Linear Function in Coordinate System

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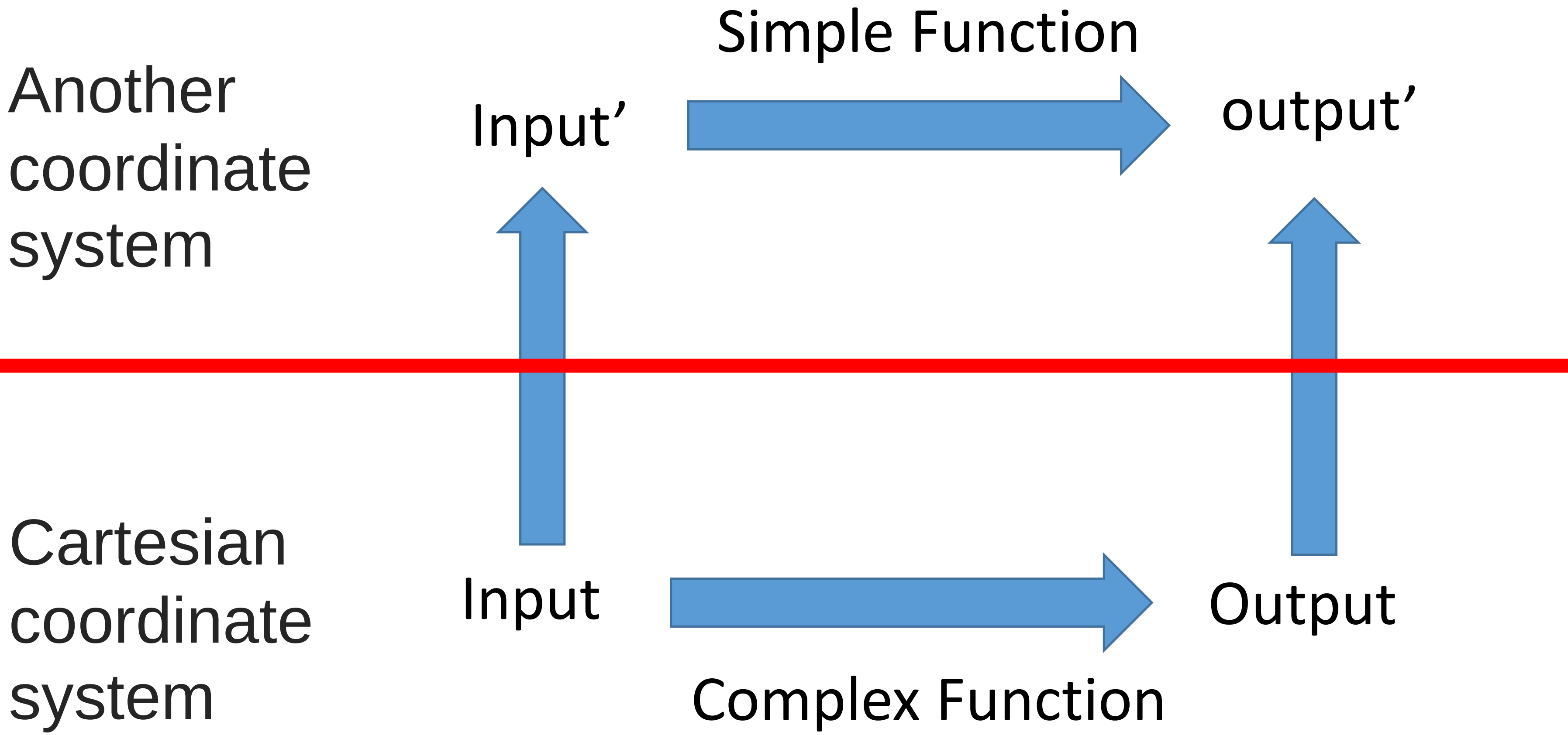
Outline

- Describing a function in a coordinate system
  - A complex function in one coordinate system can be simple in other systems.
- Reference: Textbook Chapter 4.5

