6.0 Time/Frequency Characterization of Signals/Systems

6.1 Magnitude and Phase for Signals and Systems

$$x(t),\ X(j\omega) \quad h(t),\ H(j\omega) \quad y(t),\ Y(j\omega)$$

$$x[n],\ X(e^{j\omega}) \quad h[n],\ H(e^{j\omega}) \quad y[n],\ Y(e^{j\omega})$$

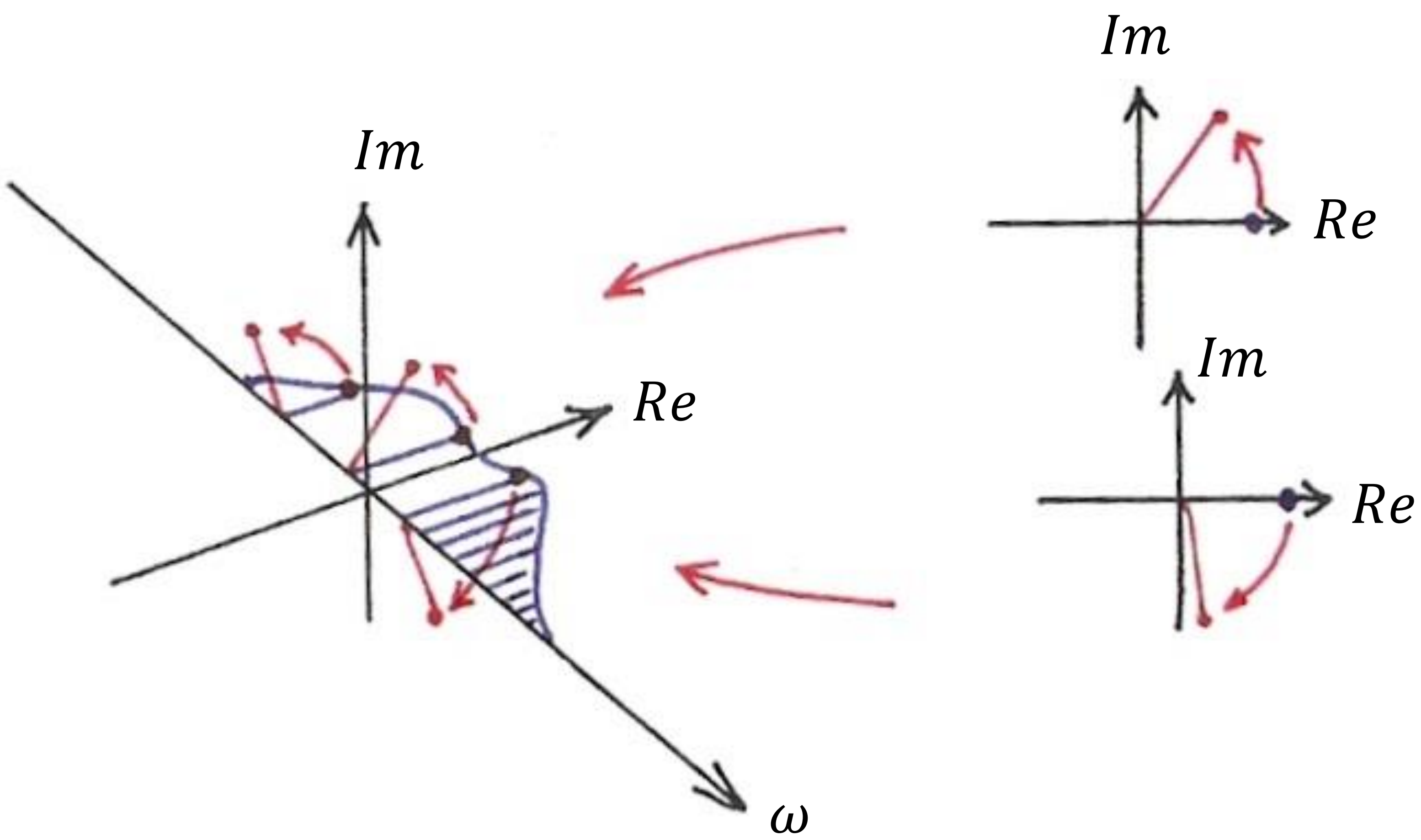
Signals

$$X(j\omega) = |X(j\omega)| e^{j\angle X(j\omega)}$$

$|X(j\omega)|$: magnitude of each frequency component

$\angle X(j\omega)$: phase of each frequency component

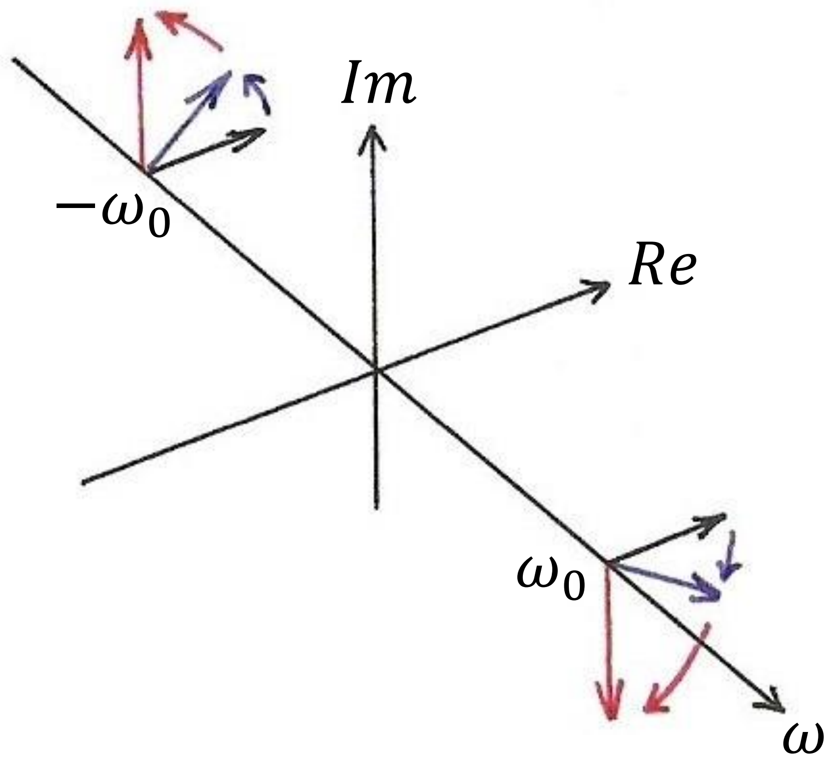
Time Shift (P.23 of 4.0)

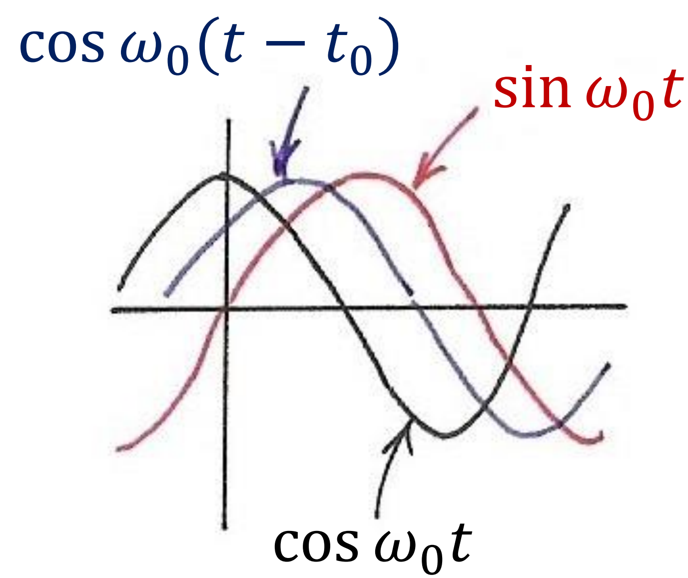


Sinusoidals (p.25 of 4.0)

$$\cos\omega_0 t \overset{F}{\leftrightarrow} \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)],\ \ \tfrac{1}{2}[e^{j\omega_0 t}+e^{-j\omega_0 t}]$$

$$\sin\omega_0 t \overset{F}{\leftrightarrow} \frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)],\ \tfrac{1}{2j}[e^{j\omega_0 t}-e^{-j\omega_0 t}]$$





$$x(t-t_0) \leftrightarrow e^{-j\omega_0 t}\cdot X(j\omega)$$

Signals

- Example

$$X(t) = 1 + \frac{1}{2}\cos(2\pi t + \phi_1) + \cos(4\pi t + \phi_2) + \frac{2}{3}\cos(6\pi t + \phi_3)$$

See Fig. 3.4, p.188 and Fig. 6.1, p.425 of text

- change in phase leads to change in time-domain characteristics
- human auditory system is relatively insensitive to phase of sound signals

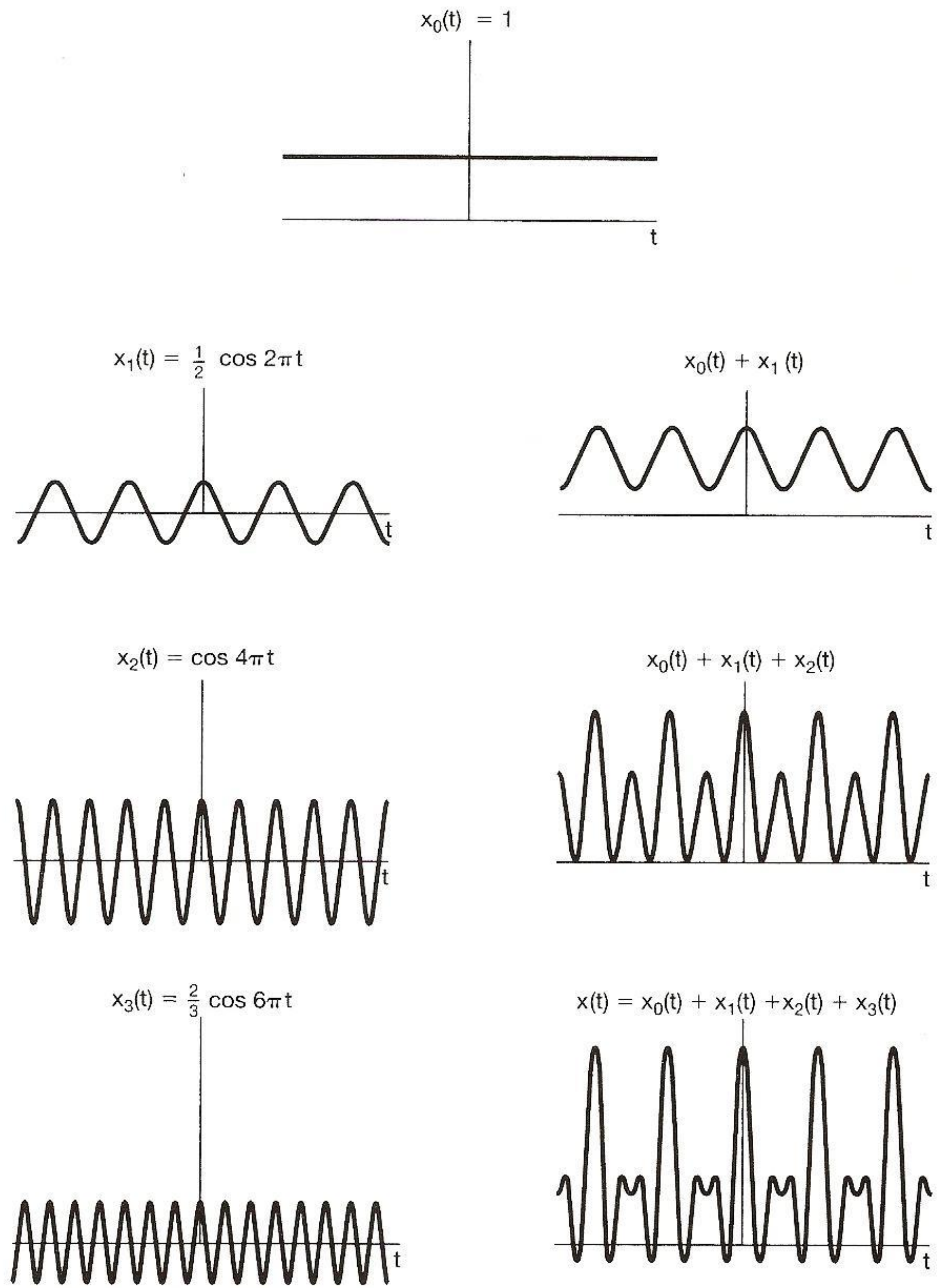


Figure 3.4 Construction of the signal $x(t)$ in Example 3.2 as a linear combination of harmonically related sinusoidal signals.

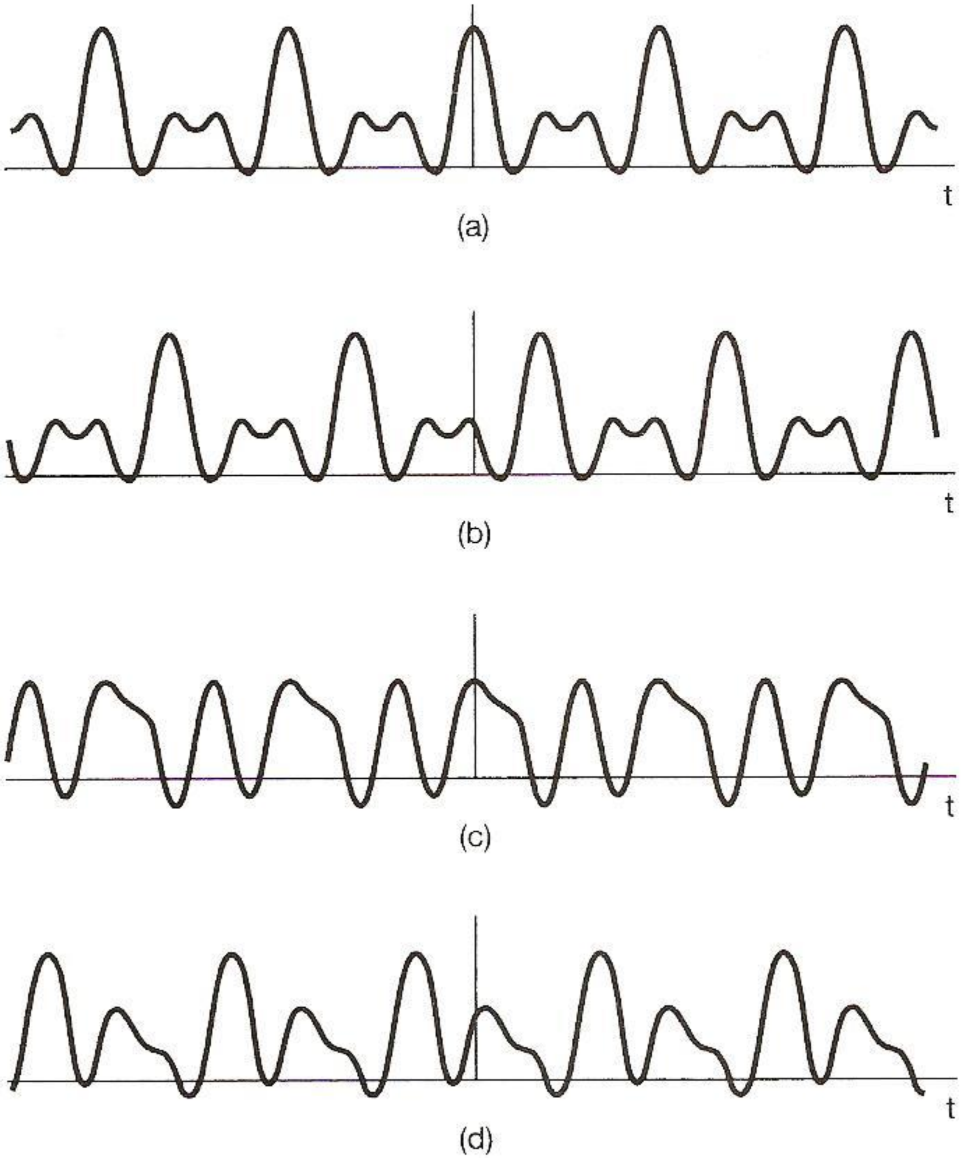


Figure 6.1 The signal $x(t)$ given in eq. (6.3) for several different choices of the phase angles ϕ_1, ϕ_2, and ϕ_3: (a) $\phi_1 = \phi_2 = \phi_3 = 0$; (b) $\phi_1 = 4$ rad, $\phi_2 = 8$ rad, $\phi_3 = 12$ rad; (c) $\phi_1 = 6$ rad, $\phi_2 = -2.7$ rad, $\phi_3 = 0.93$ rad; (d) $\phi_1 = 1.2$ rad, $\phi_2 = 4.1$ rad, $\phi_3 = -7.02$ rad.

