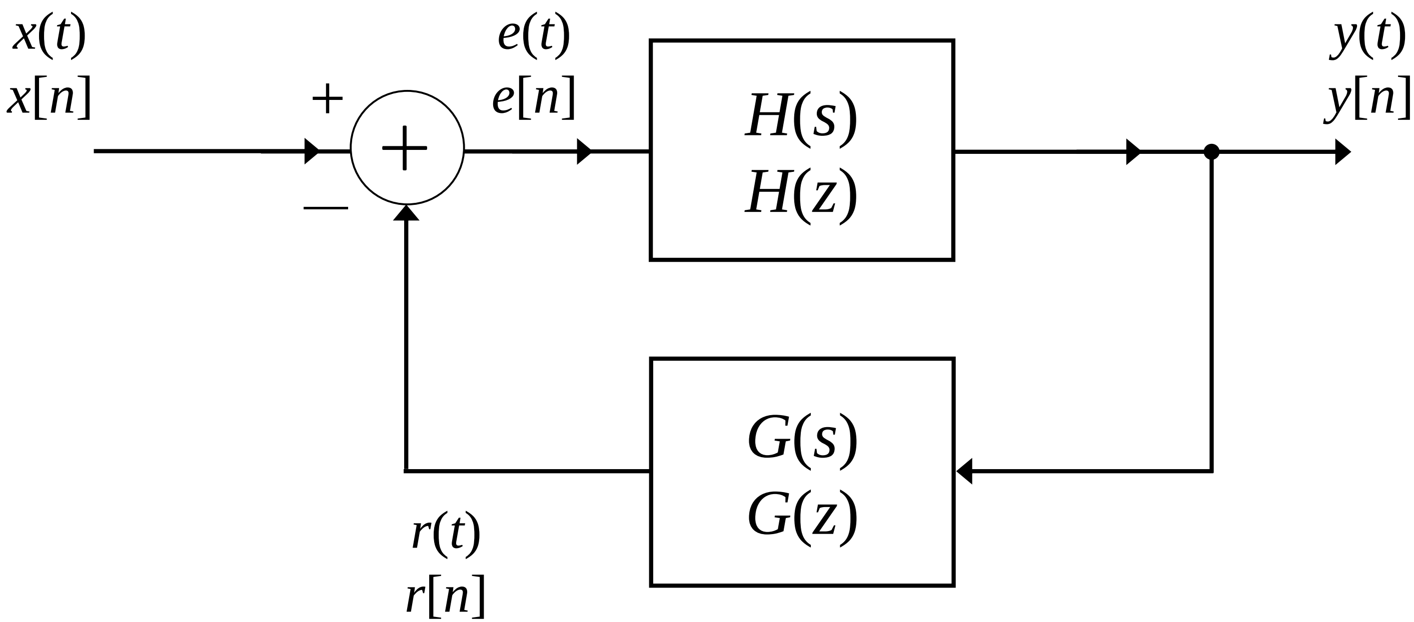
11.0 Linear Feedback Systems

11.1 Basic Principles of Linear Feedback



11.0 Linear Feedback Systems

11.1 Basic Principles of Linear Feedback

$H(s)$, $H(z)$: forward path system function

$G(s)$, $G(z)$: feedback path system function

$Q(s)$, $Q(z)$: close-loop system function

$$Q(s) = \frac{H(s)}{1 + G(s)H(s)}, \quad Q(z) = \frac{H(z)}{1 + G(z)H(z)}$$

Example 1: telescope positioning system

See Fig. 11.1, p.817 of text

close-loop system vs. open-loop system

- insensitivity to precise knowledge of the open-loop system characteristics
- insensitivity to disturbances
- tolerance to errors

