Linear Algebra:
What are we going to learn?

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What are we going to learn?

- A system of linear equations
What are we going to learn?

• A system of linear equations (多元一次联立方程式)

\[
\begin{align*}
    a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n &= b_1 \\
    a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n &= b_2 \\
    &\vdots \\
    a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n &= b_m
\end{align*}
\]

In this course, m and n can be large
What are we going to learn?

• Solving a system of linear equations

\[
\begin{align*}
    a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
    a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
    &\quad \vdots \\
    a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m
\end{align*}
\]

- Does it have solution?
- Does it have unique solution?
- How to find the solution?
- Determinants (行列式)
  - Beyond 3 X 3

Different view from high school

span
Independent
rank
Approximate Solution

Does it have solution?
Linear System

• We use a system of linear equations to describe a linear system

• What is system?
  • function, transformation, operator
  • A system has input and output
    • E.g. Speech Recognition System: The input is audio signals, and the output is recognition results.
    • E.g. Siri: You say “Hi, Siri”, it reply “What can I do for you?”
  • E.g. Communication system
  • Can have multiple inputs and outputs
Linear System

- Linear system have two properties
  - 1. Persevering Multiplication
  
  \[ x \rightarrow \text{Linear System} \rightarrow y \]
  
  \[ kx \rightarrow \text{Linear System} \rightarrow ky \]

- 2. Persevering Addition
  
  \[ x_1 \rightarrow \text{Linear System} \rightarrow y_1 \]
  
  \[ x_2 \rightarrow \text{Linear System} \rightarrow y_2 \]
  
  \[ x_1 + x_2 \rightarrow \text{Linear System} \rightarrow y_1 + y_2 \]
Linear System

- Linear system have two properties
  - 1. Persevering Multiplication
    \[ kx \neq k^2x \]
  - 2. Persevering Addition
    \[ (x_1 + x_2)^2 \neq (x_1)^2 + (x_2)^2 \]
Linear System

- Two properties
  - 1. Persevering Multiplication
  - 2. Persevering Addition
Linear System v.s.
System of Linear Equations

\[ \begin{align*}
    a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
    a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
    \vdots \\
    a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m 
\end{align*} \]
A linear system is described by a system of linear equations

\[
\begin{align*}
\begin{cases}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
\vdots & \vdots \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m
\end{cases}
\end{align*}
\]
Applications

Most systems are (assumed to) be linear

Input: voltage source, current source
Output: voltage and current on the load (燈泡、引擎)

Linear System
Applications

Most system are (assumed to) be linear
Applications

• Machine Learning
  • E.g. Prediction
Applications

- We will learn EigenXXX

\[
\begin{align*}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_n
\end{bmatrix}
\rightarrow
\begin{bmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_n
\end{bmatrix}
\end{align*}
\]

Input equals to output

\[
\begin{align*}
\begin{bmatrix}
  x_1' \\
  x_2' \\
  \vdots \\
  x_n'
\end{bmatrix}
\rightarrow
\begin{bmatrix}
  0 \\
  0 \\
  \vdots \\
  0
\end{bmatrix}
\end{align*}
\]

Output is zero
Applications — Image Compression

壓縮比: 25

壓縮比: 12.5

壓縮比: 5

壓縮比: 2.5

原圖
Linear Algebra is Important