Signals

- Example : pictures as 2-dim signals

  $x(t_1, t_2)$: 2 - dim signals

$$X(j\omega_1, j\omega_2) = \int\int_{-\infty}^{\infty} x(t_1, t_2) e^{-j(\omega_1 t_1 + \omega_2 t_2)} dt_1 dt_2$$

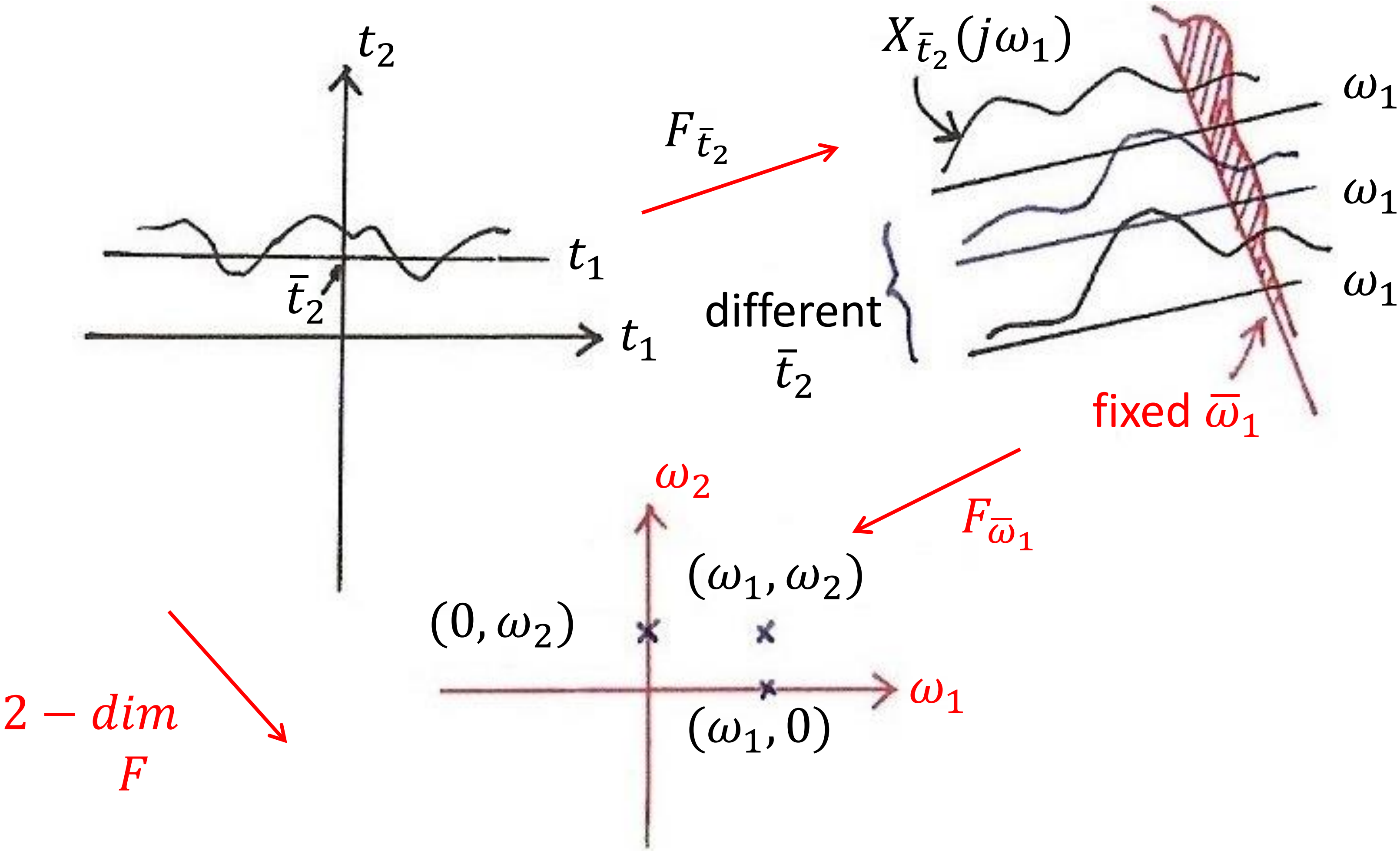
$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(t_1, t_2) e^{-j\omega_1 t_1} dt_1 \right] e^{-j\omega_2 t_2} dt_2$$

$$X_{t_2}(j\omega_1)$$

  - most important visual information in edges and regions of high contrast
  - regions of max/min intensity are where different frequency components are in phase

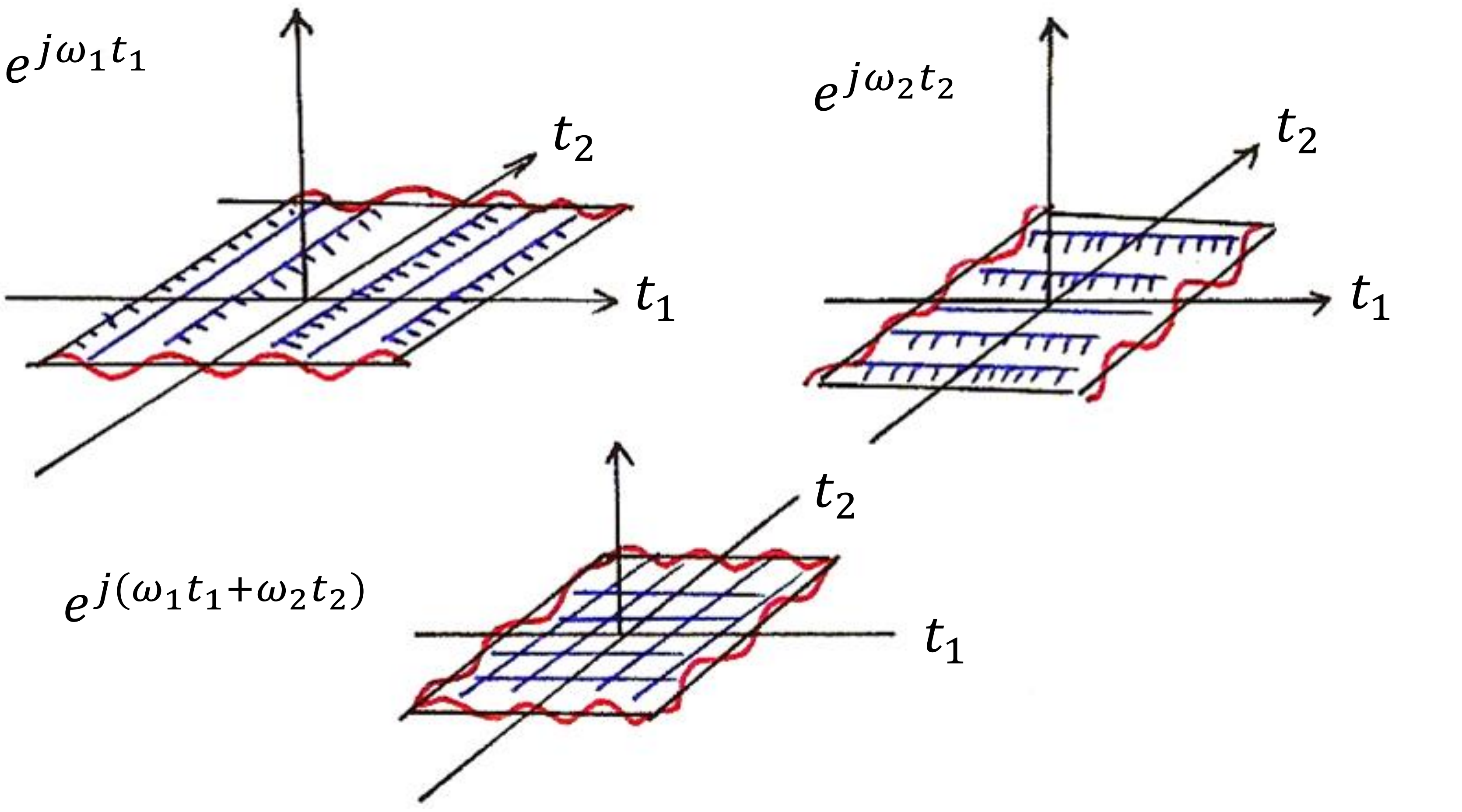
See Fig. 6.2, p.426~427 of text

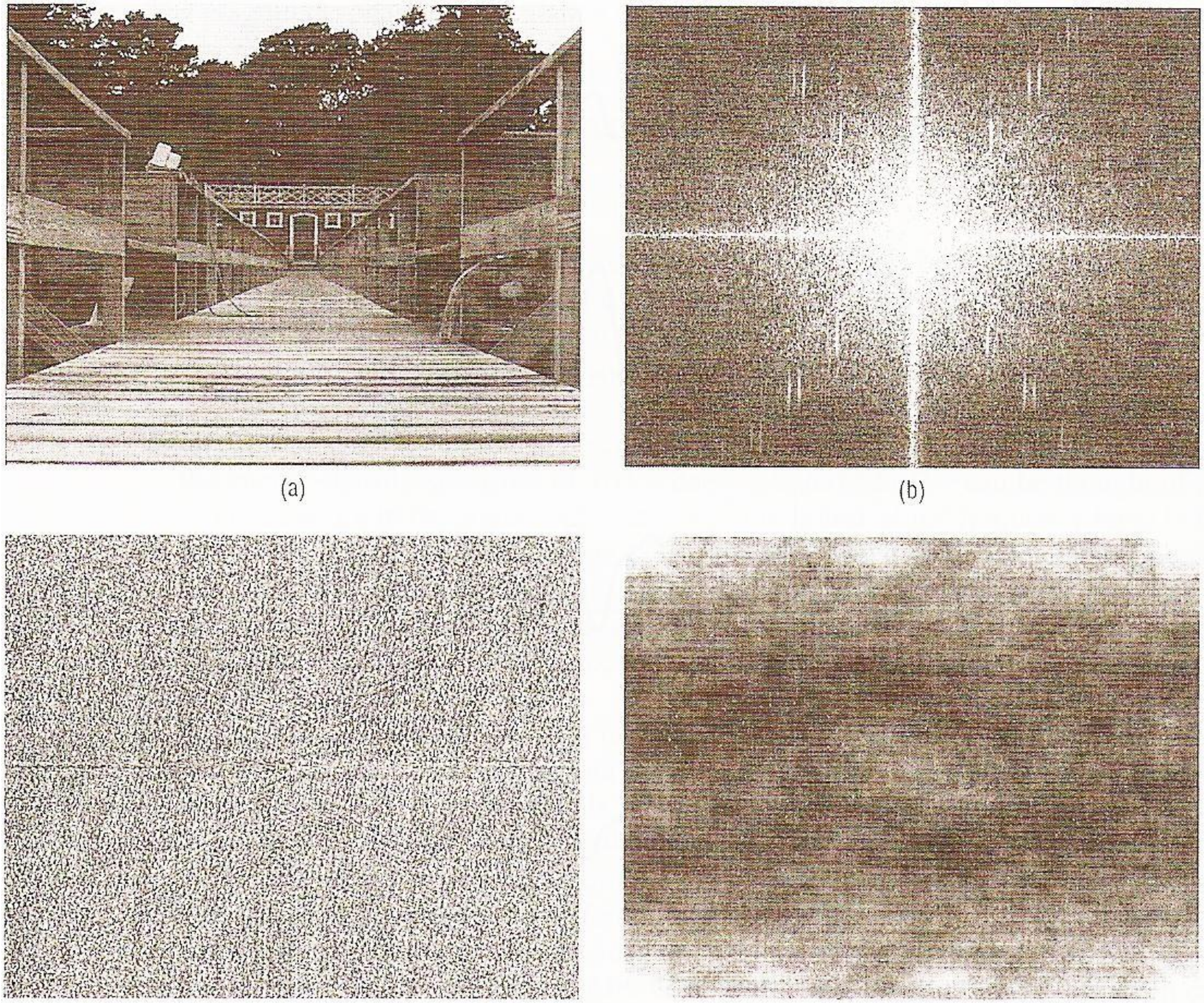
Two-dim Fourier Transform



<u>Problem 3.70</u>, p.281 of text (P.65 of 3.0)

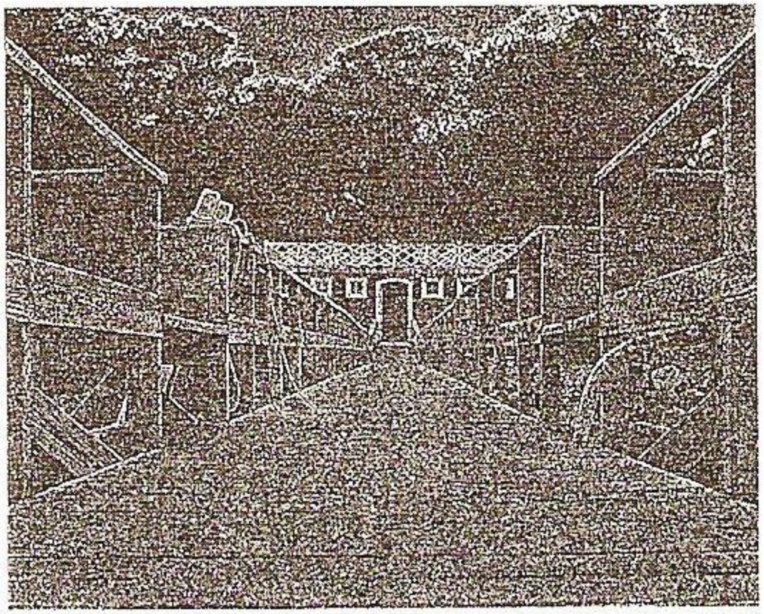
- 2-dimensional signals



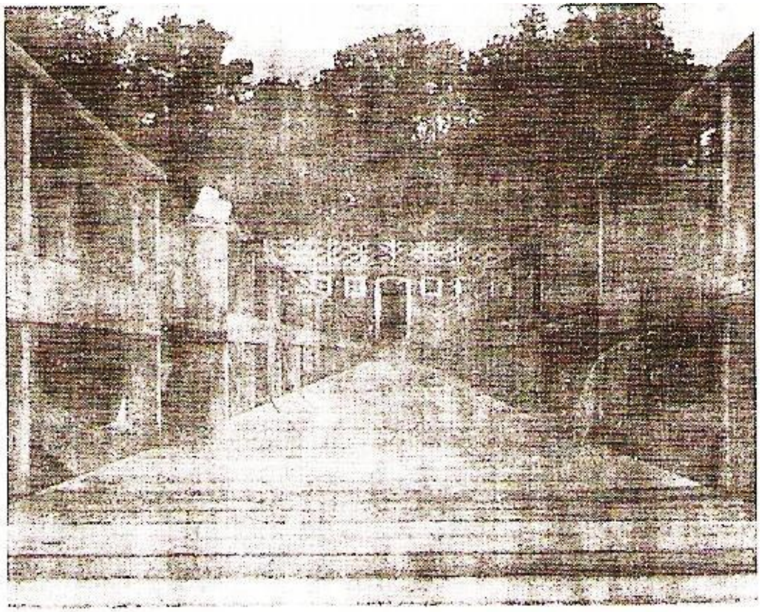


(a) (b)

(c) (d)



(e)



(f)

(g)

Figure 6.2 (a) The image shown in Figure 1.4; (b) magnitude of the two-dimensional Fourier transform of (a); (c) phase of the Fourier transform of (a); (d) picture whose Fourier transform has magnitude as in (b) and phase equal to zero; (e) picture whose Fourier transform has magnitude equal to 1 and phase as in (c); (f) picture whose Fourier transform has phase as in (c) and magnitude equal to that of the transform of the picture shown in (g).