Example 2: autopilot system for airplanes

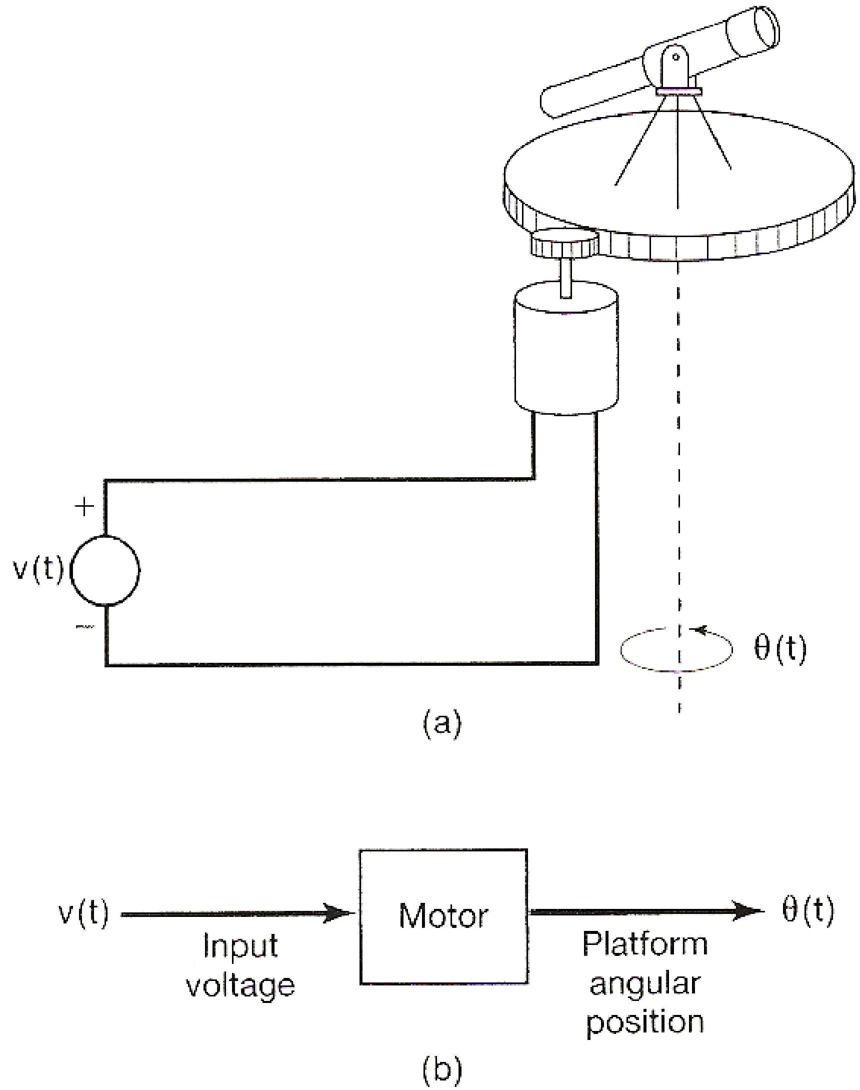