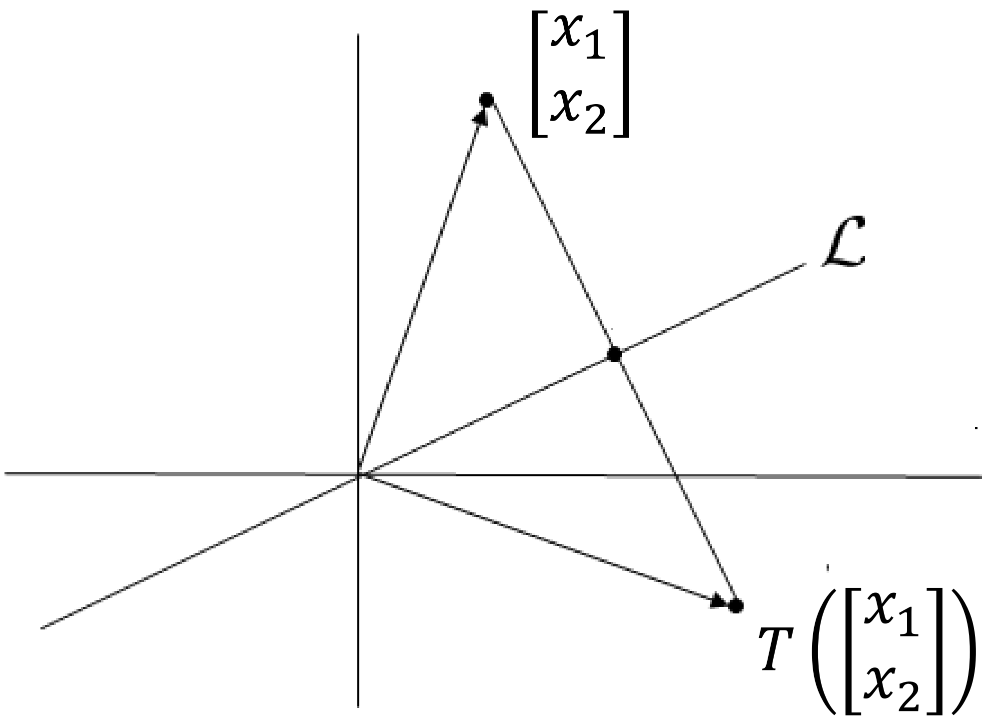
Basic Idea



Another coordinate system
Input’
Simple Function
output’
Cartesian coordinate system
Input
Output
Complex Function

Sometimes a function can be complex ……

- T: reflection about a line $\mathcal{L}$ through the origin in $\mathcal{R}^2$



$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = ?$$

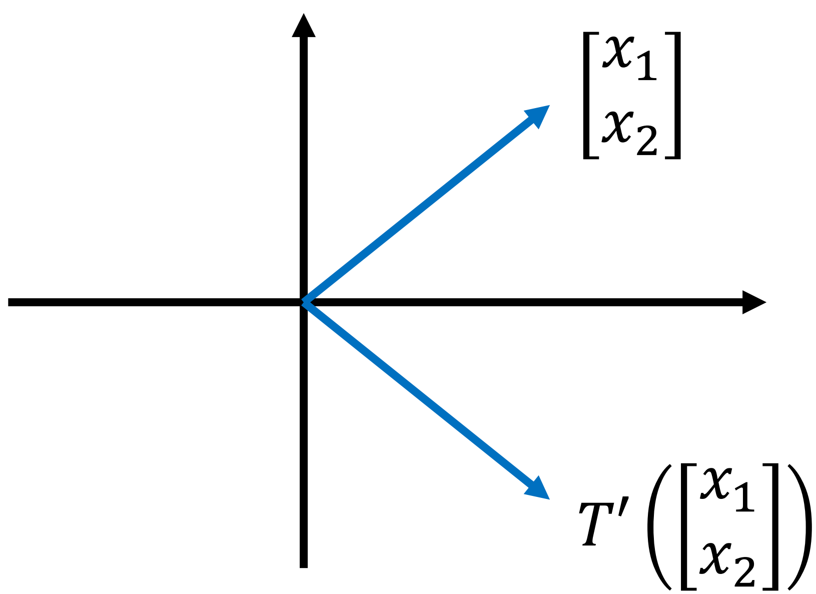
$$[\mathrm{T}] = [T(e_1) \quad T(e_2)\,]$$

$$= ?$$

Sometimes a function can be complex ……

- T: reflection about a line $\mathcal{L}$ through the origin in $\mathcal{R}^2$

special case: $\mathcal{L}$ is the *horizontal axis*



$$T'\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) =? \begin{bmatrix} x_1 \\ -x_2 \end{bmatrix}$$

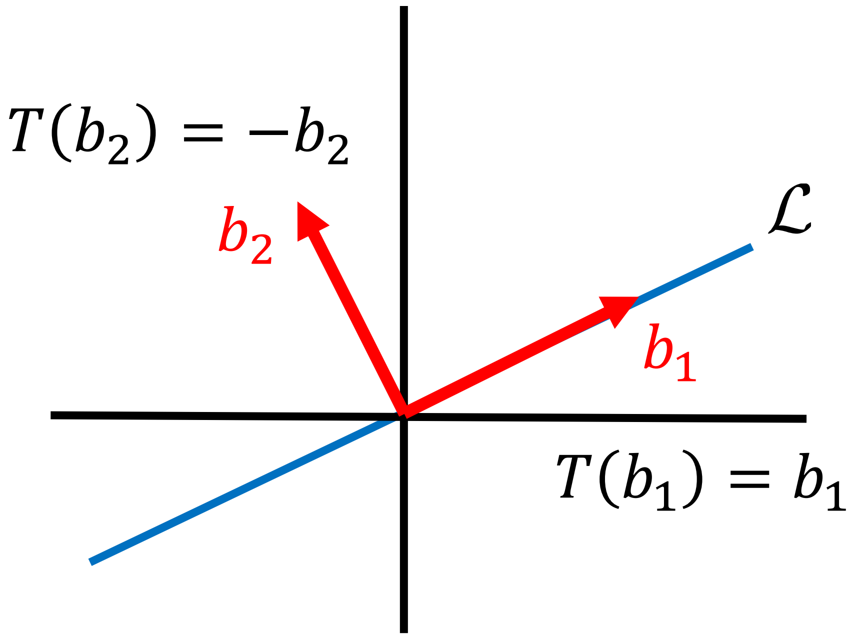
$$[T'] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$T'(e_1) = e_1$ $\quad$ $T'(e_2) = -e_2$

Describing the function in another coordinate system

- T: reflection about a line $\mathcal{L}$ through the origin in $\mathcal{R}^2$

In another coordinate system $\mathcal{B}$ ....