Systems

$$|Y(j\omega)| = |X(j\omega)||H(j\omega)|$$

$$\log|Y(j\omega)| = \log|X(j\omega)| + \log|H(j\omega)|$$

$$\angle Y(j\omega) = \angle X(j\omega) + \angle H(j\omega)$$

$|H(j\omega)|$: scaling of different frequency components

$\angle H(j\omega)$: phase shift of different frequency components

Systems

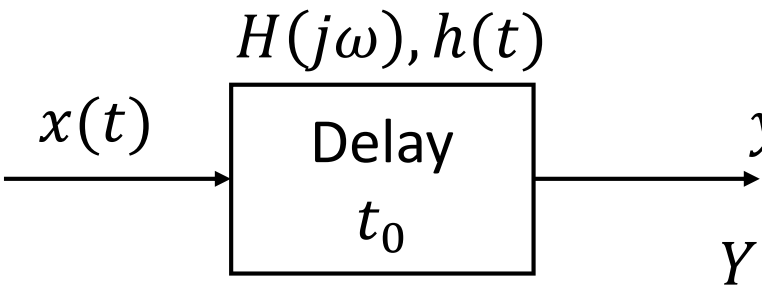
- Linear Phase (phase shift linear in frequency) means constant delay for all frequency components

$$y(t) = x(t - t_0) \rightarrow H(j\omega) = e^{-j\omega t_0},\ \angle H(j\omega) = -\omega t_0$$

$$y[n] = x[n - n_0] \rightarrow H(e^{j\omega}) = e^{-j\omega n_0},\ \angle H(j\omega) = -\omega n_0$$

  - slope of the phase function in frequency is the delay
  - a nonlinear phase may be decomposed into a linear part plus a nonlinear part

Linear Phase for Time Delay

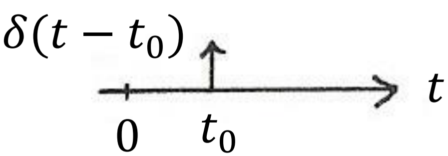


$$y(t) = x(t - t_0)$$

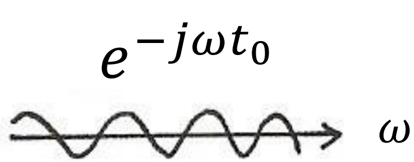
$$Y(j\omega) = e^{-j\omega t_0} \cdot X(j\omega)$$

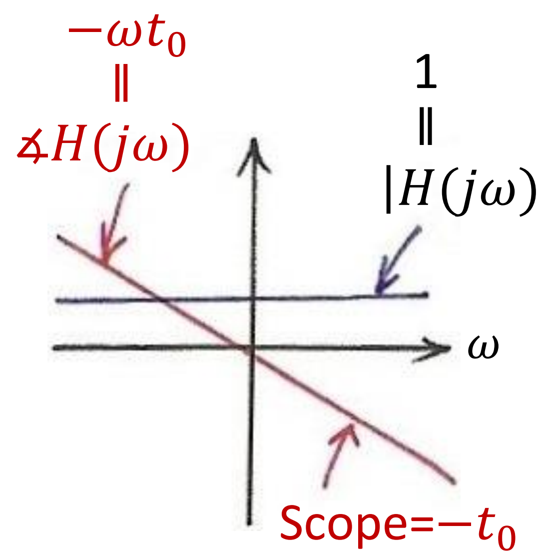
$$H(j\omega) = e^{-j\omega t_0}$$

$$h(t) = \delta(t - t_0)$$



$\overset{F}{\longleftrightarrow}$



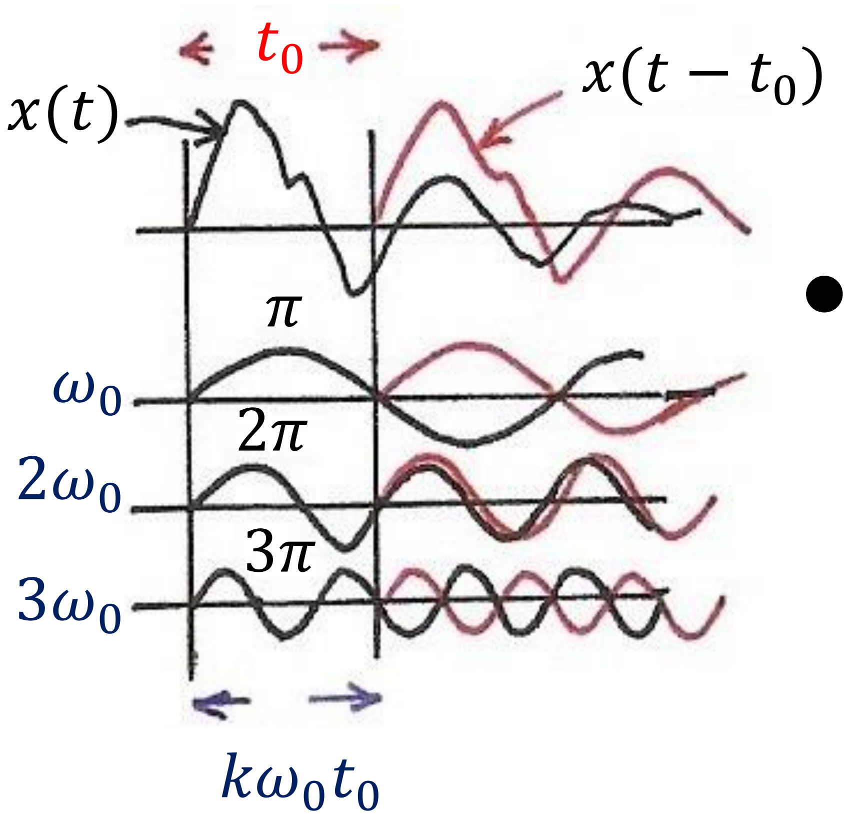


- Time Shift (P.28 of 3.0)

$$x(t-t_0) \stackrel{FS}{\longleftrightarrow} e^{-jk\omega_0 t_0} a_k$$

phase shift linear in frequency with amplitude unchanged

$$a_k\, e^{jk\omega_0(t-t_0)} = \boxed{e^{-jk\omega_0 t_0}\, a_k}\, e^{jk\omega_0 t}$$

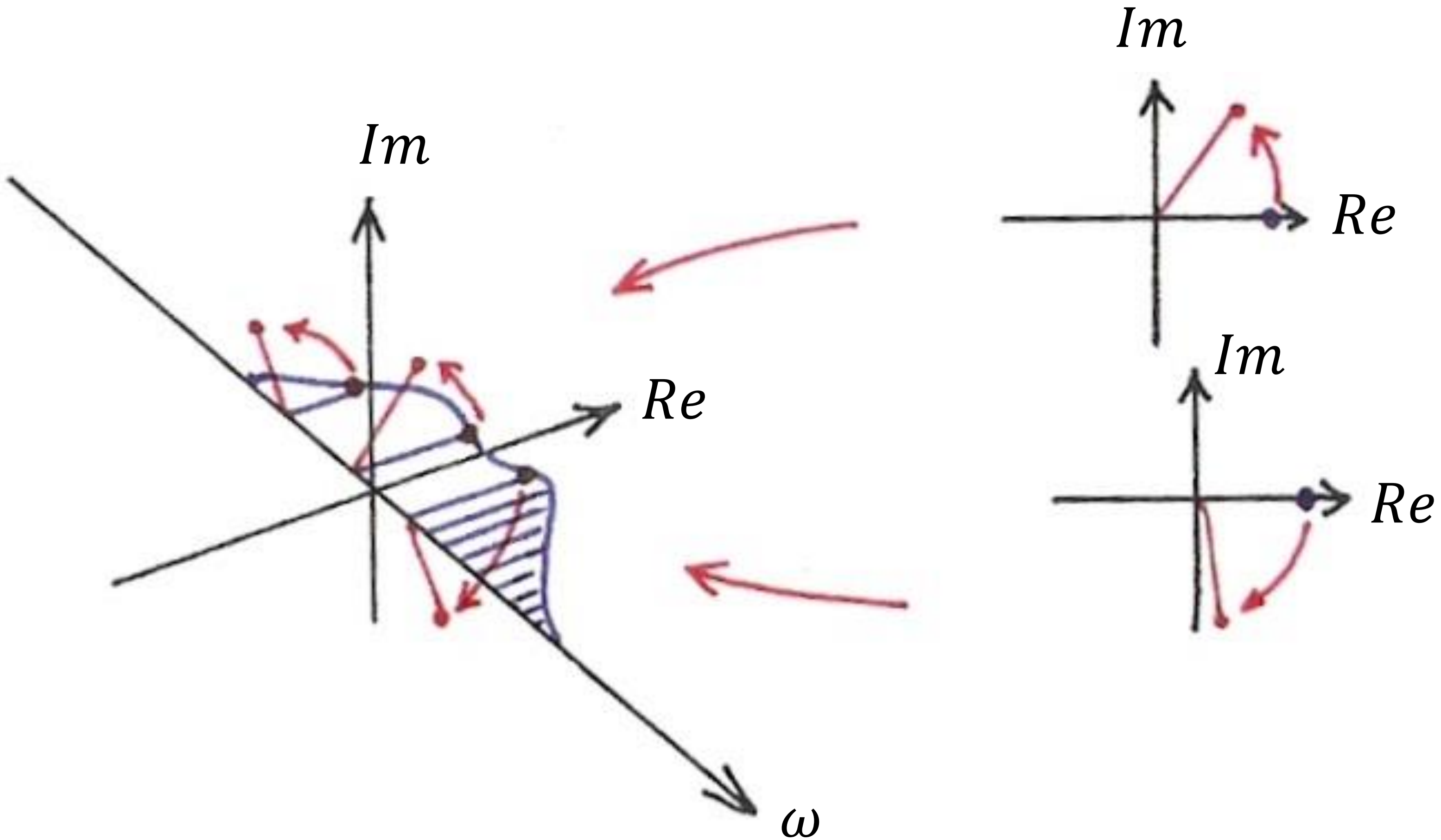


- Time Shift

$$x(t-t_0) \stackrel{F}{\longleftrightarrow} e^{-j\omega t_0} X(j\omega)$$

linear phase shift (linear in frequency) with amplitude unchanged

Time Shift (P.23 of 4.0)

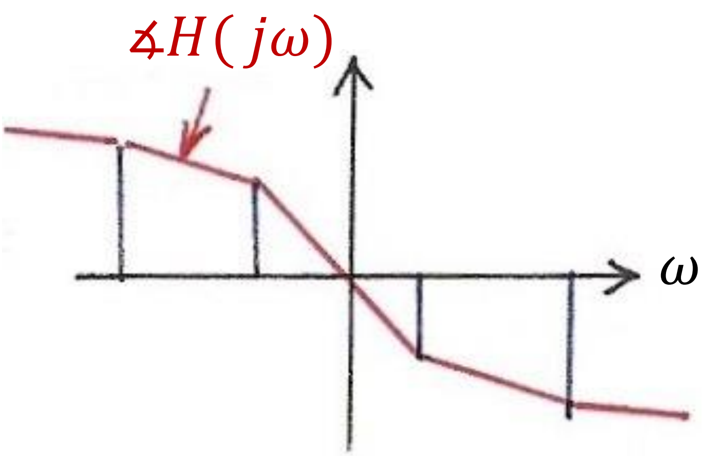


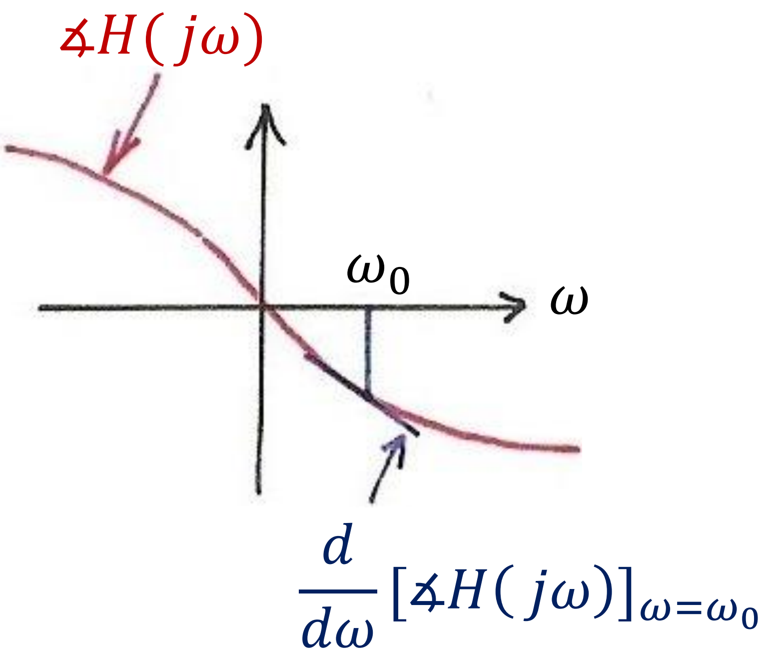
Systems

- Group Delay at frequency ω

$$\tau(\omega) = -\frac{d}{d\omega}\left[\angle H(j\omega)\right]$$

common delay for frequencies near ω





Systems

- Examples
  - *See Fig. 6.5, p.434 of text*

    Dispersion : different frequency components delayed by different intervals

    later parts of the impulse response oscillates at near 50Hz
  - Group delay/magnitude function for Bell System telephone lines

    See Fig. 6.6, p.435 of text

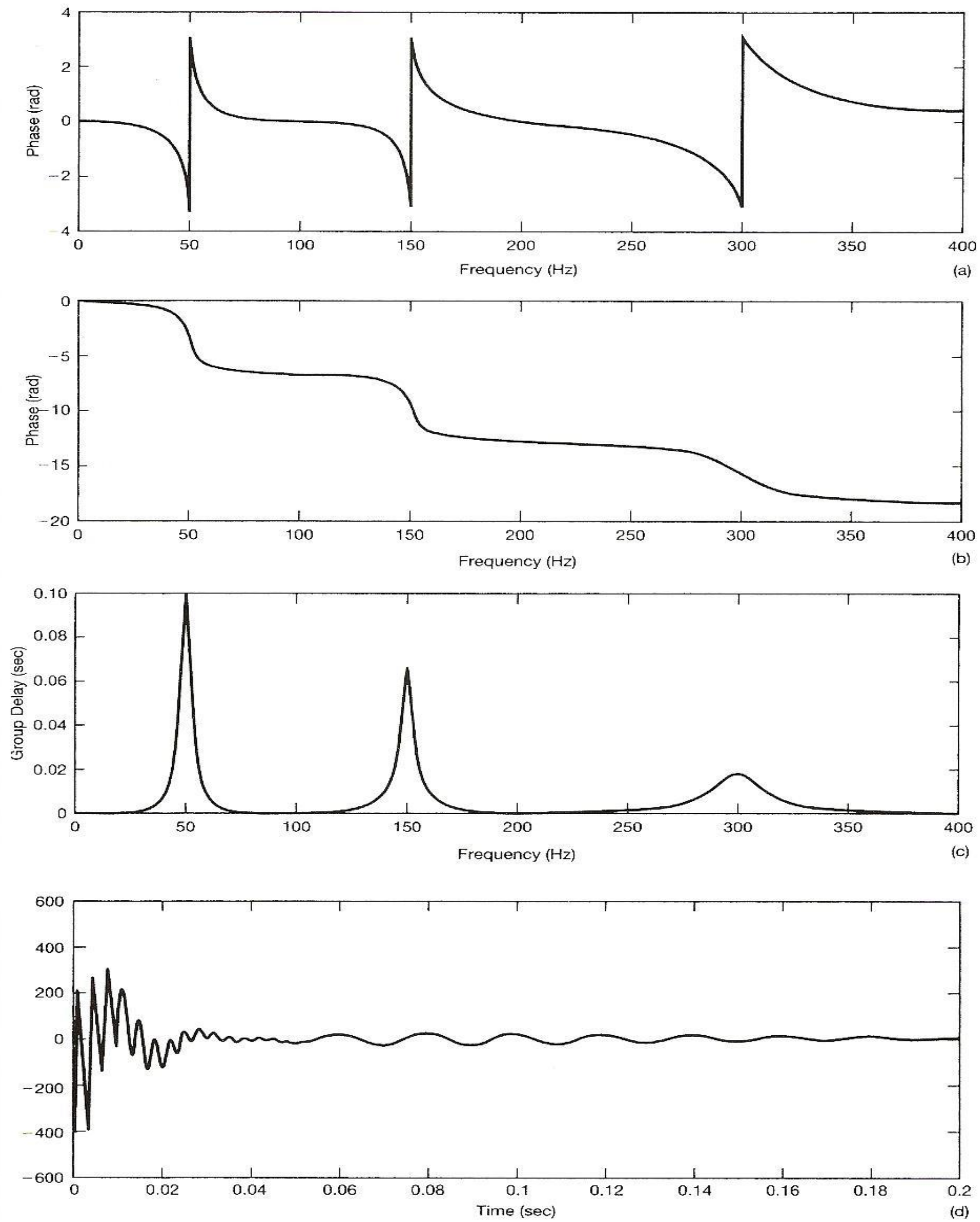


Figure 6.5 Phase, group delay, and impulse response for the all-pass system of Example 6.1: (a) principal phase; (b) unwrapped phase; (c) group delay; (d) impulse response. Each of these quantities is plotted versus frequency measured in Hertz.