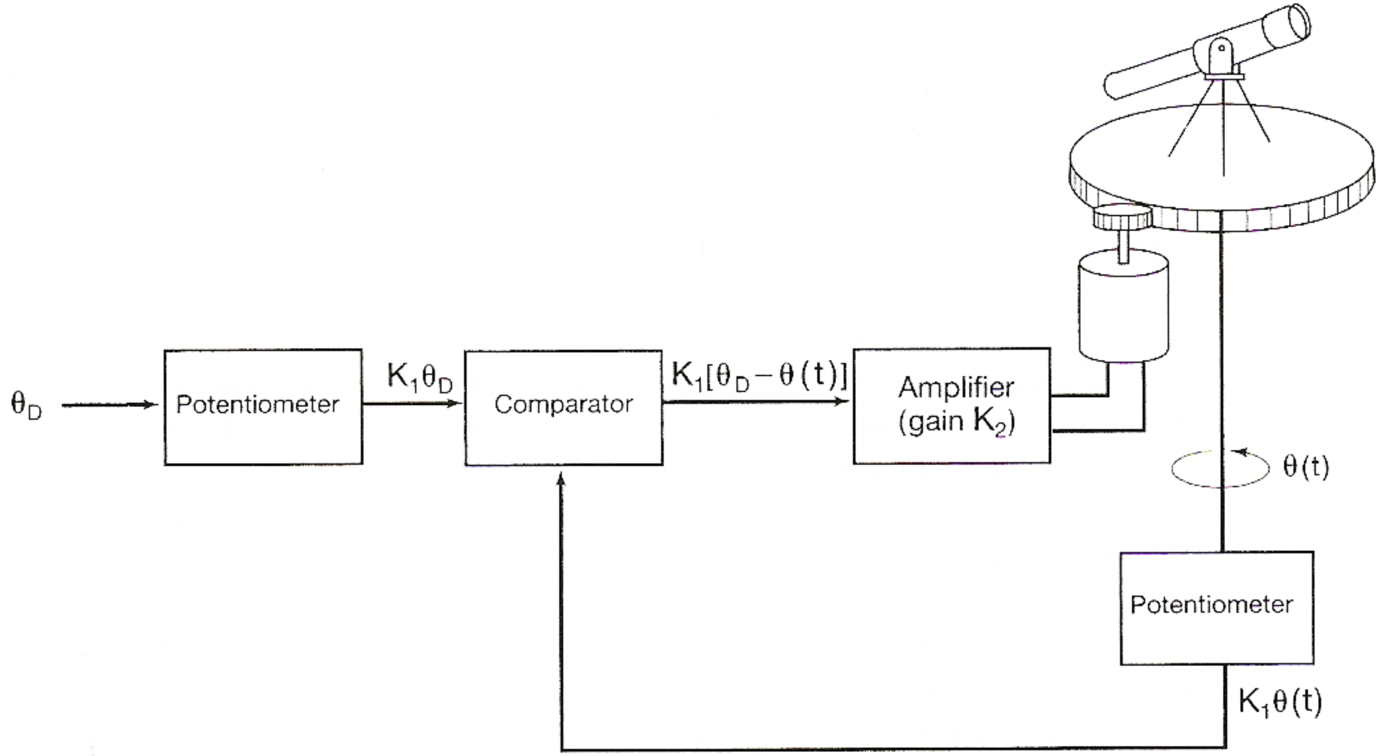
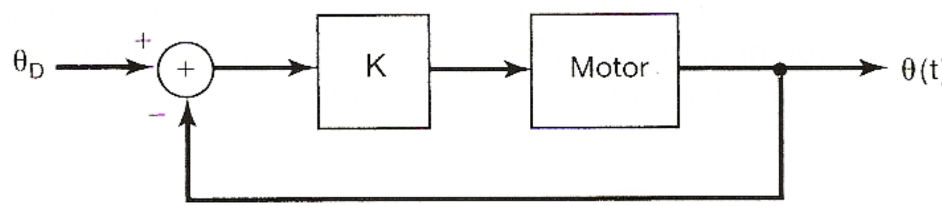


