Digital Speech Processing Homework #1

Implementing Discrete Hidden Markov Model

蔡翔陞

October 14, 2020

Due on 23:59, November 6, 2020

Outline

HMM in Speech Recognition

Speech Recognition 1/2

In acoustic model

- each word consists of syllables
- each syllable consists of phonemes
- each phoneme consists of some (hypothetical) states

$$\text{“語音”} \rightarrow \text{“語 (ㄩˇ) 音 (ㄧㄣ)”} \rightarrow \text{“ㄩ”} \rightarrow \{s_1, s_2, \ldots\}$$

Each phoneme can be described by a HMM (acoustic model). Given a sequence of observation (MFCC vectors), each of them can be mapped to a corresponding state.

Speech Recognition 2/2

Hence, there are

- state transition probabilities (a_{ij}) and
- observation distribution ($b_j[o_t]$)

in each phoneme acoustic model (HMM).

Usually in speech recognition we restrict the HMM to be a *left-to-right model*, and the observation distribution is assumed to be a continuous Gaussian mixture model.

Review

- Left-to-right
- The observation distribution is a continuous Gaussian mixture model.

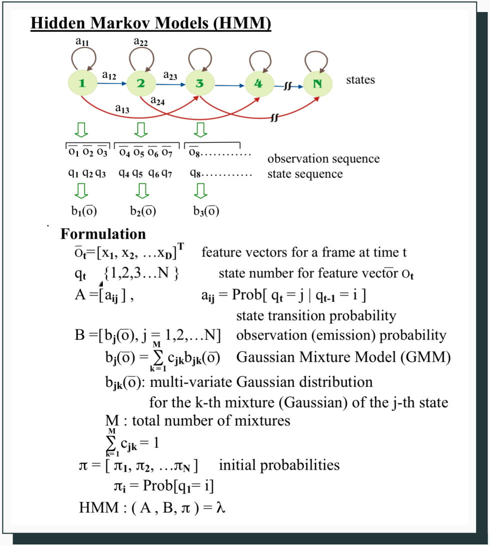


Figure 1: HMM from lecture 2.0

General Discrete HMM

$$a_{ij} = P(q_{t+1} = j \mid q_t = i), \forall\, t, i, j \tag{1}$$

$$b_j(A) = P(o_t = A \mid q_t = j), \forall\, t, A, j \tag{2}$$

Given q_t, the probability distributions of q_{t+1} and o_t are completely determined. (independent of other states or observation)

HW1 v.s. Speech Recognition

	Homework	*Speech Recognition*
λ **set**	5 models	initial-final
λ	model_01-05	“ㄩ”
$\{o_t\}$	A, B, C, D, E, F	39-dim MFCC
unit	an alphabet	a time frame
observation	sequence	voice wave

Homework of HMM

Flowchart

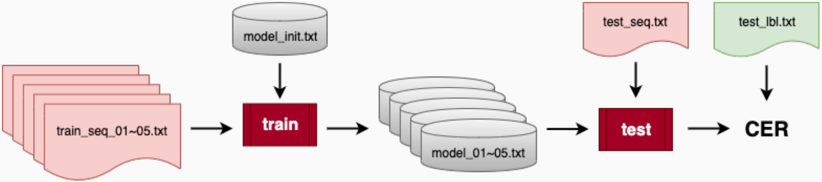


Figure 2: Training and testing models

Problems of HMM

Training

- Basic problem 3 in lecture 4.0
  - Given observations O and an initial model $\lambda = (A, B, \pi)$, adjust λ to maximize $P(O \mid \lambda)$.

$$A_{ij} = a_{ij},\ B_{jt} = b_j[o_t],\ \pi_i = P(q_1 = i)$$

- Baum-Welch algorithm

Testing

- Basic problem 2 in lecture 4.0
  - Given λ and O, find the best state sequences to maximize $P(O \mid \lambda, q)$.
- Viterbi algorithm