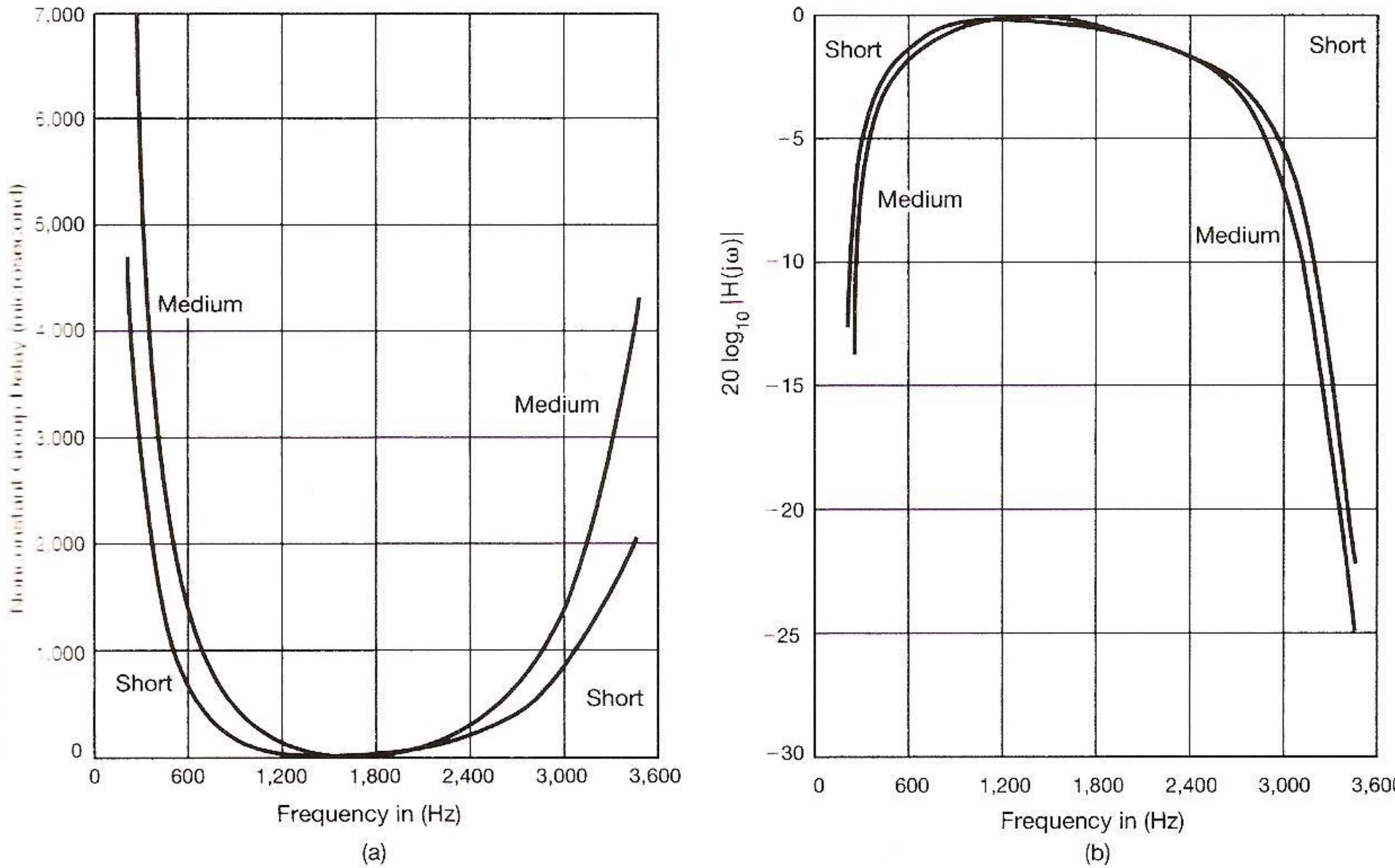


Figure 6.6 (a) Non-constant portion of the group delay; and (b) frequency response magnitude as functions of frequency for short- and medium-distance toll calls in switched telecommunications networks [after Duffy and Thatcher]. Each of these quantities is plotted versus frequency measured in Hertz. Also, as is commonly done in practice, the magnitudes of the frequency responses are plotted using a logarithmic scale in units of *decibels*. That is, what is plotted in (b) is $20\log_{10}|H(j\omega)|$ for the frequency responses corresponding to short- and medium-distance toll calls. The use of this logarithmic scale for the frequency-response magnitudes is discussed in detail in Section 6.2.3.

Systems

- Bode Plots

$$\log\left| H\left(j\omega\right)\right|, \ \angle H\left(j\omega\right) \quad \text{vs.} \ \log\omega$$

- Discrete-time Case

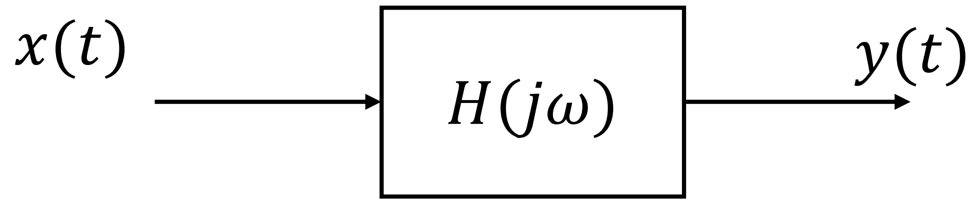
ω not in log scale, finite within [-π, π]

- Condition for Distortionless

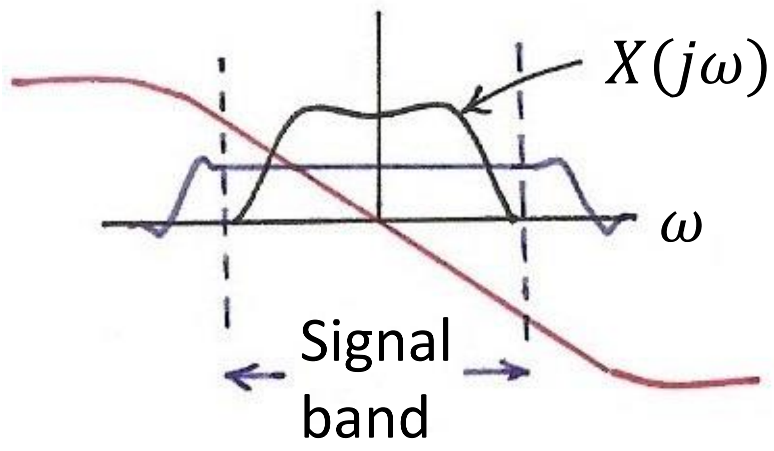
$$y(t) = kx(t - t_0), \qquad y[n] = kx[n - n_0]$$

– Constant magnitude plus linear phase in signal band

Distortionless



$$Y(j\omega) = H(j\omega)\,X(j\omega)$$



6.2 Filtering

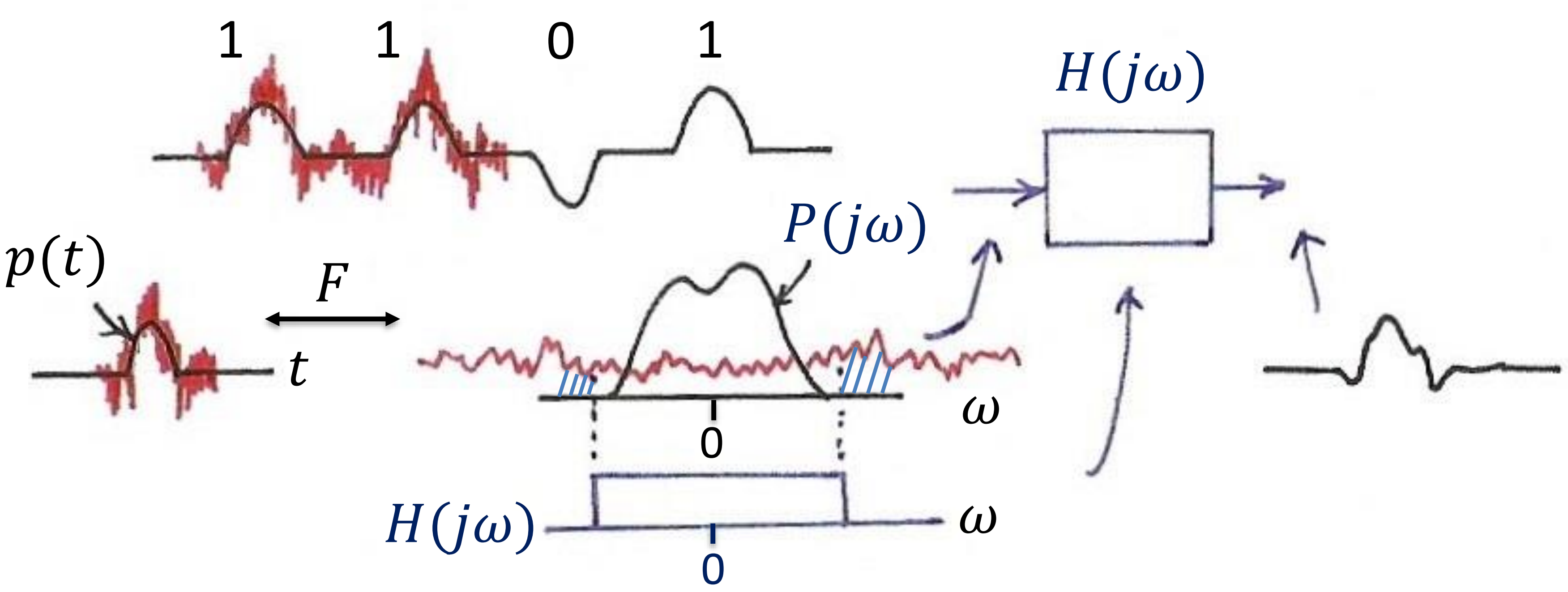
Ideal Filters

- Low pass Filters as an example
  - Continuous-time or Discrete-time

    See Figs. 6.10, 6.12, pp.440,442 of text

    mainlobe, sidelobe, mainlobe width $\alpha \dfrac{1}{\omega_c}$

Filtering of Signals (p.59 of 4.0)



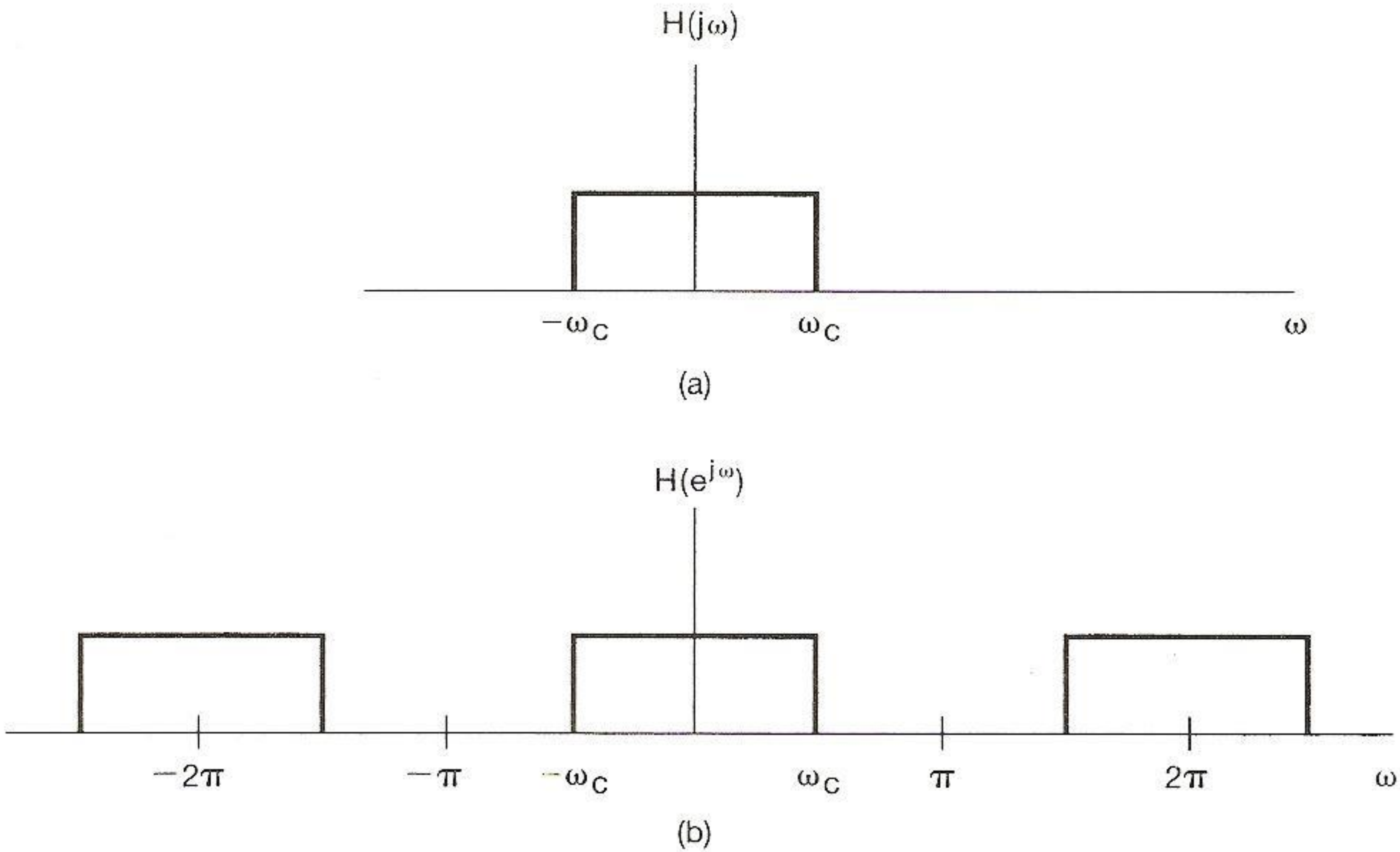


Figure 6.10 (a) The frequency response of a continuous-time ideal lowpass filter; (b) the frequency response of a discrete-time ideal lowpass filter.