Figure 11.1 Use of feedback to control the angular position of a telescope: (a) dc motor-driven telescope platform; (b) block diagram of the system in (a); (c) feedback system for pointing the telescope; (d) block diagram of the system in (c) (here, $K = K_1K_2$).



(c)



(d)

Figure 11.1 Use of feedback to control the angular position of a telescope: (a) dc motor-driven telescope platform; (b) block diagram of the system in (a); (c) feedback system for pointing the telescope; (d) block diagram of the system in (c) (here, $K = K_1 K_2$).

- Stablization of Unstable Systems

Example 3:

$$H(s) = \frac{b}{s-a}, \text{ unstable if } a > 0$$

$$Q(s) = \frac{H(s)}{1 + KH(s)} = \frac{b}{s - a + Kb}$$

$$\text{stable if } K > \frac{a}{b}$$

- Stablization of Unstable Systems

Example 4:

$$H(s) = \frac{b}{s^2 + a}, \text{ unstable}$$

$$G(s) = K_1 + K_2 s$$

$$\text{stable if } \ bK_2 > 0, \ a + K_1 b > 0$$

- Destablization Caused by Feedback

Example 5: audio feedback

See Fig. 11.8, p.831 of text

$$\text{unstable if } K_1 K_2 > 1$$

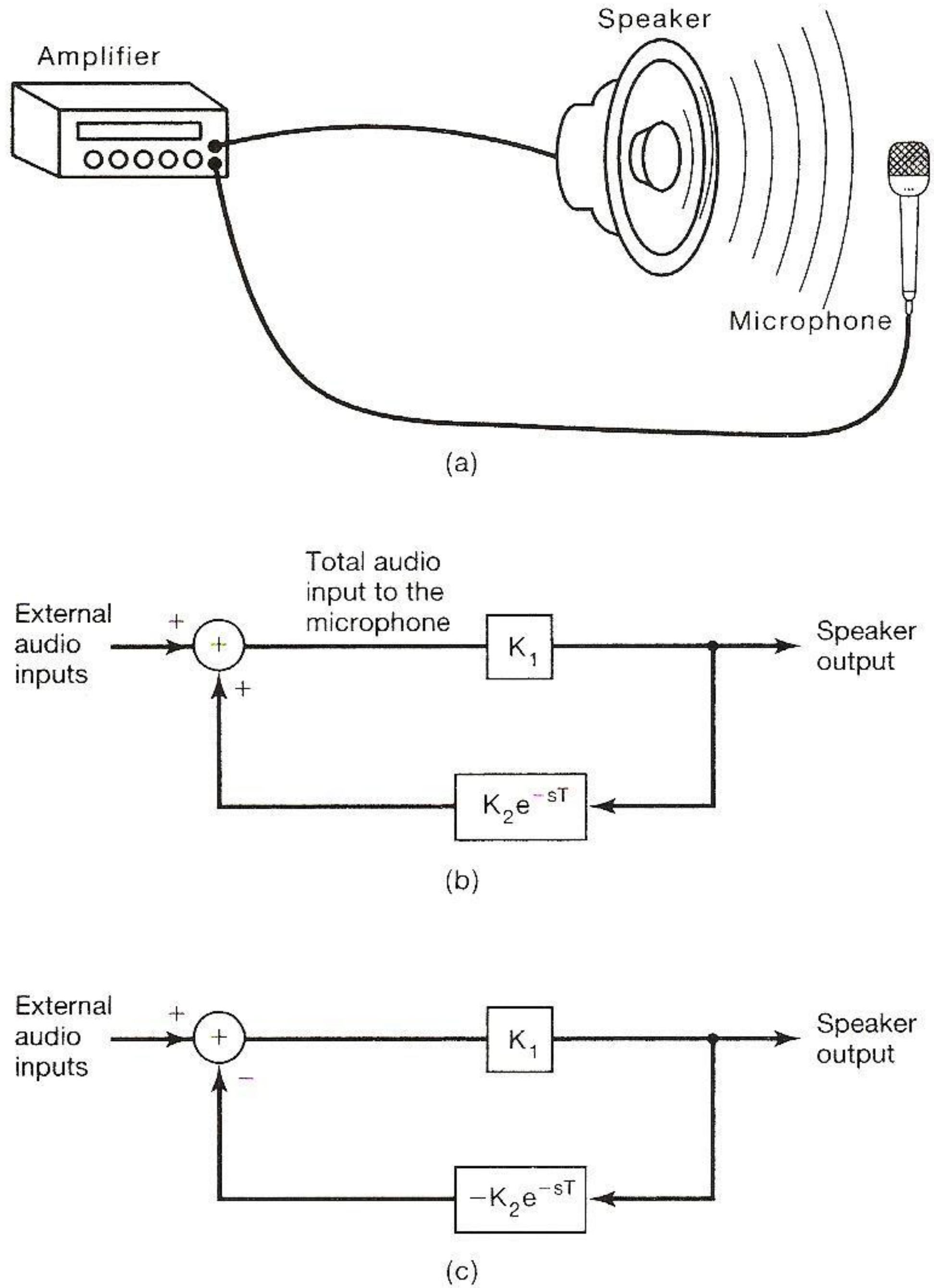


Figure 11.8 (a) Pictorial representation of the phenomenon of audio feedback; (b) block diagram representation of (a); (c) block diagram in (b) redrawn as a negative feedback system. (*Note:* e^{-sT} is the system function of a T-second time delay.)

