL
  - http://doradca.oeiizk.waw.pl/survey.pdf