$\mathcal{B} = \{b_1, b_2\}$

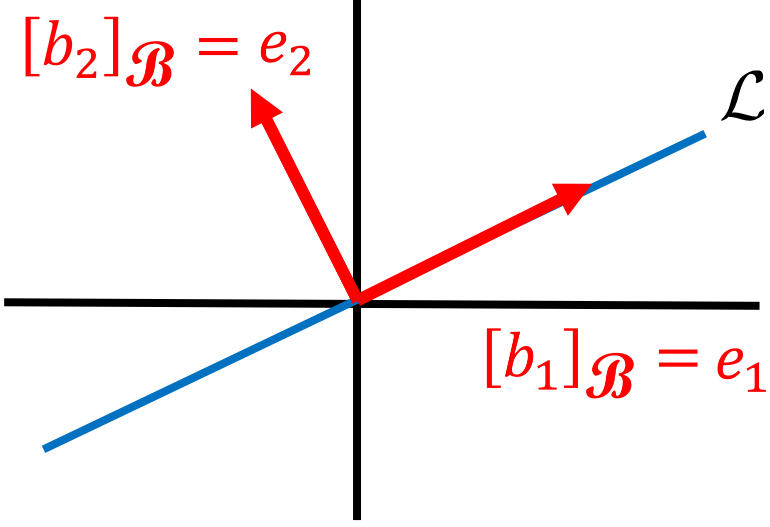
Describing the function in another coordinate system

- T: reflection about a line $\mathcal{L}$ through the origin in $\mathcal{R}^2$

In another coordinate system $\mathcal{B}$ ….

$$[T]_{\mathcal{B}} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Input and output are both in $\mathcal{B}$