- Sampled-Data Feedback System

  discrete-time feedback for continuous-time systems

  See Fig. 11.6, p.827 of text

- The Adjustable Gain can be on either arm

  See Fig. 11.10, p.835 of text

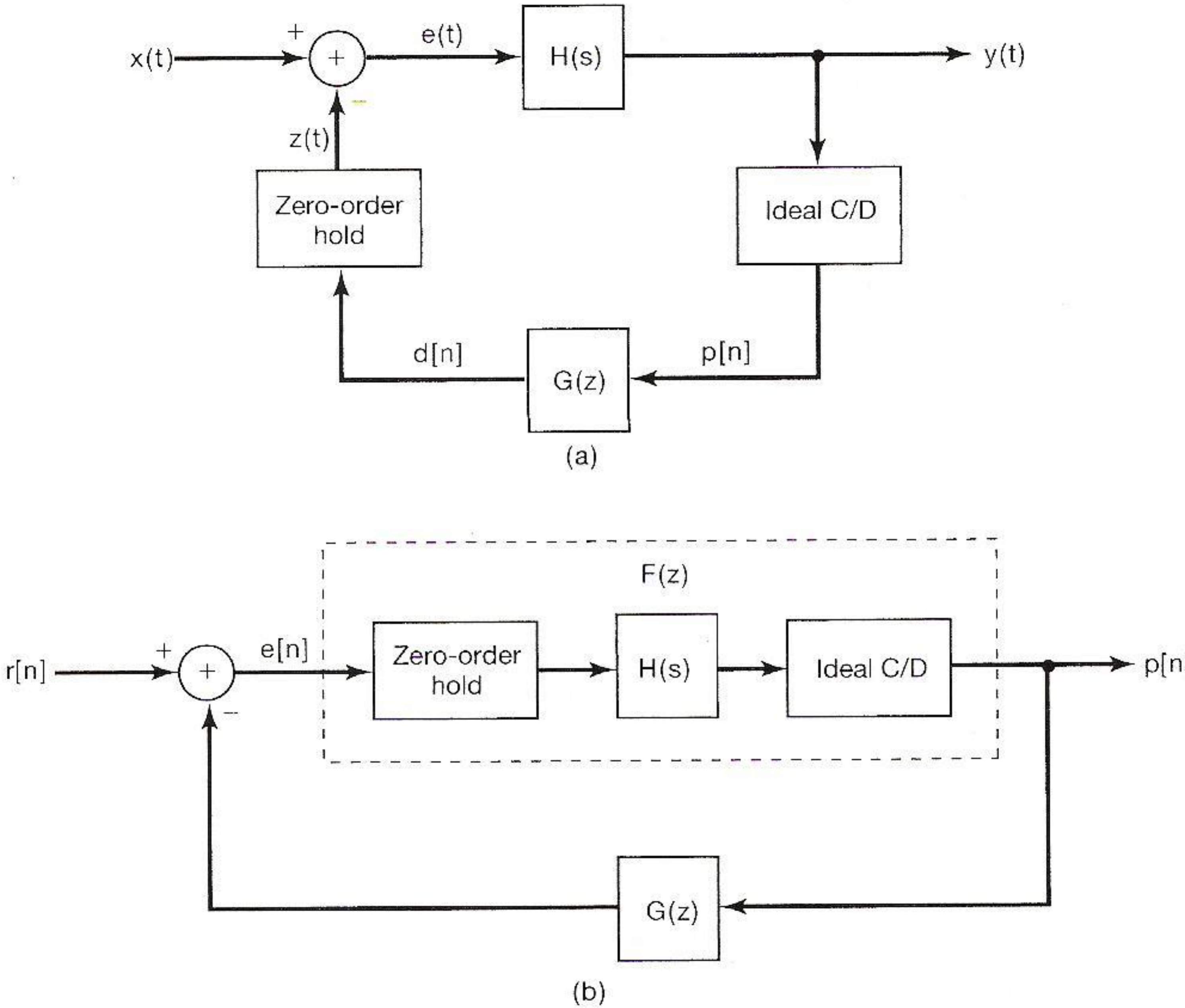
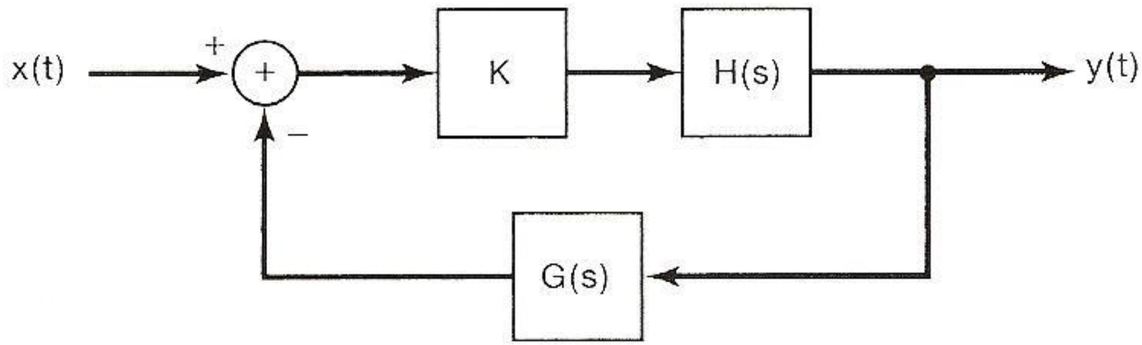
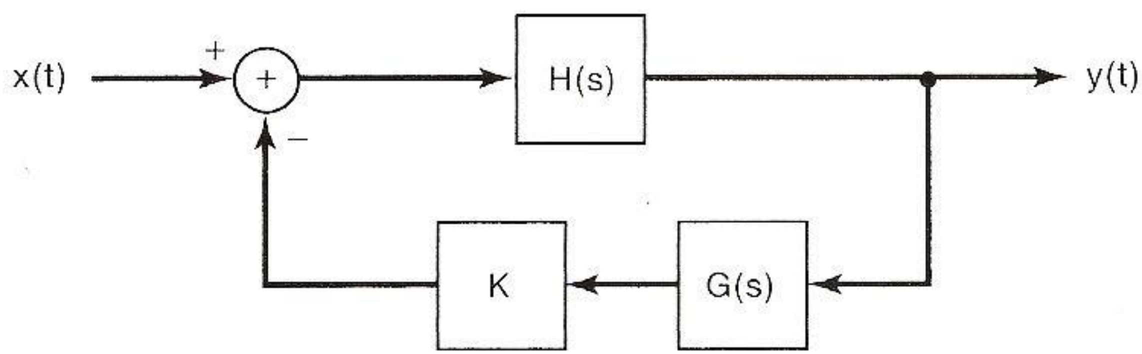