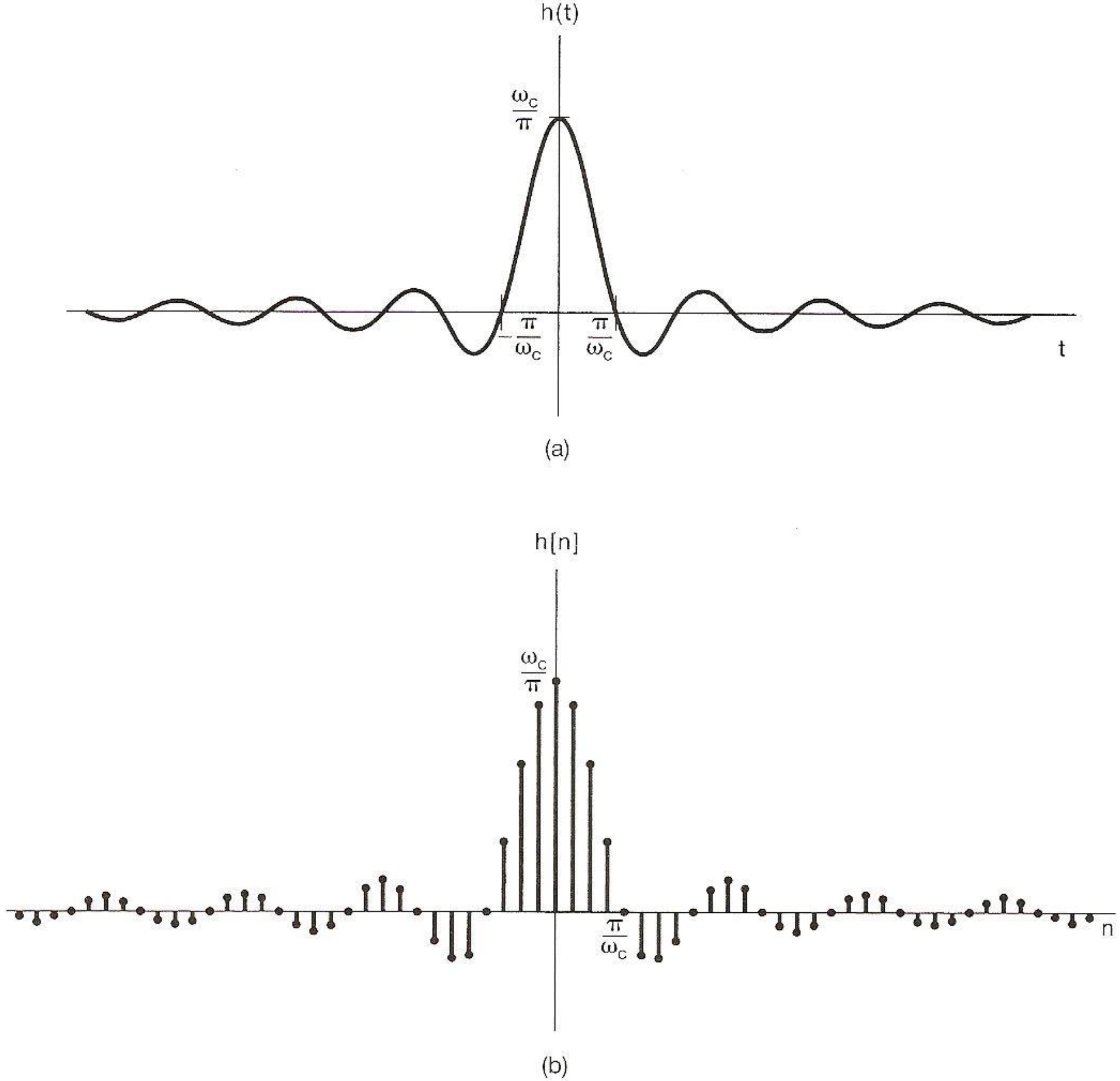


Figure 6.12 (a) The impulse response of the continuous-time ideal lowpass filter of Figure 6.10(a); (b) the impulse response of the discrete-time ideal lowpass filter of Figure 6.10(b) with $\omega_c = \pi/4$.

Ideal Filters

- Low pass Filters as an example
  - with a linear phase or constant delay

    See Figs. 6.11, 6.13, pp.441,442 of text
  - causality issue
  - implementation issues

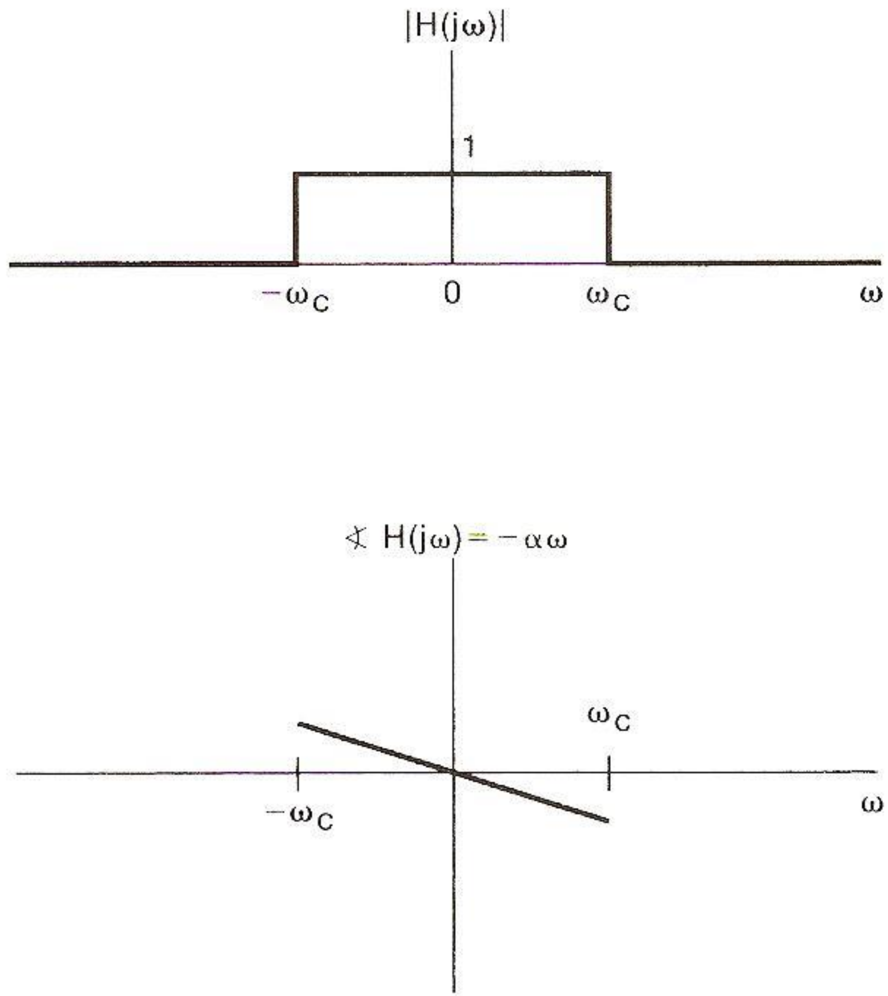


Figure 6.11 Continuous-time ideal lowpass filter with linear phase characteristic.

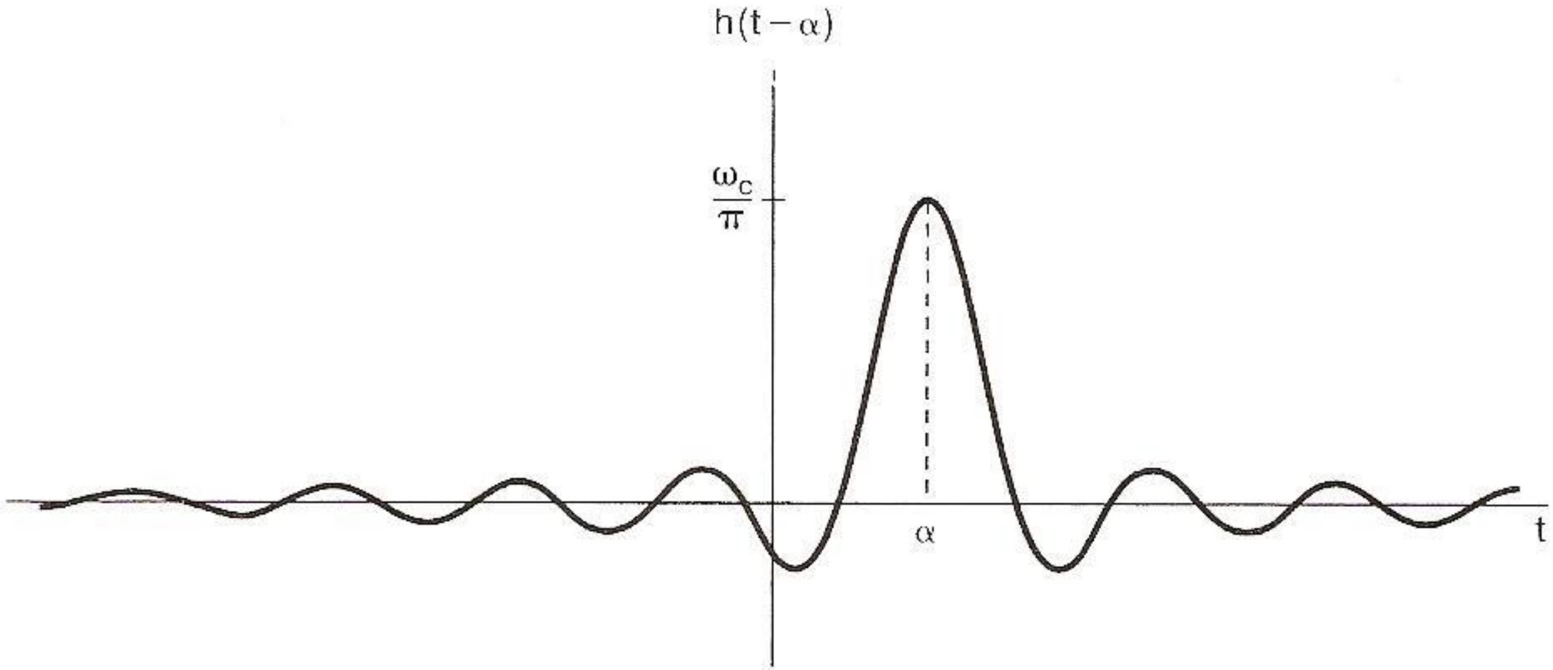


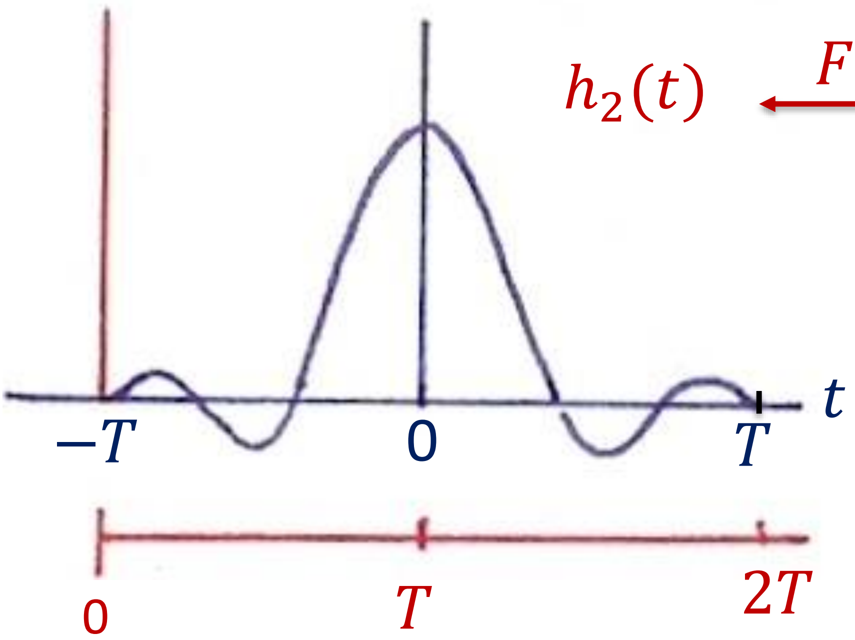
Figure 6.13 Impulse response of an ideal lowpass filter with magnitude and phase shown in Figure 6.11.

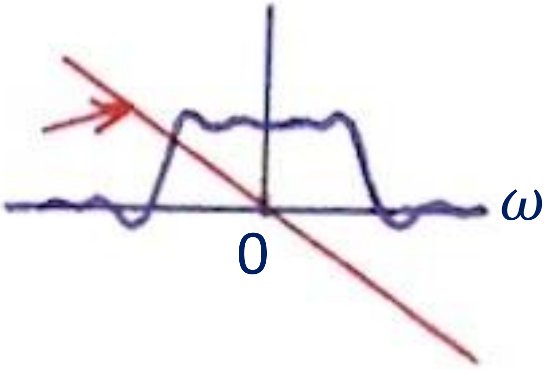
Realizable Lowpass Filter (p.62 of 4.0)

$$h_1(t) \overset{F}{\longleftrightarrow} |H_1(j\omega)|$$

$$\|$$

$$h_2(t) \overset{F}{\longleftrightarrow} \begin{cases} |H_2(j\omega)| \\ \angle H_2(j\omega) \end{cases}$$





Ideal Filters

- Low pass Filters as an example
  - step response

$$s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau, \quad s[n] = \sum_{m=-\infty}^{n} h[m]$$

See Fig. 6.14, p.443 of text

overshoot, oscillatory behavior, rise time $\alpha \dfrac{1}{\omega_c}$

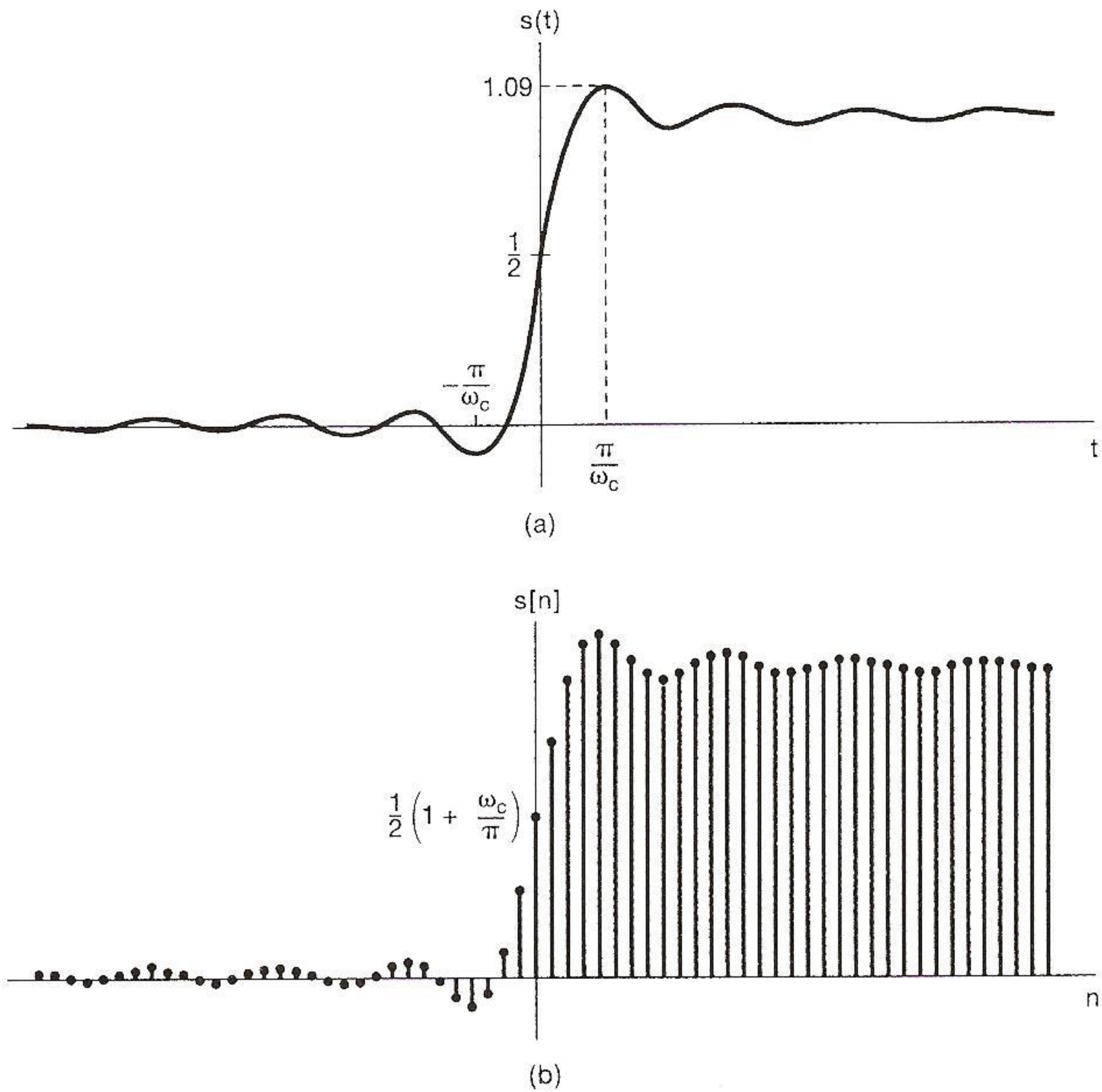


Figure 6.14 (a) Step response of a continuous-time ideal lowpass filter; (b) step response of a discrete-time ideal lowpass filter.