$[T]b_1 = b_1$

⇒ $[T]_{\mathcal{B}}([b_1]_{\mathcal{B}}) = [b_1]_{\mathcal{B}}$

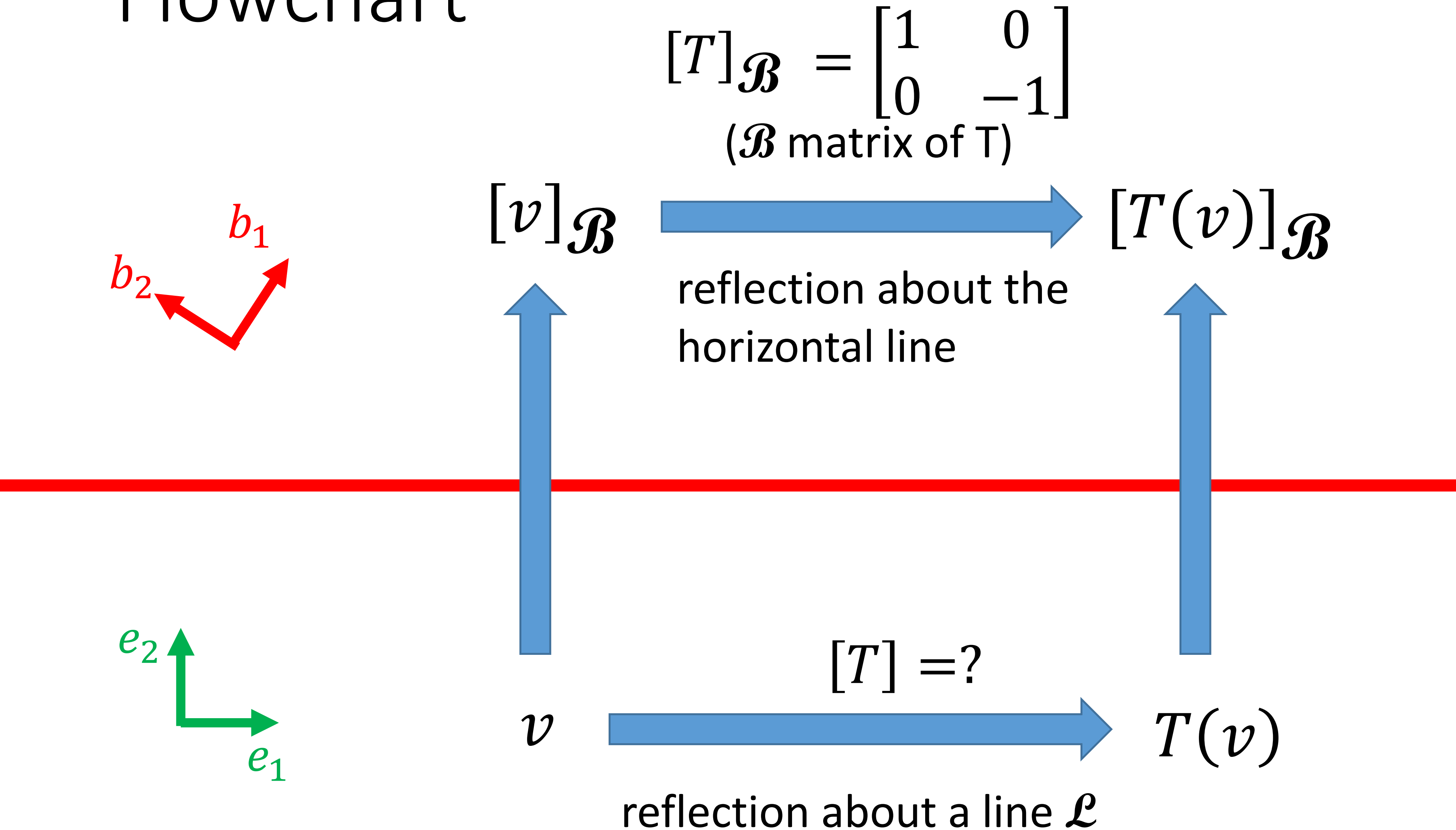
⇒ $[T]_{\mathcal{B}}(e_1) = e_1$

$[T]b_2 = -b_2$

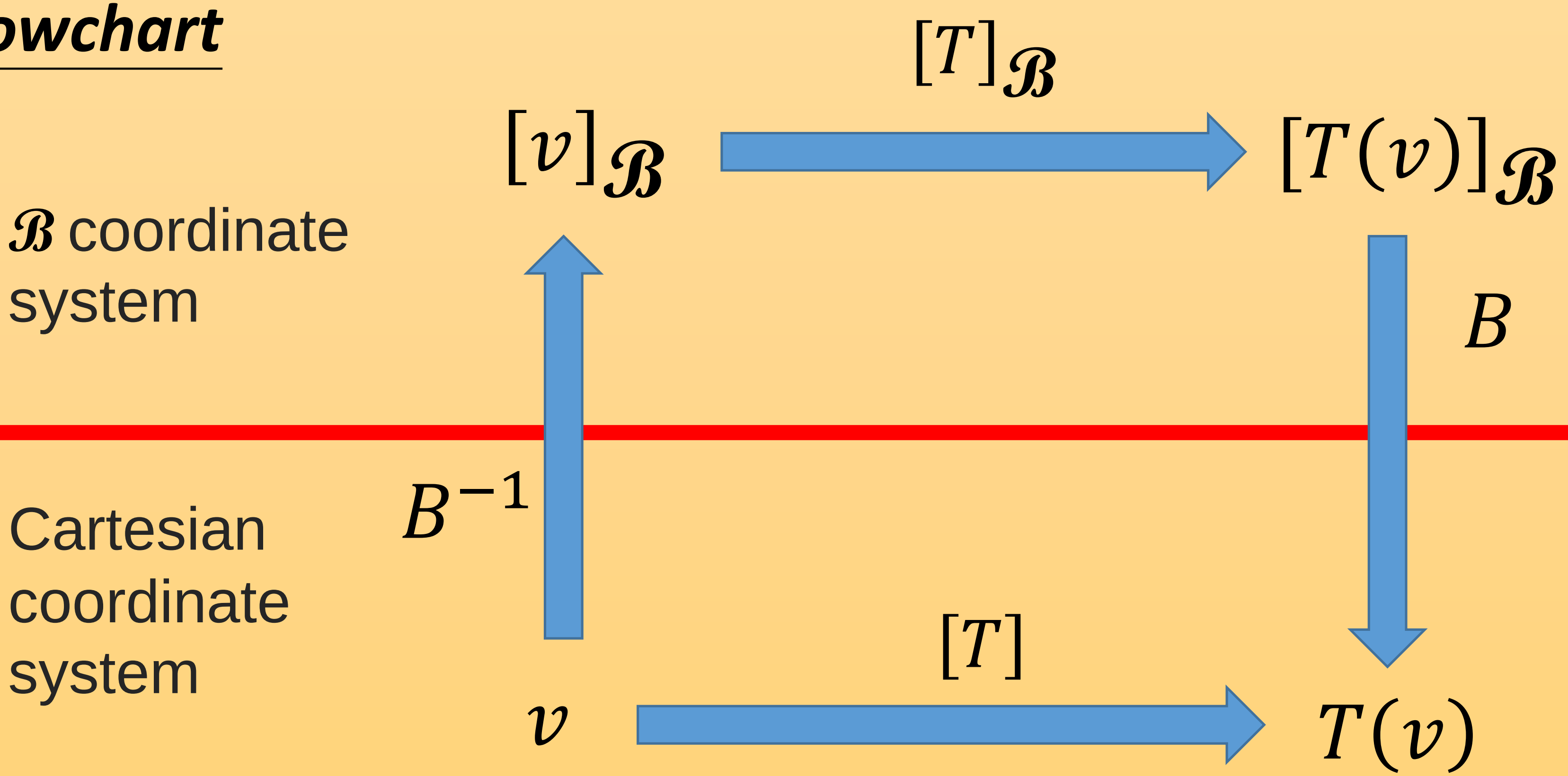
⇒ $[T]_{\mathcal{B}}([b_2]_{\mathcal{B}}) = [-b_2]_{\mathcal{B}}$

⇒ $[T]_{\mathcal{B}}(e_2) = -e_2$

Flowchart


$[T]_{\mathcal{B}} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
($\mathcal{B}$ matrix of T)
$[v]_{\mathcal{B}}$
$[T(v)]_{\mathcal{B}}$
reflection about the horizontal line
b_1
b_2
e_2
e_1
$[T] = ?$
v
$T(v)$
reflection about a line $\mathcal{L}$