Figure 11.6 (a) A sampled-data feedback system using a zero-order hold; (b) equivalent discrete-time system.



$$Q(s) = \frac{KH(s)}{1 + KH(s)G(s)}$$

(a)



$$Q(s) = \frac{H(s)}{1 + KH(s)G(s)}$$

(b)

Figure 11.10 Feedback systems containing an adjustable gain: (a) system in which the gain is located in the forward path; (b) system with the gain in the feedback path.

- Root-locus Analysis for Feedback Systems
  - the locus (path) for the poles of the close-loop system when an adjustable gain is varied

<u>Example 6:</u>

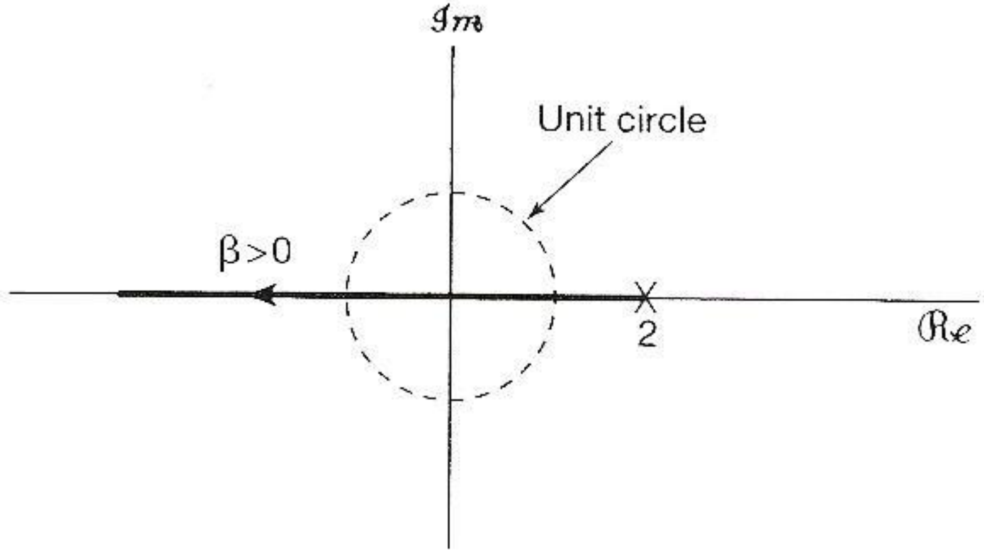
$$H(z) = \frac{1}{1-2z^{-1}} = \frac{z}{z-2}$$

$$G(z) = 2\beta z^{-1} = \frac{2\beta}{z}$$

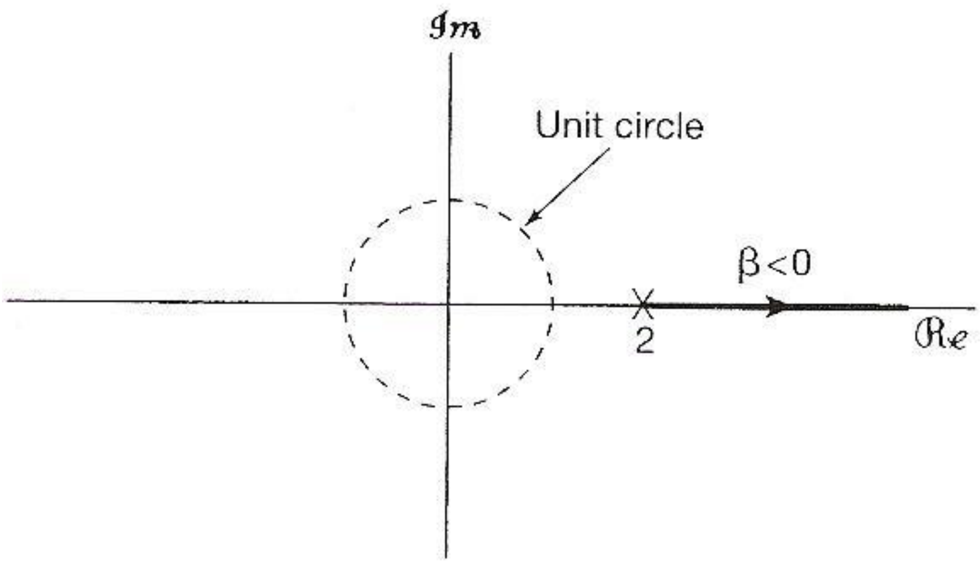
$$Q(z) = \frac{1}{1-2(1-\beta)z^{-1}} = \frac{z}{z-2(1-\beta)}$$

pole at $z = 2(1-\beta)$

See Fig. 11.9, p.834 of text



(a)



(b)

Figure 11.9 Root locus for the closed-loop system of eq. (11.40) as the value of β is varied: (a) $\beta > 0$; (b) $\beta < 0$. Note that we have marked the point $z = 2$ that corresponds to the pole location when $\beta = 0$.