Homework of HMM

Training

Training

- Basic problem 3
- *Baum-Welch algorithm*: A generalized expectation-maximization (EM) algorithm[1]
    - Calculate α (forward probabilities) and β (backward probabilities) given the observations
    - Find temporary variables ϵ and γ from α and β
    - Update model parameters $\lambda' = (A', B', \pi')$

[1] `http://en.wikipedia.org/wiki/Baum-Welch_algorithm`

Forward Procedure

Forward algorithm: define a forward variable $\alpha_t(i)$

$$\alpha_t(i) = P(o_1, o_2, \ldots, o_t,\ q_t = i \mid \lambda) \tag{3}$$

$$= \text{Prob}[\text{ observing } o_1, o_2, \ldots, o_t,\ \text{state } i \text{ at time } t \mid \lambda\ ] \tag{4}$$

Initialization

$$\alpha_1(i) = \pi_i b_i(o_1),\ 1 \leq i \leq N \tag{5}$$

Induction

$$\alpha_{t+1}(j) = \left[\sum_{i=1}^{N} \alpha_t(i) a_{ij} \right] \cdot b_j(o_{t+1}),$$

$$1 \leq t \leq T-1,\ 1 \leq j \leq N \tag{6}$$

Termination

$$P\left(\bar{O} \mid \lambda\right) = \sum_{i=1}^{N} \alpha_T(i) \tag{7}$$

Backward Procedure

Backward algorithm: define a backward variable $\beta_t(i)$

$$\beta_t(i) = P(o_{t+1}, o_{t+2}, \ldots, o_T \mid q_t = i, \lambda) \tag{8}$$
$$= \text{Prob}[\text{ observing } o_{t+1}, o_{t+2}, \ldots, o_T \mid \text{state } i \text{ at time } t,\ \lambda\ ] \tag{9}$$

Initialization

$$\beta_T(i) = 1,\ 1 \leq i \leq N \tag{10}$$

Induction

$$\beta_t(i) = \sum_{j=1}^{N} a_{ij}\ b_j(o_{t+1})\ \beta_{t+1}(j),$$
$$t = \{T-1, T-2, \ldots, 1\},\ 1 \leq i \leq N \tag{11}$$

Calculate γ

Define a temporary variable $\gamma_t(i) = P\left(q_t = i \mid \bar{O}, \lambda\right)$

$$\gamma_t(i) = \frac{\alpha_t(i)\beta_t(i)}{\sum_{i=1}^{N} \alpha_t(i)\beta_t(i)} = \frac{P\left(\bar{O}, q_t = i \mid \lambda\right)}{P\left(\bar{O} \mid \lambda\right)} \tag{12}$$

It should be a $N \times T$ matrix!

Calculate ε

The probability of transition from state i to state j given observation and model.

$$\epsilon_t(i,j) = P\left(q_t = i, q_{t+1} = j \mid \bar{O}, \lambda\right) \tag{13}$$

$$= \frac{\alpha_t(i)\ a_{ij}\ b_j(o_{t+1})\ \beta_{t+1}(j)}{\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_t(i)\ a_{ij}\ b_j(o_{t+1})\ \beta_{t+1}(j)} \tag{14}$$

$$= \frac{\text{Prob}\left[\bar{O},\ q_t = i,\ q_{t+1} = j \mid \lambda\right]}{P\left(\bar{O} \mid \lambda\right)} \tag{15}$$

In total $T - 1$ matrices (each $N \times N$)

Accumulate ε and γ

Recall $\gamma_t(i) = P\left(q_t = i \mid \bar{O}, \lambda\right)$

$$\begin{aligned}
\sum_{t=1}^{T-1} \gamma_t(i) &= \text{expected number of times that state } i \\
&\quad \text{is visited in } \bar{O} \text{ from } t = 1 \text{ to } t = T - 1 \qquad (16) \\
\sum_{t=1}^{T-1} \epsilon_t(i,j) &= \text{expected number of transitions from} \\
&\quad \text{state } i \text{ to state } j \text{ in } \bar{O} \qquad (17)
\end{aligned}$$