Flowchart



$$[T] = B[T]_{\mathscr{B}}B^{-1}$$

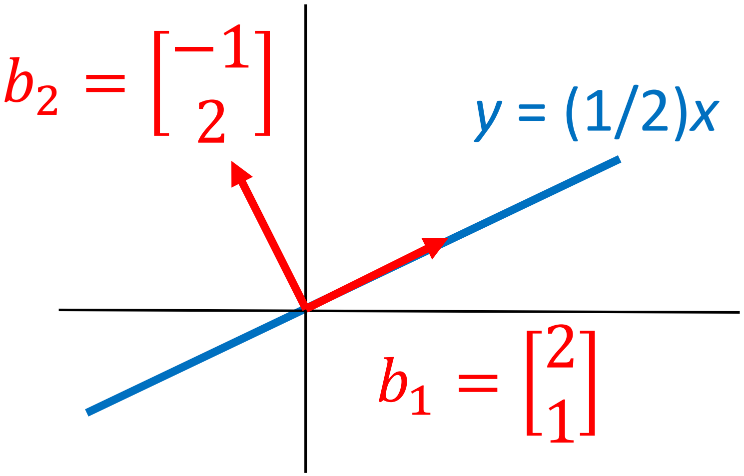
similar

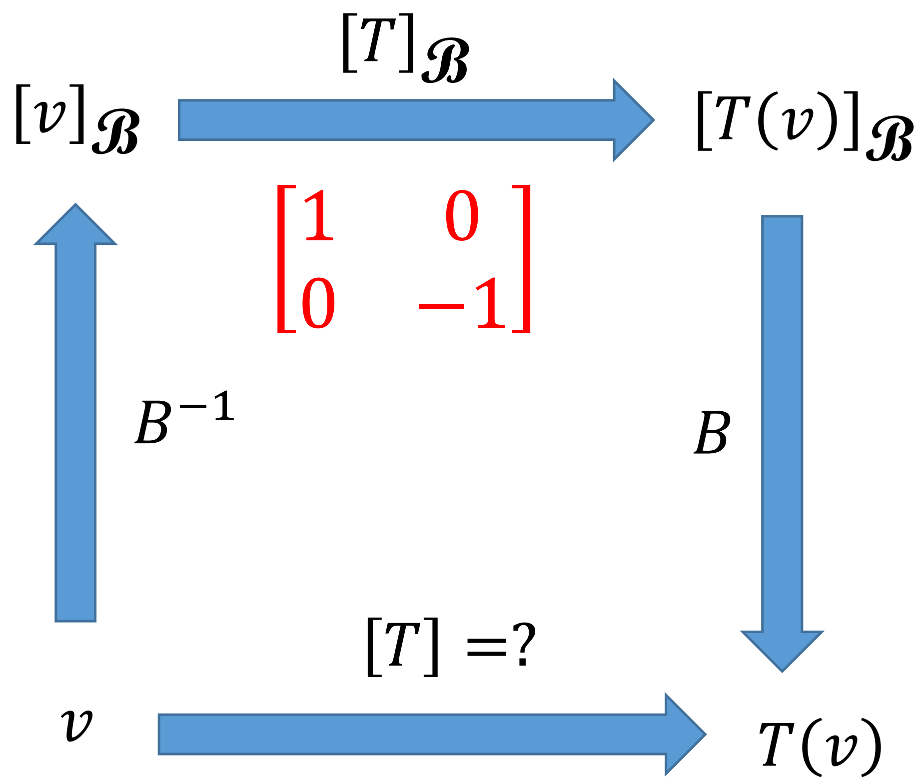
$$[T]_{\mathscr{B}} = B^{-1}[T]B$$

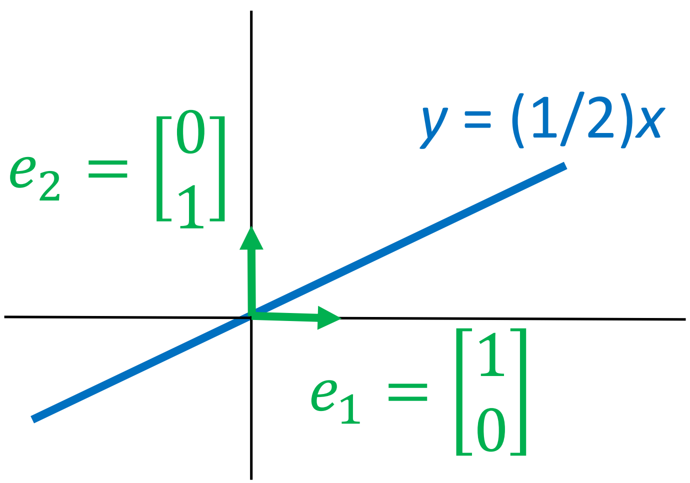
similar

- Example: reflection operator *T* about the line $y = (1/2)x$

$$B^{-1} = \begin{bmatrix} 0.4 & 0.2 \\ -0.2 & 0.4 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$