- Root-locus Analysis for Feedback Systems
  - the locus (path) for the poles of the close-loop system when an adjustable gain is varied

<u>Example 7:</u>

$$H(z)=\frac{K}{1-\frac{1}{2}z^{-1}}, \quad G(z)=\frac{z^{-1}}{z-\frac{1}{4}z^{-1}}$$

See Fig. 11.16, p.846 of text

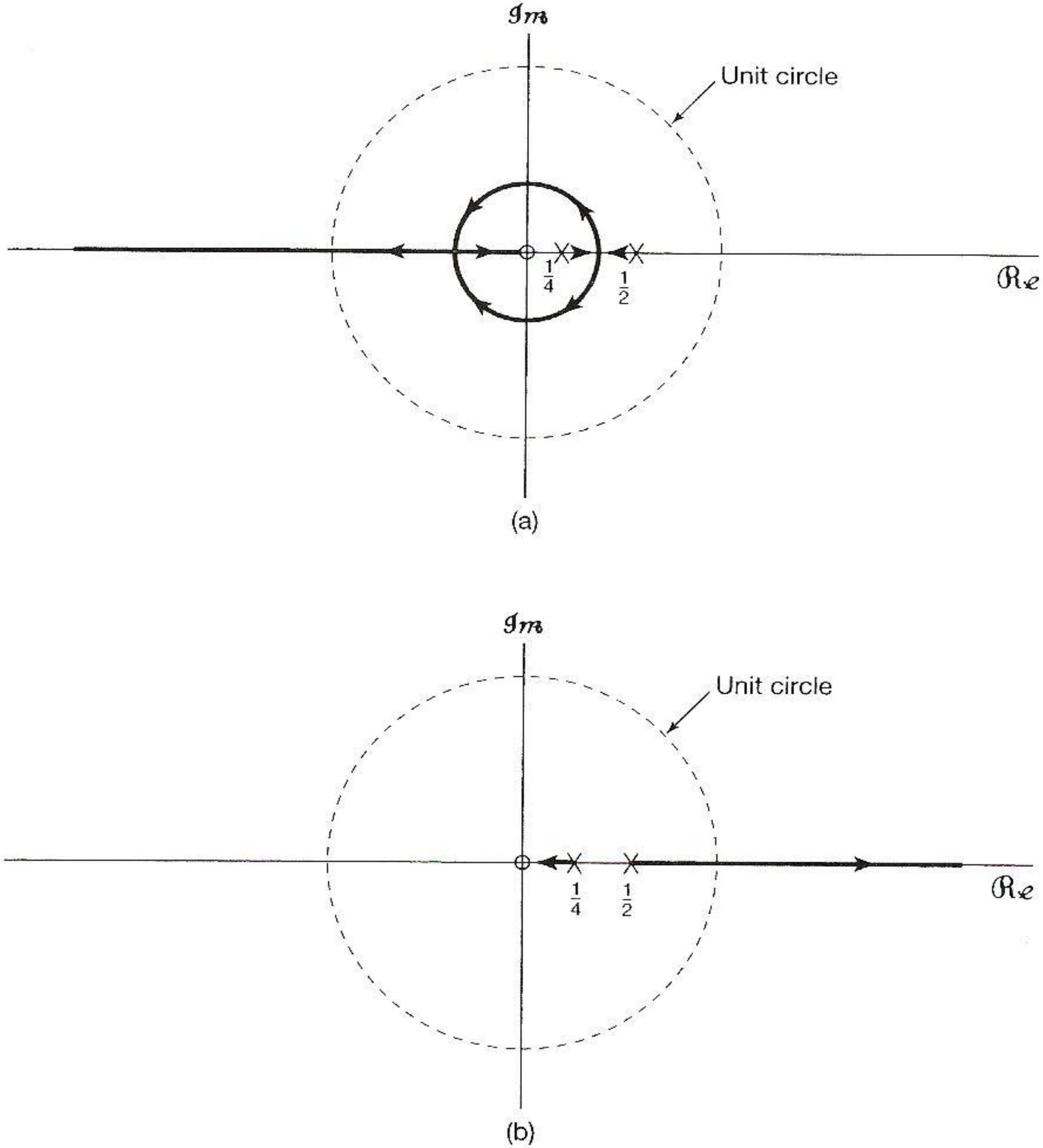


Figure 11.16 Root locus for Example 11.3: (a) $K > 0$; (b) $K < 0$. The poles of $G(z)H(z)$ at $z = 1/4$ and $z = 1/2$ and the zero of $G(z)H(z)$ at $z = 0$ are indicated in the figure.

END Of THIS COURSE