Nonideal Filters

- Frequency Domain Specification
  (Lowpass as an example)

$$\left.\begin{array}{l}\delta_1\text{(passband ripple)}, \ \delta_2\text{(stopband ripple)} \\[2ex] \omega_p\text{(passband edge)}, \ \omega_s\text{(stopband edge)} \\[2ex] \omega_s - \omega_p\text{(transition band)}\end{array}\right\} \begin{array}{l}\text{magnitude} \\ \text{characteristics}\end{array}$$

phase characteristics

See Fig. 6.16, p.445 of text

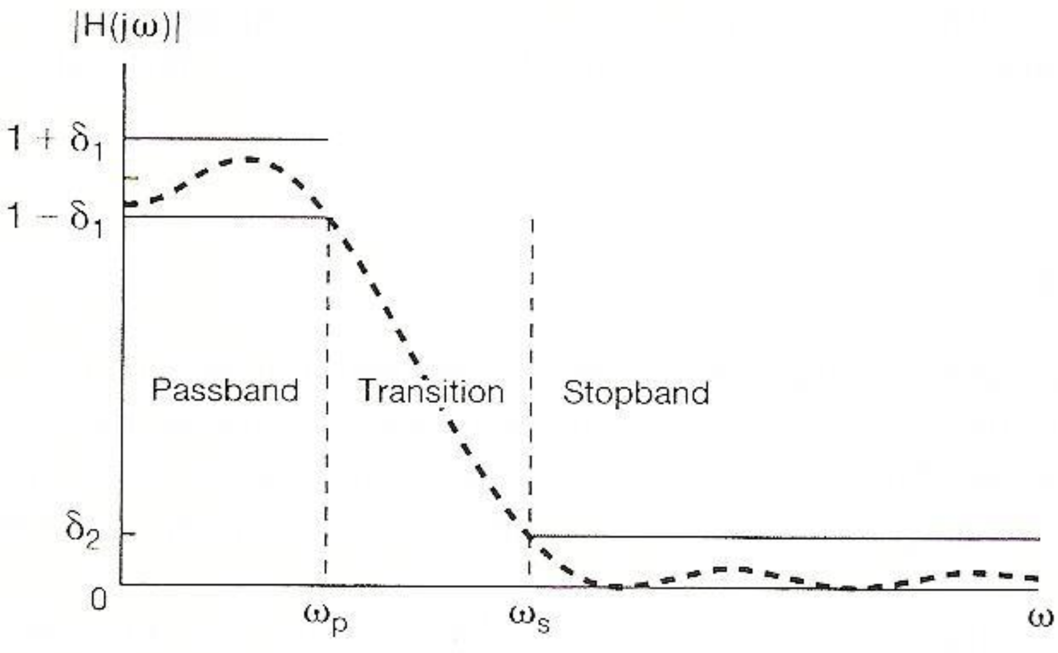


Figure 6.16 Tolerances for the magnitude characteristic of a lowpass filter. The allowable passband ripple is δ_1 and stopband ripple is δ_2. The dashed curve illustrates one possible frequency response that stays within the tolerable limits.

$$\left.\begin{array}{l} \delta_1\text{(passband ripple)}, \ \delta_2\text{(stopband ripple)} \\ \omega_p\text{(passband edge)}, \ \omega_s\text{(stopband edge)} \\ \omega_s - \omega_p\text{(transition band)} \end{array}\right\} \begin{array}{l}\text{magnitude} \\ \text{characteristics}\end{array}$$

Nonideal Filters

- Time Domain Behavior : step response

  t_r(rise time), δ(ripple), Δ(overshoot)

  ω_r(ringing frequency), t_s(settling time)

  See Fig. 6.17, p.446 of text

t_r(rise time), δ(ripple), Δ(overshoot)

ω_r(ringing frequency), t_s(settling time)

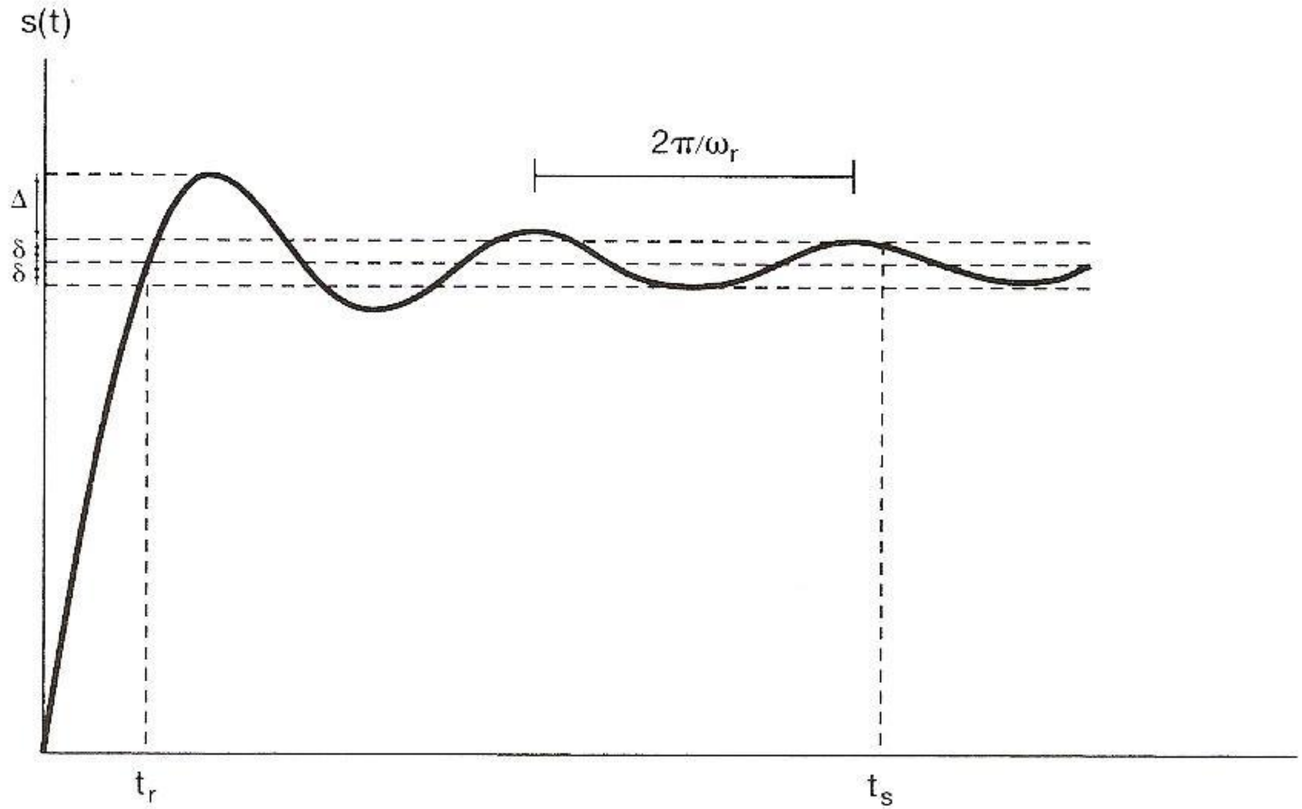


Figure 6.17 Step response of a continuous-time lowpass filter, indicating the rise time t_r, overshoot Δ, ringing frequency ω_r, and settling time t_s—i.e., the time at which the step response settles to within $\pm\delta$ of its final value.

Nonideal Filters

- Example: tradeoff between transition band width (frequency domain) and settling time (time domain)

  See Fig. 6.18, p.447 of text

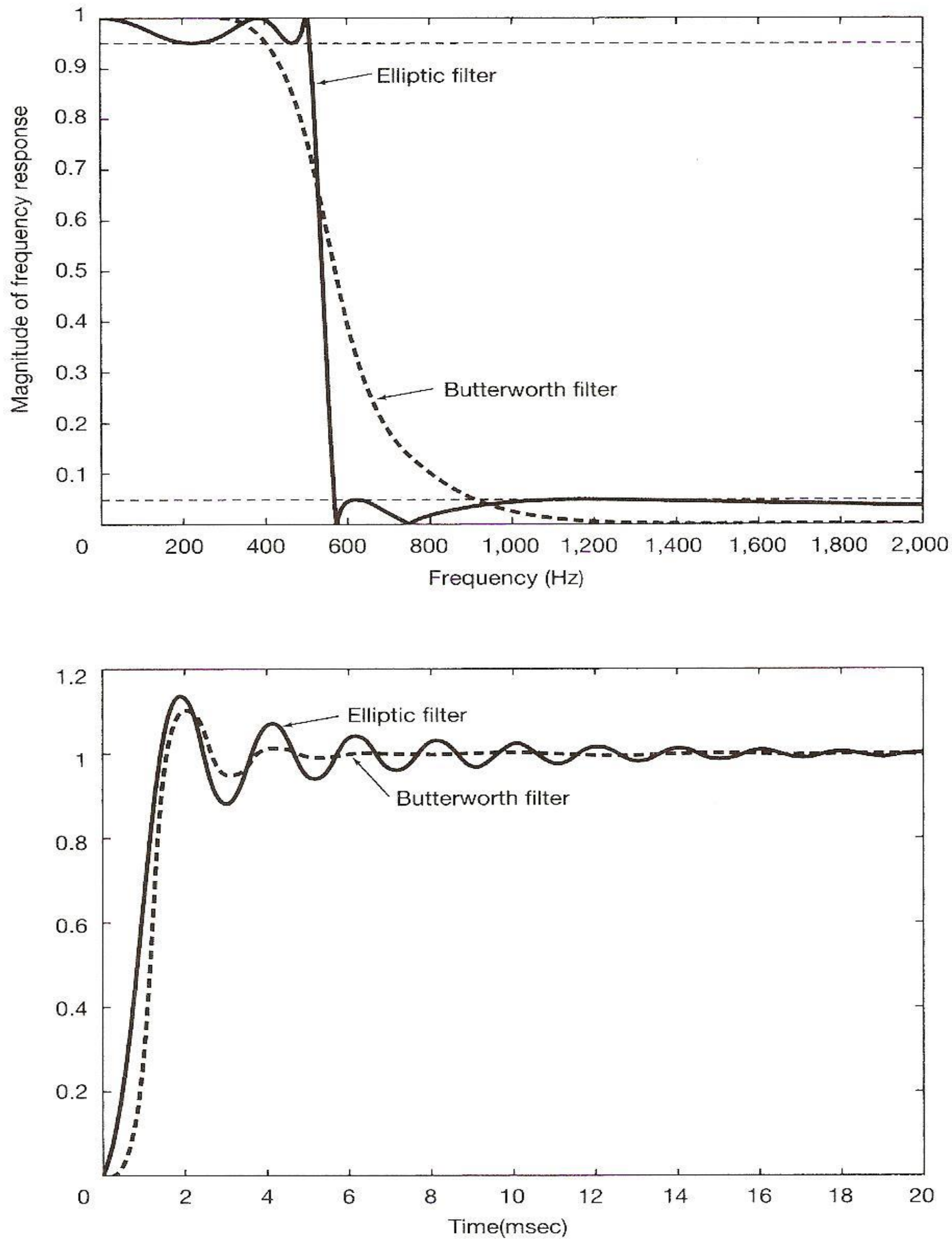


Figure 6.18 Example of a fifth-order Butterworth filter and a fifth-order elliptic filter designed to have the same passband and stopband ripple and the same cutoff frequency: (a) magnitudes of the frequency responses plotted versus frequency measured in Hertz; (b) step responses.

Nonideal Filters

- Examples

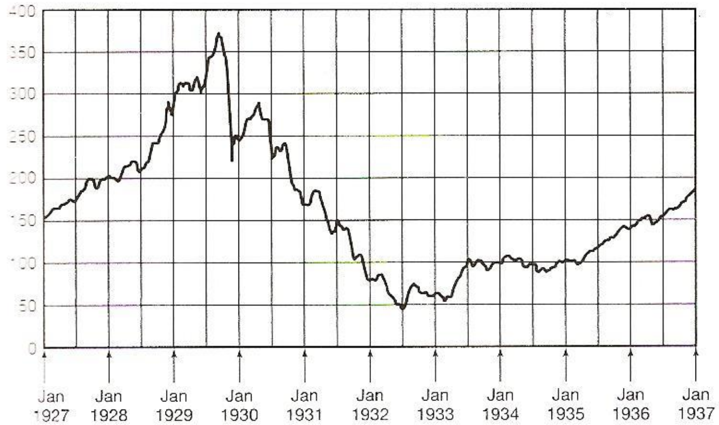
$$y[n] = \frac{1}{N+M+1}\sum_{k=-N}^{M} x[n-k]$$

$$H\left(e^{j\omega}\right) = \frac{1}{N+M+1} e^{j\omega[(N-M)/2]} \frac{\sin[\omega(M+N+1)/2]}{\sin(\omega/2)}$$

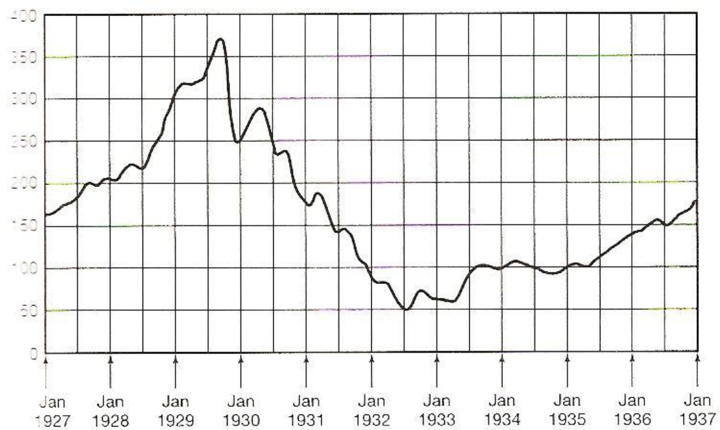
linear phase

See Figs. 6.35, 6.36, pp.478,479 of text

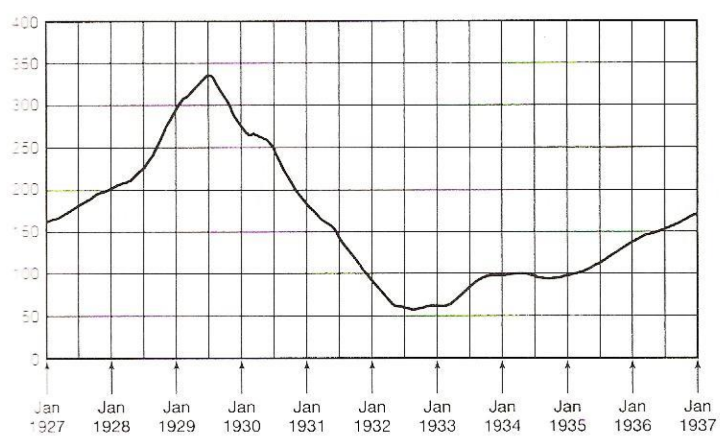
A narrower passband requires longer impulse response



(a)



(b)



(c)

Figure 6.36 Effect of lowpass filtering on the Dow Jones weekly stock market index over a 10-year period using moving-average filters: (a) weekly index; (b) 51-day moving average applied to (a); (c) 201-day moving average applied to (a). The weekly stock market index and the two moving averages are discrete-time sequences. For clarity in the graphical display, the three sequences are shown here with their individual values connected by straight lines to form a continuous curve.