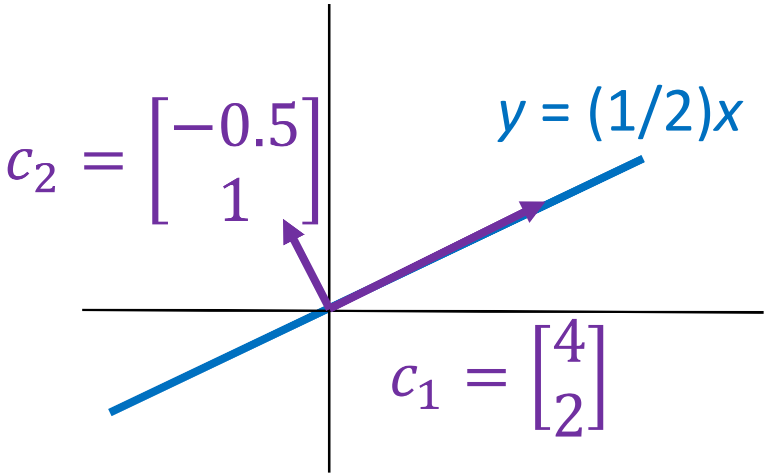


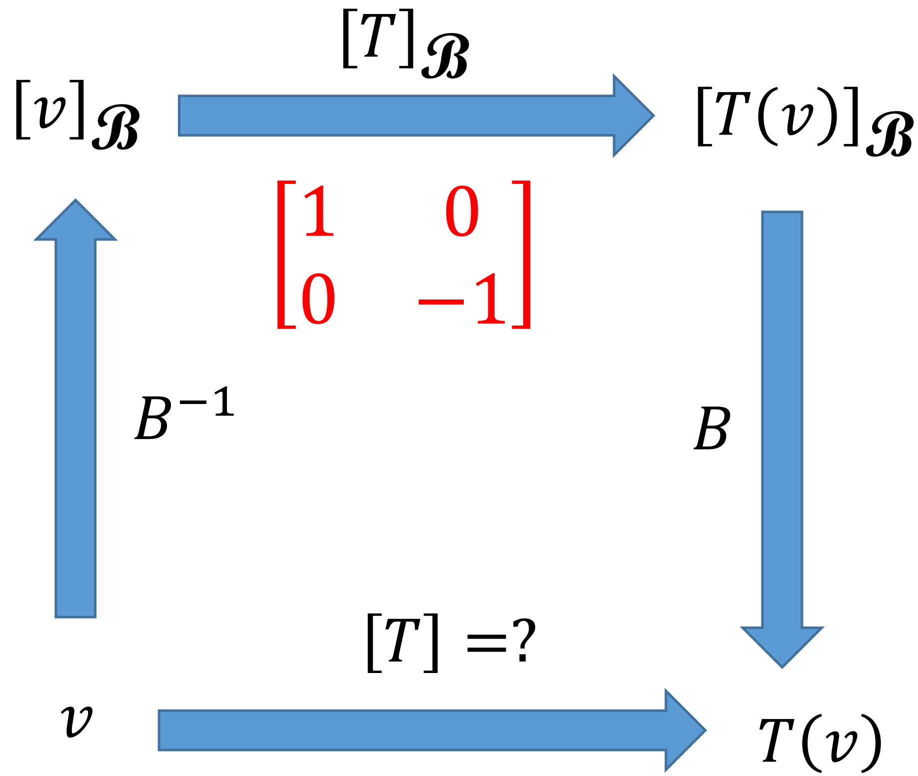
$$[T] = B[T]_{\mathcal{B}}B^{-1}$$

- Example: reflection operator T about the line $y = (1/2)x$

$$[T] = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 0.4 & 0.2 \\ -0.2 & 0.4 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$





$$[T] = C[T]_C C^{-1}$$

$$= \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$$

$$[T] = B[T]_{\mathcal{B}} B^{-1}$$

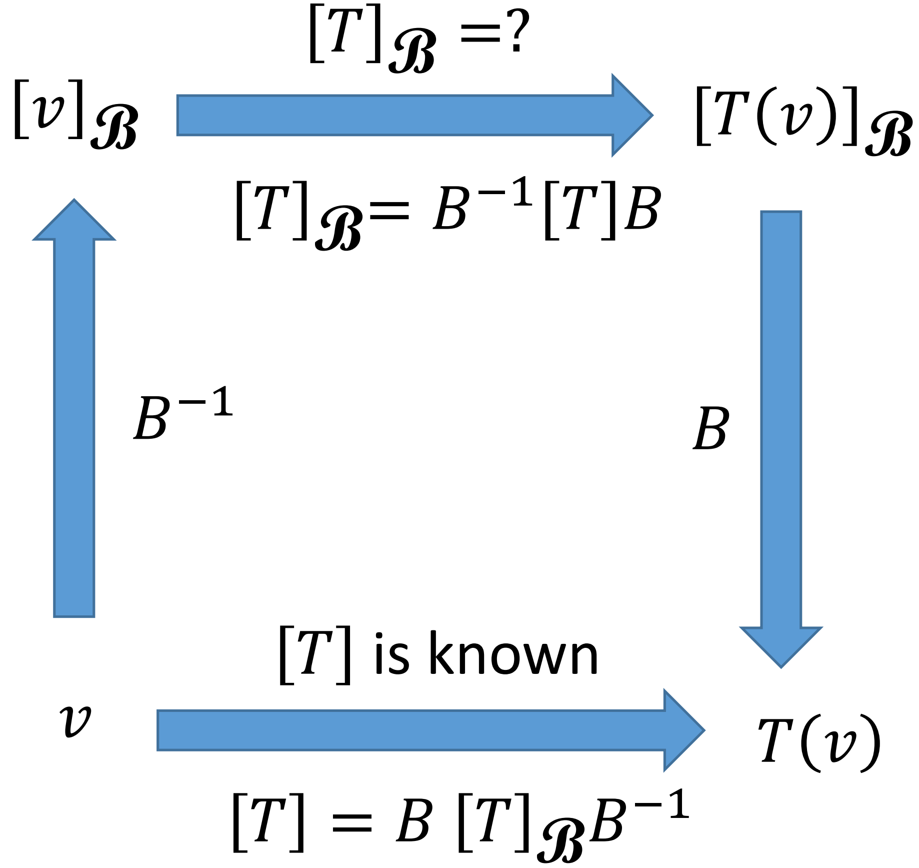
Example (P279)