Re-estimate Model Parameters

Now we could update the parameters with εand γ

$$\lambda' = (A', B', \pi') \tag{18}$$

$$\pi_i' = \gamma_1(i) = p(\text{the first state is i}) \tag{19}$$

$$a_{ij}' = \frac{\sum_{t=1}^{T-1} \epsilon_t(i,j)}{\sum_{t=1}^{T-1} \gamma_t(i)} = \frac{\mathbb{E}\ [\text{ Number of transition from } i \text{ to } j\ ]}{\mathbb{E}\ [\text{ Number of visiting state } i\ ]} \tag{20}$$

$$b_i'(k) = \frac{\sum_{O_t=k} \gamma_t(i)}{\sum_{t=1}^{T} \gamma_t(i)} = \frac{\mathbb{E}\ [\text{ Number of observation } O = k \text{ in state } i\ ]}{\mathbb{E}\ [\text{ Number of visiting state } i\ ]} \tag{21}$$

Re-estimate Model Parameters with Multiple Sequences

With N training sequences, we should update parameters from all those sequences

$$\pi_i' = \frac{\sum_{n=1}^{N} \gamma_{n,1}(i)}{N} \tag{22}$$

$$a_{ij}' = \frac{\sum_{n=1}^{N} \sum_{t=1}^{T-1} \epsilon_{n,t}(i,j)}{\sum_{n=1}^{N} \sum_{t=1}^{T-1} \gamma_{n,t}(i)} \tag{23}$$

$$b_i'(k) = \frac{\sum_{n=1}^{N} \sum_{O_{n,t}=k} \gamma_{n,t}(i)}{\sum_{n=1}^{N} \sum_{t=1}^{T} \gamma_{n,t}(i)} \tag{24}$$

Homework of HMM

Testing

Testing

- Basic problem 2
  - Given λ and O, find the best state sequences to maximize $P(O \mid \lambda, q)$.
- Calculate $P(O \mid \lambda) \approx \max P(O \mid \lambda, q)$ for each of the five models
- The model with the highest probability for the most probable path usually also has the highest probability for all possible paths.

Viterbi Algorithm

Complete procedure for Viterbi algorithm[2]

Initialization

$$\delta_1(i) = \pi_i b_i(o_1),\ 1 \le i \le N \tag{25}$$

Recursion

$$\delta_t(j) = \max_{1 \le i \le N} [\ \delta_{t-1}(i)\ a_{ij}\ ] \cdot b_j(o_t),\ 2 \le t \le T,\ 1 \le j \le N \tag{26}$$

Termination

$$P^* = \max_{1 \le i \le N} [\ \delta_T(i)\ ] \tag{27}$$

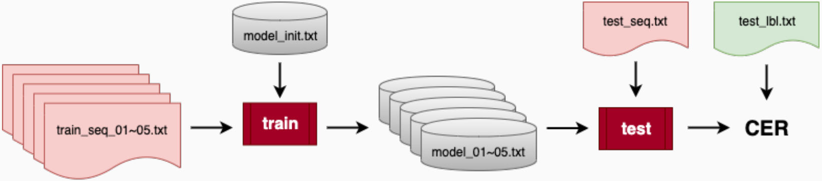
$$\delta_t(i) = \max_{q_1,\ldots,q_{t-1}} P[\ q_1, q_2, \ldots, q_{t-1}, q_t = i, o_1, o_2, \ldots, o_t \mid \lambda\ ] \tag{28}$$

$$= \text{the highest probability along a certain single path ending at state } i \text{ at time } t \text{ for the first } t \text{ observations, given } \lambda \tag{29}$$

[2] `http://en.wikipedia.org/wiki/Viterbi_algorithm`

Requirements

Recall: Flowchart



Provided Files

`data/train_seq_0X.txt`

- Training data (10K observation sequences)

`data/test_lbl.txt`

- Testing labels

`data/test_seq.txt`

- Testing data (2.5K observation sequences)

`inc/hmm.h`

- Provided by TA, please work with it!
- You can load/dump models with functions within.