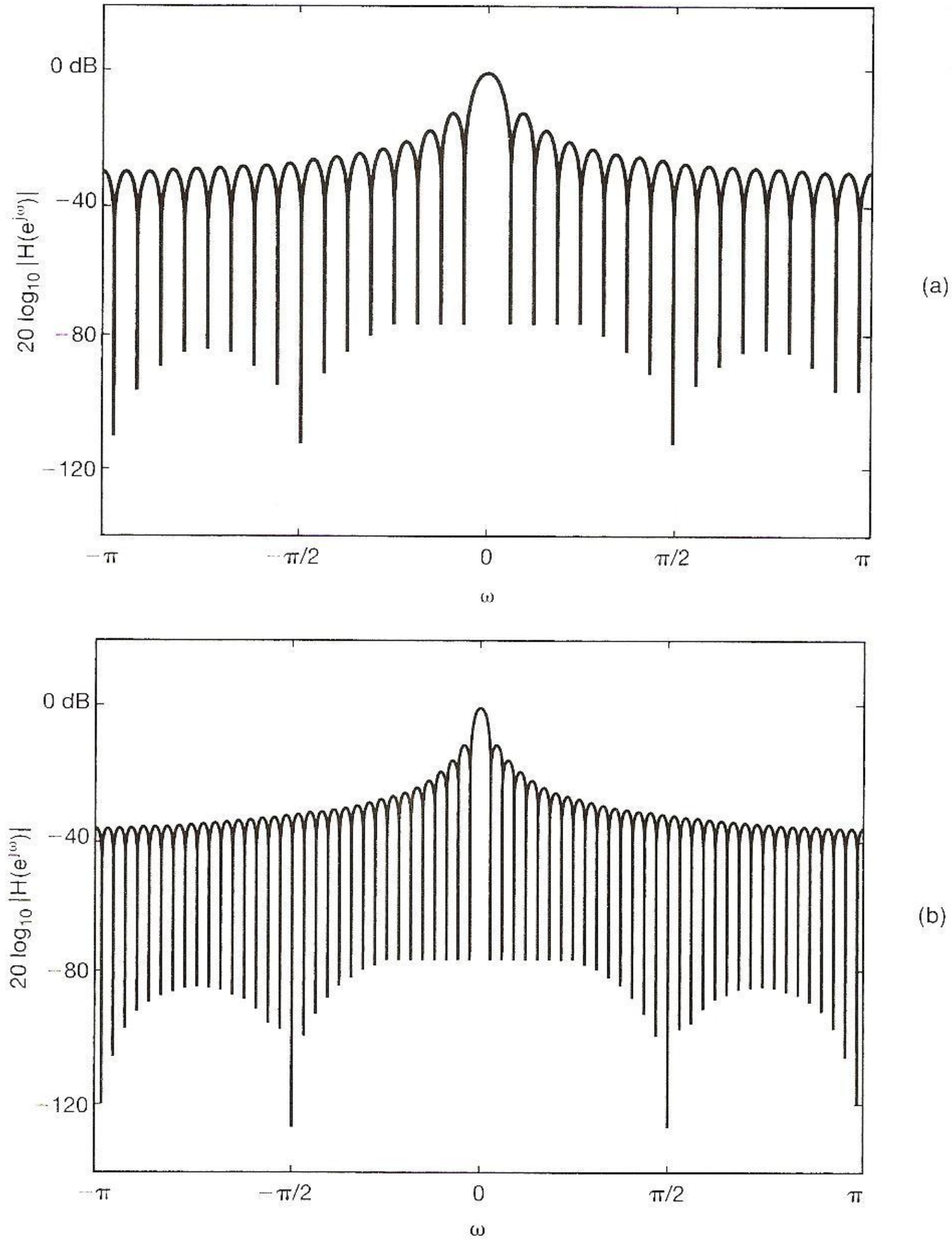


Figure 6.35 Log-magnitude plots for the moving-average filter of eqs. (6.78) and (6.79) for (a) $M + N + 1 = 33$ and (b) $M + N + 1 = 65$.

Nonideal Filters

- Examples
  - A more general form

$$y[n] = \sum_{k=-N}^{M} b_k x[n-k]$$

$$h[k] = b_k = \frac{\sin(2\pi k/33)}{\pi k}, \quad |k| \le 32$$

$$= 0 \qquad , \quad |k| > 32$$

See Figs. 6.37, 6.38, p.480,481 of text

A truncated impulse response for ideal lowpass filter gives much sharper transition

See Figs. 6.39, p.482 of text

very sharp transition is possible

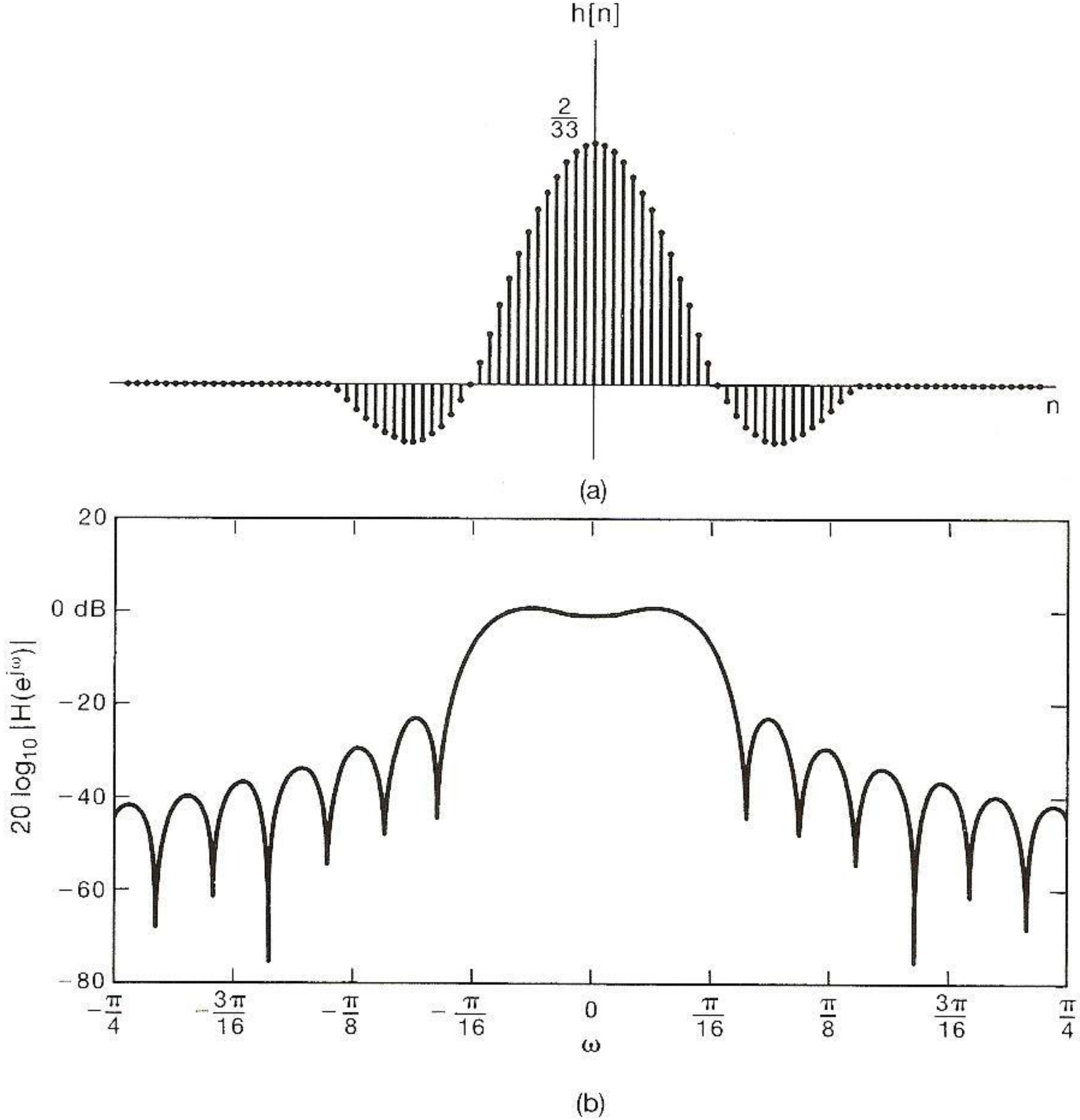


Figure 6.37 (a) Impulse response for the nonrecursive filter of eq. (6.82); (b) log magnitude of the frequency response of the filter.

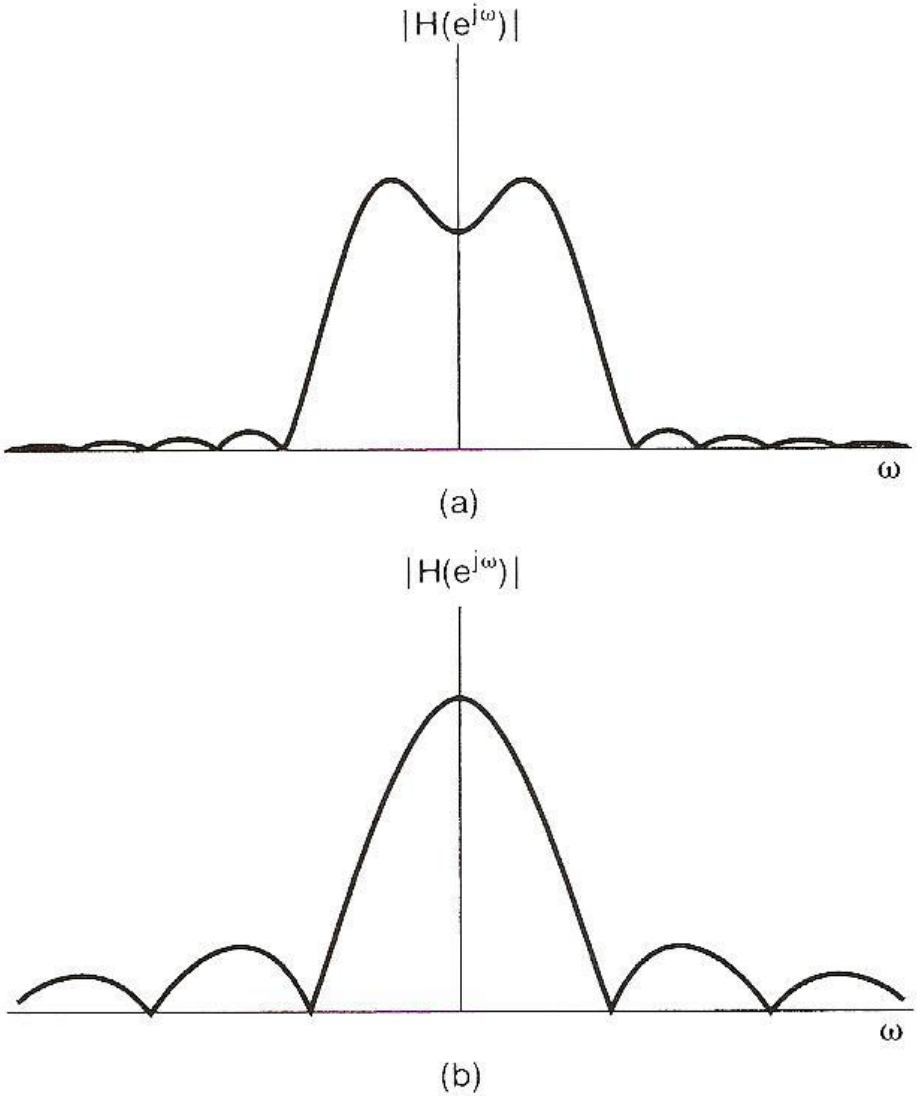


Figure 6.38 Comparison, on a linear amplitude scale, of the frequency responses of (a) Figure 6.37 and (b) Figure 6.35.

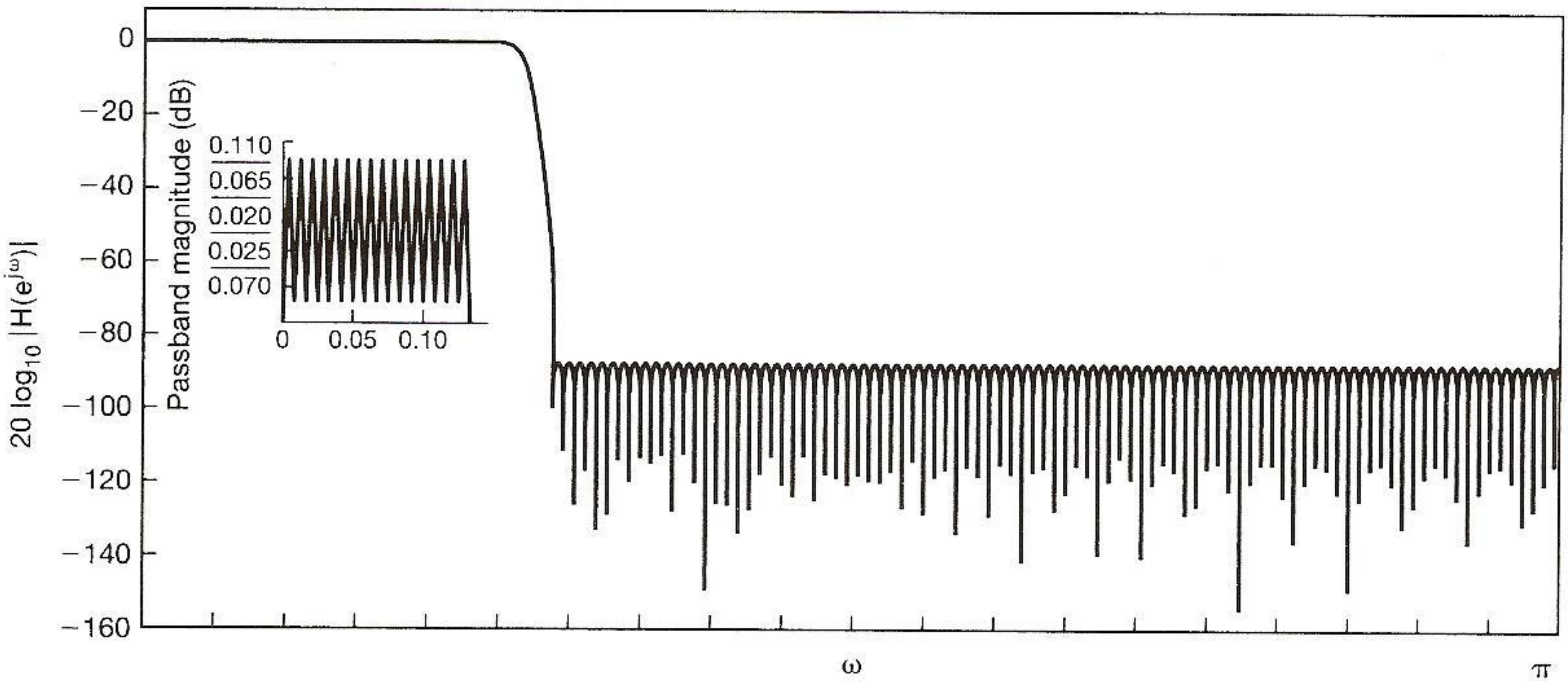


Figure 6.39 Lowpass nonrecursive filter with 251 coefficients designed to obtain the sharpest possible cutoff.