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 + x_3 \\ x_1 + x_2 \\ -x_1 - x_2 + 3x_3 \end{bmatrix} \quad \mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}$$

$$[T] = \begin{bmatrix} 3 & 0 & 1 \\ 1 & 1 & 0 \\ -1 & -1 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$$

$$[T]_{\mathcal{B}} = \begin{bmatrix} 3 & -9 & 8 \\ -1 & 3 & -3 \\ 1 & 6 & 1 \end{bmatrix}$$



Example (P279) Determine T

$$T\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right)=\begin{bmatrix}1\\2\\1\end{bmatrix} \qquad T\left(\begin{bmatrix}1\\0\\1\end{bmatrix}\right)=\begin{bmatrix}3\\-1\\1\end{bmatrix} \qquad T\left(\begin{bmatrix}0\\1\\1\end{bmatrix}\right)=\begin{bmatrix}2\\0\\1\end{bmatrix}$$

$\mathbf{b}_1$ $\mathbf{c}_1$ $\mathbf{b}_2$ $\mathbf{c}_2$ $\mathbf{b}_3$ $\mathbf{c}_3$

$\mathbf{e}_1$ $\mathbf{e}_2$ $\mathbf{e}_3$ $[v]_{\mathcal{B}}$ → $[T]_{\mathcal{B}}$ → $[T(v)]_{\mathcal{B}}$ $\mathbf{B}^{-1}\mathbf{c}_1$ $\mathbf{B}^{-1}\mathbf{c}_2$ $\mathbf{B}^{-1}\mathbf{c}_3$

B^{-1} ↑ B^{-1} ↑

$\mathbf{b}_1$ $\mathbf{b}_2$ $\mathbf{b}_3$ v → $[T]$ → $T(v)$ $\mathbf{c}_1$ $\mathbf{c}_2$ $\mathbf{c}_3$

$\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{b}_3$ as a coordinate system