`model_init.txt`

- Initial model parameters

`modellist.txt`

- Paths to model files

`src/test_hmm.c`

- A showcase of the usage of `hmm.h`

```
dsp_hw1
├── data
│   ├── test_lbl.txt
│   ├── test_seq.txt
│   ├── train_seq_01.txt
│   ├── train_seq_02.txt
│   ├── train_seq_03.txt
│   ├── train_seq_04.txt
│   └── train_seq_05.txt
├── inc
│   └── hmm.h
├── Makefile
├── model_init.txt
├── modellist.txt
└── src
    └── test_hmm.c
```

Input and Output of Your Program

Training

Input
1. number of iterations
2. initial model (`model_init.txt`)
3. observation sequences (`train_seq_01~05.txt`)

Output Five files of parameters for 5 models, each contains $\lambda = (A, B, \pi)$ (e.g. `model_01~05.txt`)

Testing

Input
1. a file of paths to the models trained in the previous step (`modellist.txt`)
2. observation sequences (`test_seq.txt`)

Output best answer labels and $P(O \mid \lambda)$ (e.g. `result.txt`)

```
./train <iter> <model_init_path> <seq_path> <output_model_path>
```

`iter`	# of iterations
`model_init_path`	path to the initial model params
`seq_path`	path to sequence data
`output_model_path`	path to dump trained models

Testing Details

```
./test <models_list_path> <seq_path> <output_result_path>
```

`models_list_path`	path to the model list file
`seq_path`	path to sequence data
`output_result_path`	path to output testing result

Program Execution Example

Compiling

```
make  # type this in the root directory of the project
```

Training

```
./train 100 model_init.txt data/train_seq_01.txt model_01.txt
```

Testing

```
./test modellist.txt data/test_seq.txt result.txt
```

Notice!

Command-line arguments are not fixed, read them during runtime. (e.g. Use `argv` in main function to pass the arguments.)

Also the paths in arguments need to be variable path.

Requirements

File Format

Observation Sequence Format

The given `data/train_seq_01~05.txt` and `data/test_seq.txt` look like this.

```
ACCDDDDFFCCCCBCFFFCCCCCEDADCCAEFCCCACDDFFCCDDFFCCD
CABACCAFCCFFCCCDFFCCCCCDFFCDDDDFCDDCCFCCCEFFCCCCBC
ABACCCDDCCCDDDDFBCCCCCDDAACFBCCBCCCCCCCFFFCCCCCDBF
AAABBBCCFFBDCDDFFACDCDFCDDFFFFFCDFFFCCCDCFFFFCCCCD
AACCDCCCCCCCDCEDCBFFFCDCDCDAFBCDCFFCCDCCCEACDBAFFF
...
```

Each of the former has 10000 sequences and the latter has 2500 sequences.

Model Format 1/2

`initial: 6`

$$\pi = [\ \pi_1,\ \pi_2,\ \pi_3,\ \pi_4,\ \pi_5,\ \pi_6\ ]$$

`transition: 6`

$$A = \begin{bmatrix} a_{11} & \dots & a_{16} \\ \vdots & \ddots & \vdots \\ a_{61} & \dots & a_{66} \end{bmatrix}$$

`observation:` [3]`6`

$$B = \begin{bmatrix} b_1(o_1) & \dots & b_6(o_1) \\ \vdots & \ddots & \vdots \\ b_1(o_6) & \dots & b_6(o_6) \end{bmatrix}$$

[3] The sum of column is 1 here.