$\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ is a basis of R^3

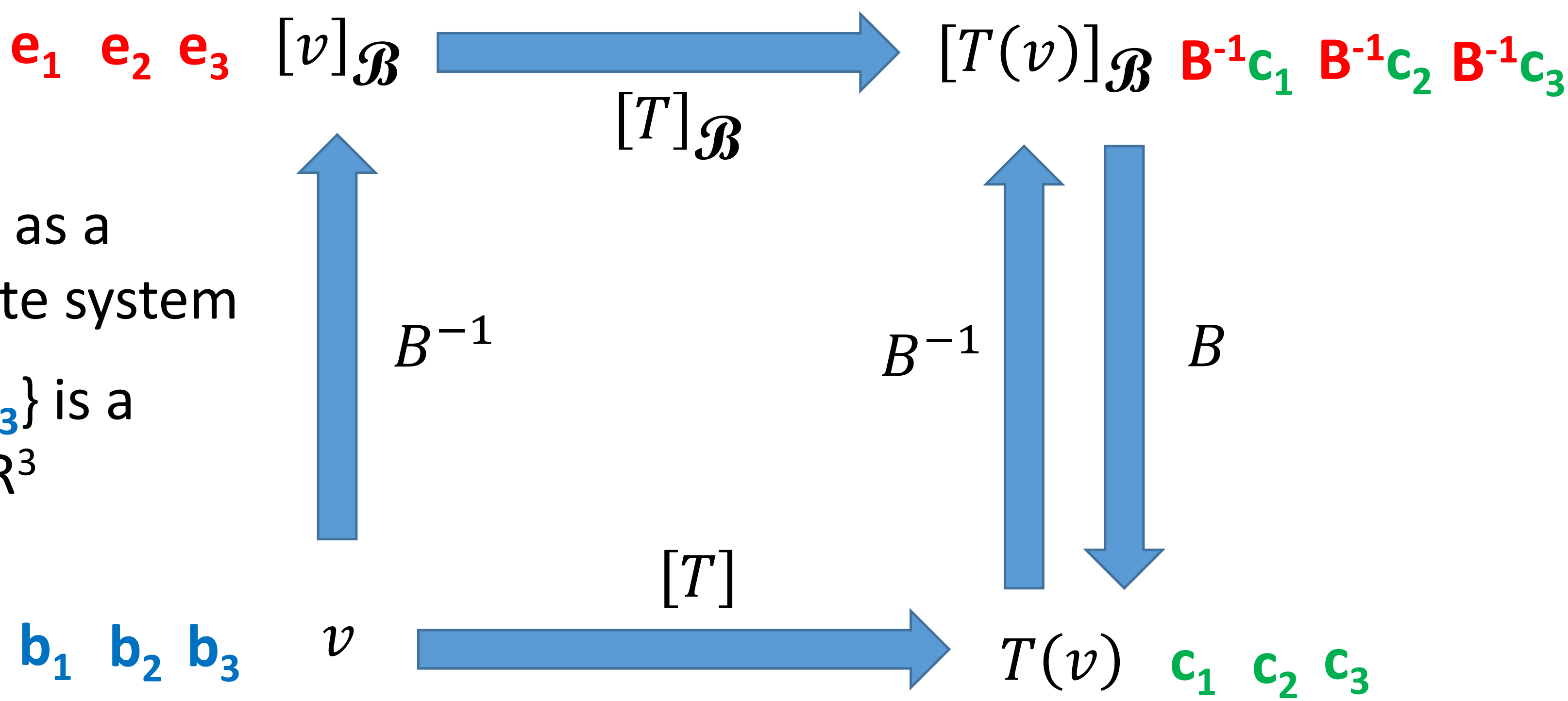
Example 3 (P279) Determine T

$$[T]_{\mathcal{B}} = [B^{-1}c_1 \quad B^{-1}c_2 \quad B^{-1}c_3] = B^{-1}C$$

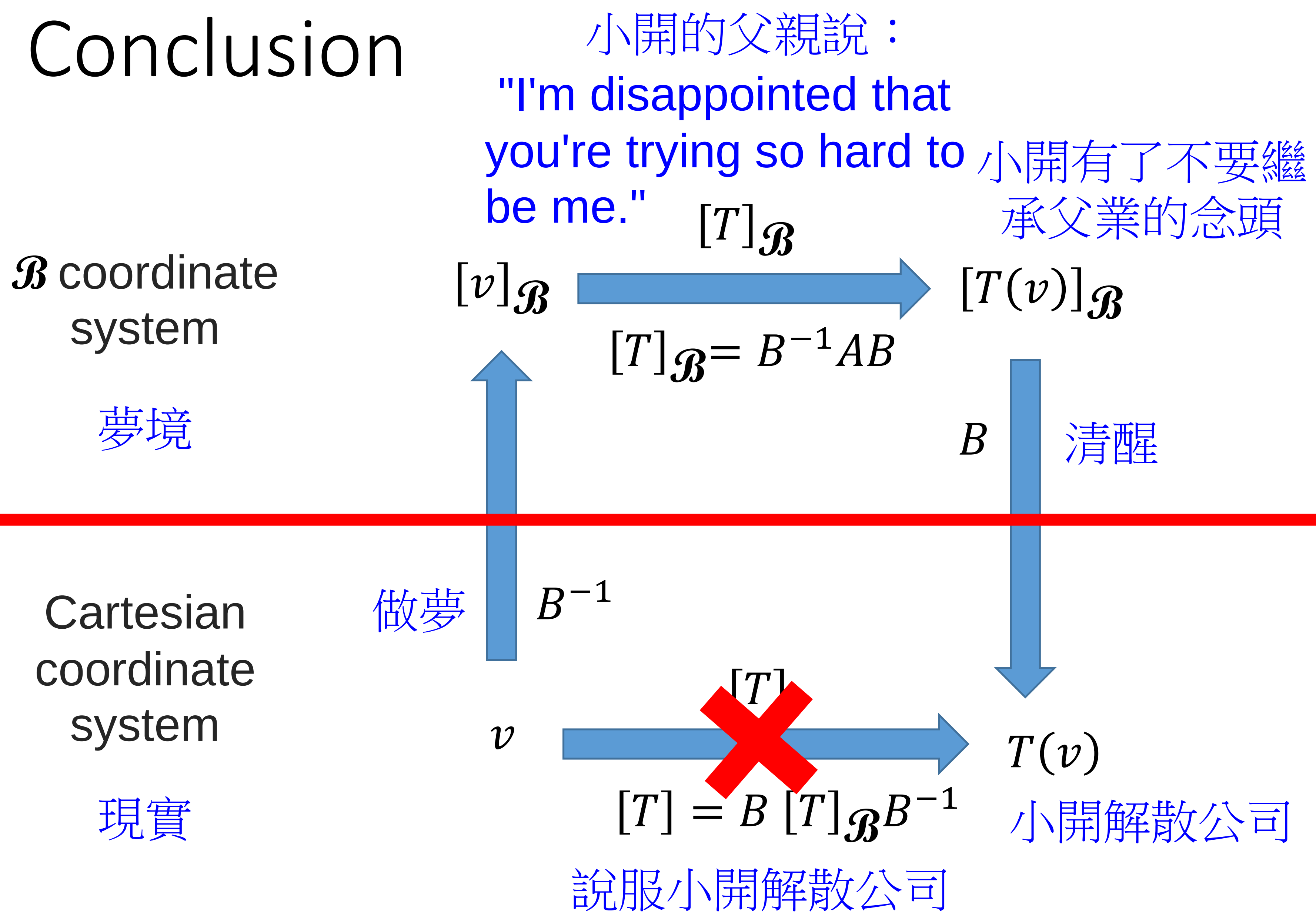
$$[T] = B[T]_{\mathcal{B}}B^{-1} = BB^{-1}CB^{-1} = CB^{-1}$$

$\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{b}_3$ as a coordinate system

{$\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{b}_3$} is a basis of R^3



Conclusion



小開的父親說：
"I'm disappointed that you're trying so hard to be me."
小開有了不要繼承父業的念頭
$\mathcal{B}$ coordinate system
夢境
$[v]_{\mathcal{B}}$
$[T]_{\mathcal{B}}$
$[T(v)]_{\mathcal{B}}$
$[T]_{\mathcal{B}} = B^{-1}AB$
B
清醒
Cartesian coordinate system
現實
做夢
B^{-1}
v
$[T]$
$T(v)$
$[T] = B\,[T]_{\mathcal{B}}B^{-1}$
小開解散公司
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