Model Format 2/2

A model file (e.g. `model_0X.txt`) should look like this.

```
initial: 6
0.2     0.1     0.2     0.2     0.2     0.1

transition: 6
0.3     0.3     0.1     0.1     0.1     0.1
0.1     0.3     0.3     0.1     0.1     0.1
0.1     0.1     0.3     0.3     0.1     0.1
0.1     0.1     0.1     0.3     0.3     0.1
0.1     0.1     0.1     0.1     0.3     0.3
0.3     0.1     0.1     0.1     0.1     0.3

observation: 6
0.2     0.2     0.1     0.1     0.1     0.1
0.2     0.2     0.2     0.2     0.1     0.1
0.2     0.2     0.2     0.2     0.2     0.2
0.2     0.2     0.2     0.2     0.2     0.2
0.1     0.1     0.2     0.2     0.2     0.2
0.1     0.1     0.1     0.1     0.2     0.2
```

Model List Format

The given `modellist.txt` looks like this.

```
model_01.txt
model_02.txt
model_03.txt
model_04.txt
model_05.txt
```

Your testing program should be able to read a list like this and load models from the specified paths for testing. (Don't worry! If you use `hmm.h`, all of these are done by calling function `load_models()`. For more details please refer to `hmm.h`.)

Output Format

Your testing program should output these to the specific path (e.g. `result.txt`) given as a command-line argument while executing the program.

```
model_01.txt 7.822367e-34
model_05.txt 1.094896e-40
model_01.txt 7.928724e-33
model_02.txt 4.262100e-37
model_02.txt 5.914689e-42
...
```

Each line consists of the hypothesis model and its likelihood. They should be separated by a space.

Label File Format

The first few lines of the given `data/test_lbl.txt` looks like this.

```
model_01.txt
model_05.txt
model_01.txt
model_02.txt
model_02.txt
...
```

Makefile Format

The Makefile you submit should be capable to compile your program using `make`. The provided one can compile `train.c` and `test.c` in directory `src` into two executables `train` and `test`.

```
.PHONY: all clean run
CC=gcc
CFLAGS=-std=c99 -O2
LDFLAGS=-lm
TARGET=train test

all: $(TARGET)

train: src/train.c
	$(CC) -o $@ $^ $(CFLAGS) $(LDFLAGS) -Iinc

test: src/test.c
	$(CC) -o $@ $^ $(CFLAGS) $(LDFLAGS) -Iinc

clean:
	rm -f $(TARGET)
```

Report Format

Please write a **one-page** report in **PDF** format, name it `report.pdf` and submit with your source code.

State your name, student ID and any challenges you encounter or attempts you try. A good report may grant you bonus of extra 5%.

File Structure

All of your source code files must be placed under `inc/` and `src/`.

Let's say you only have two implementation files and use the functions provided in `hmm.h`. You should put your source code under `src/` and leave `hmm.h` in `inc/`.

```
.
├── inc
│   ├── hmm.h
│   └── [other *.h or *.hpp]
├── Makefile
├── model_init.txt
├── report.pdf
└── src
    ├── test.c
    ├── train.c
    └── [other *.c, *.cc or *.cpp]
```

Requirements

Submission Requirement

Submission Requirement 1/2

1. Create a directory named `hw1_[STUDENT_ID]`[4].
2. Put
   - `inc/`
   - `Makefile`
   - `model_init.txt`
   - `report.pdf`
   - `src/`

   into the directory. [5]
3. Compress the directory into a ZIP file named `hw1_[STUDENT_ID].zip`.
4. Upload this ZIP file to CEIBA.

[4] lowercase
[5] Put every source code file in `inc/` and `src/`.

Submission Requirement 2/2

Let's say your student ID is `r01234567`.

```
hw1_r01234567.zip
└── hw1_r01234567
    ├── inc
    │   └── [*.h or *.hpp]
    ├── Makefile
    ├── model_init.txt
    ├── report.pdf
    └── src
        └── [*.c, *.cc or *.cpp]
```

Grading

Grading Method

Your training and testing program will be tested respectively. We will specify `100` as the number of iterations while testing your training program. And each of your program is allowed to run for 1 min.

Here's TA's environment.

Kernel	Linux 5.4.0-48-generic
Processors	Intel Core i7-6700 (8 Cores)
RAM	32 GB
GCC Version	9.3.0

Grading Policy

File Format 20%

- ZIP file name
- directory name
- separated header and implementation files
- `Makefile`
- `model_init.txt`

Program 20%

- compiled and executed without error
- output files generated after execution

Report 10%

and bonus of extra 5% for the impressive ones

Accuracy 50%

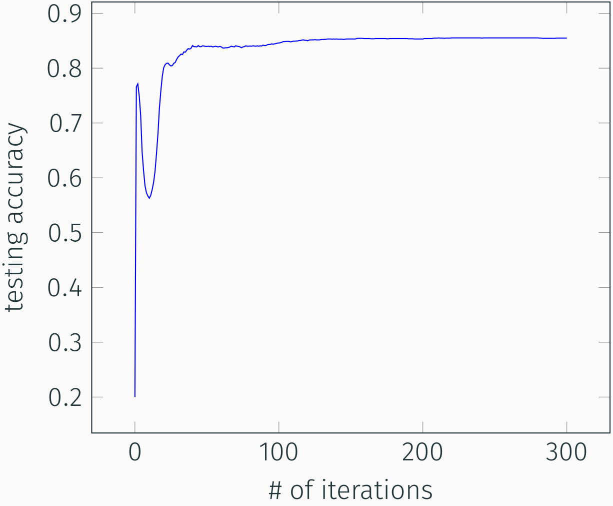
30% for provided test set, 20% for private test set[6]

[6] details are provided at the next page

Accuracy vs Credit

Accuracy	Credit
$80\% \leq \text{accuracy}$	100%
$60\% \leq \text{accuracy} < 80\%$	75%
$40\% \leq \text{accuracy} < 60\%$	50%
$\text{accuracy} < 40\%$	0%

Test Accuracy v.s. # of Training Iterations



Late Submission

Due on 23:59, November 6, 2020

You are still allowed to submit after the due date. The penalty for late submission is an exponential decay with decay rate 1.5%[7] of the maximum grade applicable for the assignment, for each hour that the assignment is late.

An assignment submitted more than 3 days after the deadline will have a grade of zero recorded for that assignment.

$$\mathrm{SCORE}_{final}(hr) = \begin{cases} \mathrm{SCORE}_{original} \times 0.985^{hr} & , \ hr \leq 72 \\ 0 & , \ hr > 72 \end{cases}$$

[7] less than 70% after 24 hrs, 48% for 48 hrs and 33% for 72 hrs

Please Note...

File Format

- All of your source code files should be placed under `inc/` and `src/`.
- `model_init.txt` must be submitted, even if it's not needed for your program.

Program

- Only `C/C++` is allowed.
- Make sure your program can be compiled with the `Makefile` you submit.
- The paths in command-line arguments have to be relative path.
- Each of your program is allowed to run for 1 min.

Accuracy

- Make sure your training program saves models within time limit.
- Public and private test set come from same models.

If you have any questions, please read the FAQ[8] first.

[8] http://speech.ee.ntu.edu.tw/DSP2020Autumn/hw1/FAQ.html

DO NOT CHEAT

Any form of cheating, lying, or plagiarism will not be tolerated.

Questions?

Contact TAs

If you have any question or need help,

- send email to *ntu-dsp-2020-ta@googlegroups.com*
- and use “**`[HW1]`**” as the subject line prefix

Or come to EE2 R531, and don’t forget to inform us by email, thanks!

蔡翔陞	Thr.	9:00 - 12:00

Office hours