Nonideal Filters

- To make a FIR filter causal

$$
\begin{aligned}
h[n] &= 0, \ |n| > N \\
h_1[n] &= h[n-N] \\
H_1(e^{j\omega}) &= H(e^{j\omega}) e^{-j\omega N}
\end{aligned}
$$

6.3 First/Second-Order Systems Described by Differential/Difference Equations

Higher-order systems can usually be represented as combinations of 1st/2nd-order systems

- Continuous-time Systems by Differential Equations
  - first-order

$$\tau \frac{dy(t)}{dt} + y(t) = x(t)$$

  - second-order

$$m \frac{d^2 y(t)}{dt^2} = x(t) - ky(t) - b \frac{dy(t)}{dt}$$

See Fig. 6.21, p.451 of text

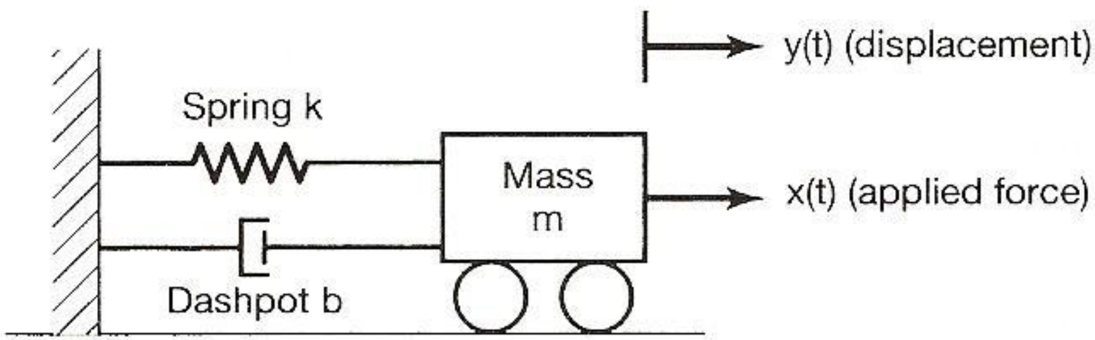


Figure 6.21 Second-order system consisting of a spring and dashpot attached to a moveable mass and a fixed support.

- Discrete-time Systems by Differential Equations
  - first-order

$$y[n] - ay[n-1] = x[n]$$

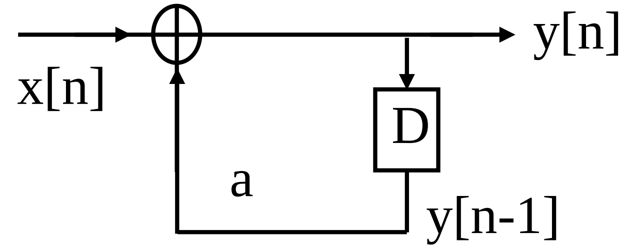
See Fig. 6.26, 27, 28, p.462, 463, 464, 465 of text

  - second-order

$$y[n] - 2r\cos\theta y[n-1] + r^2 y[n-2] = x[n]$$

- First-order Discrete-time system (P.461 of text)

$$y[n] - ay[n-1] = x[n], \quad |a| < 1$$



$$H(e^{j\omega}) = \frac{1}{1 - ae^{-j\omega}}$$

$$h[n] = a^n u[n]$$

$$s[n] = h[n] * u[n] = \left(\frac{1 - a^{n+1}}{1 - a}\right)u[n]$$

$$\left|H(e^{j\omega})\right| = \frac{1}{(1 + a^2 - 2a\cos\omega)^{1/2}}$$

$$\angle H(e^{j\omega}) = -\tan^{-1}\left(\frac{a\sin\omega}{1 - a\cos\omega}\right)$$

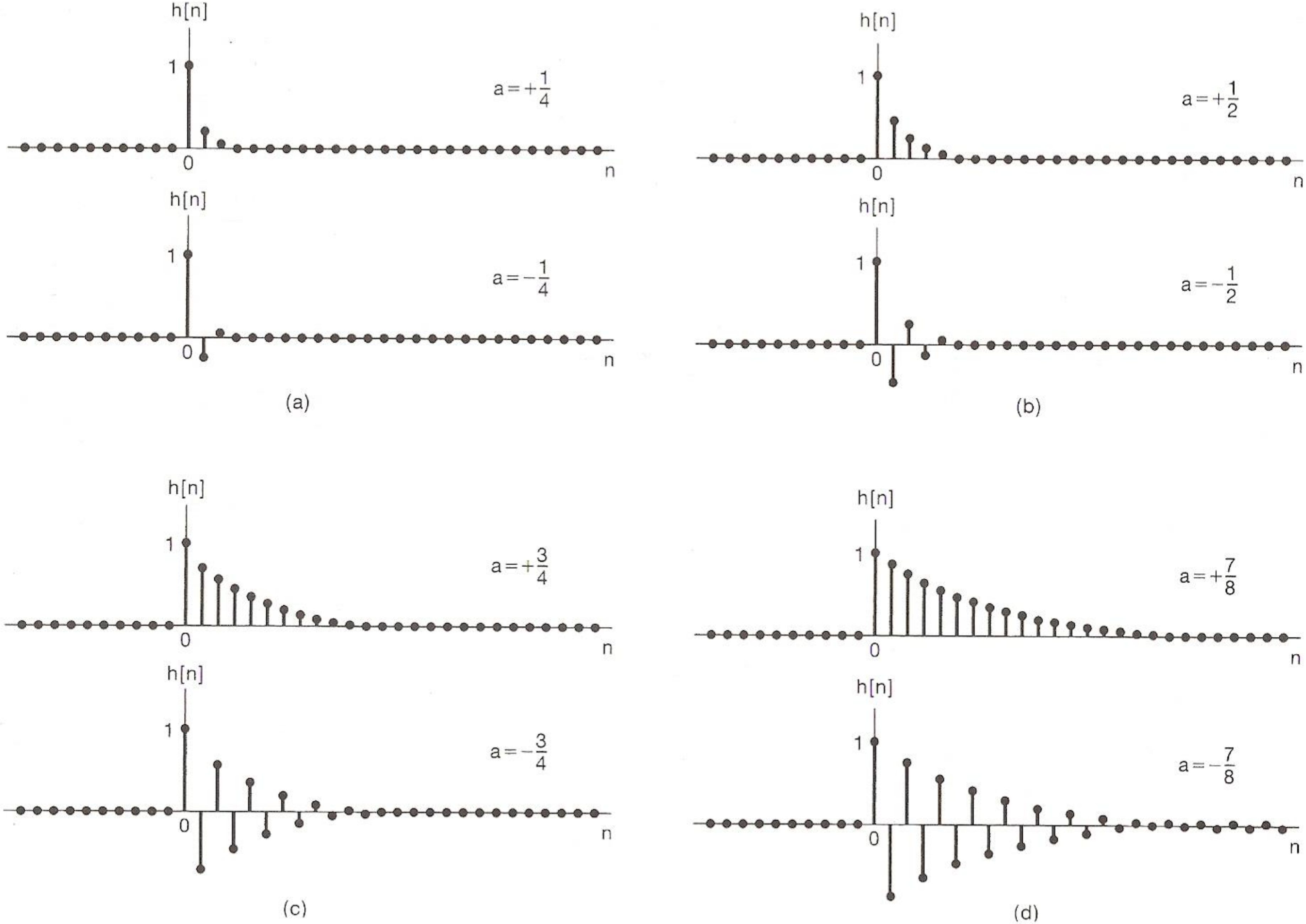


Figure 6.26 Impulse response $h[n] = a^n u[n]$ of a first-order system: (a) $a = \pm 1/4$; (b) $a = \pm 1/2$; (c) $a = \pm 3/4$; (d) $a = \pm 7/8$.

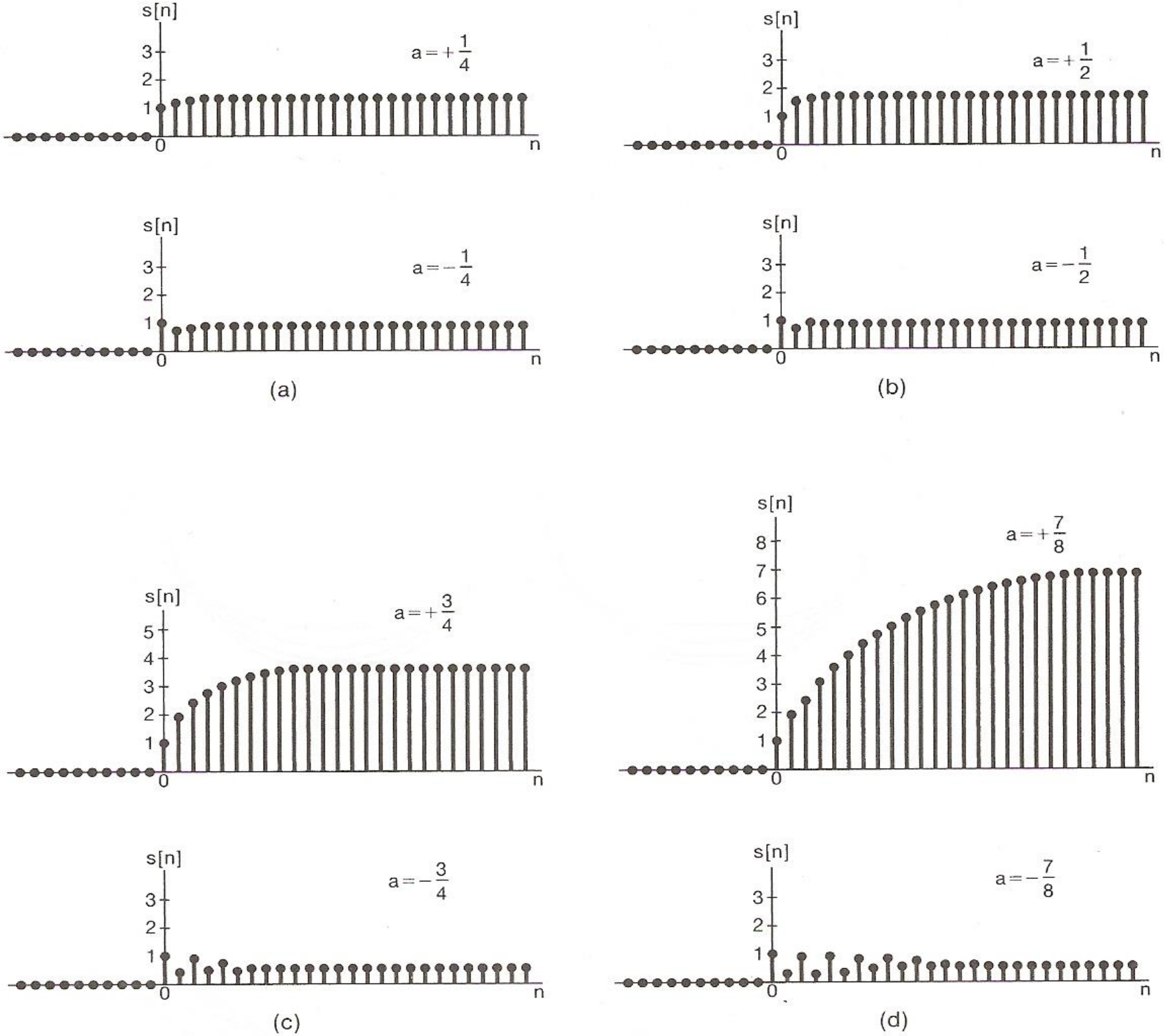


Figure 6.27 Step response $s[n]$ of a first-order system: (a) $a = \pm 1/4$; (b) $a = \pm 1/2$; (c) $a = \pm 3/4$; (d) $a = \pm 7/8$.

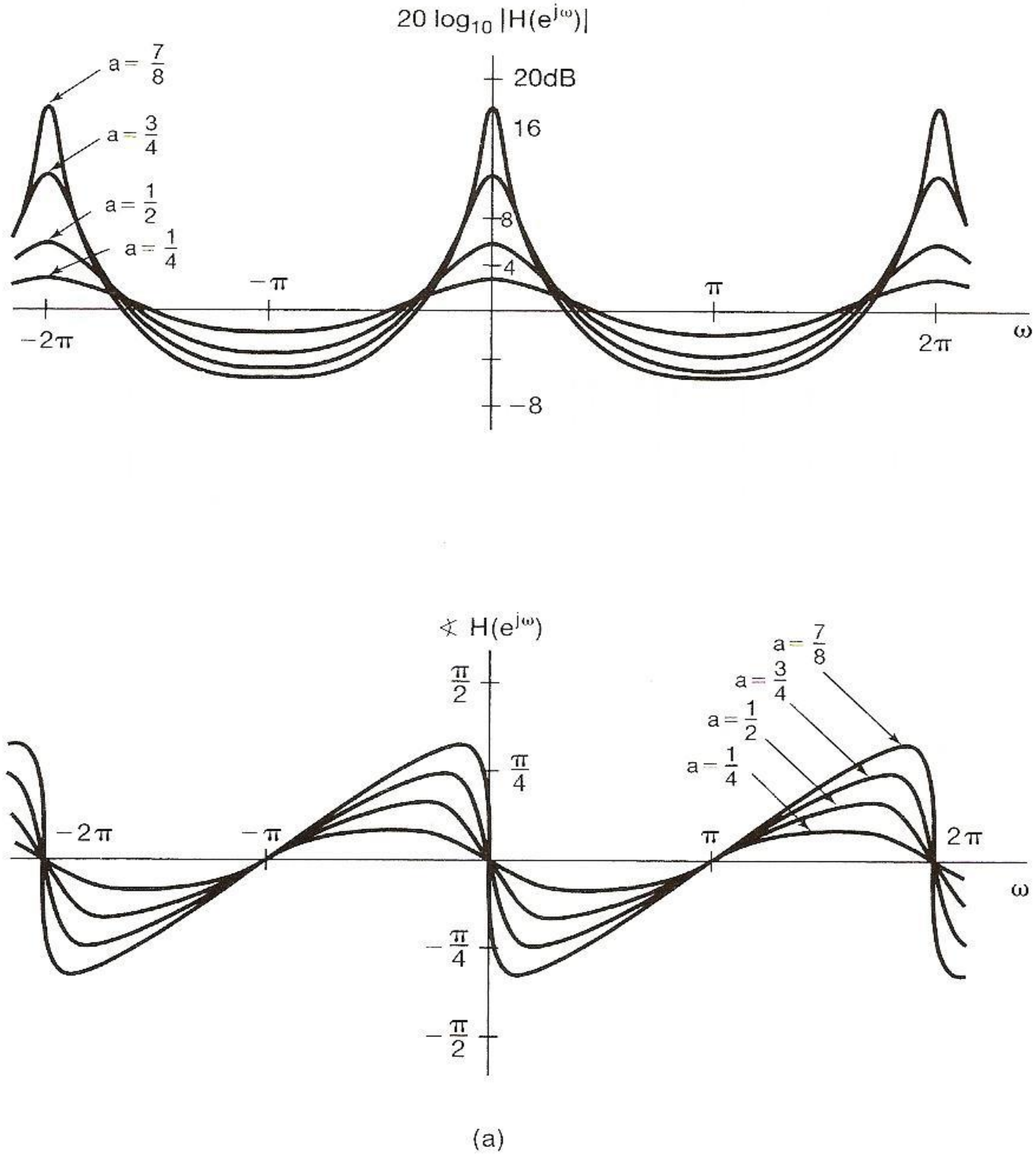


Figure 6.28 Magnitude and phase of the frequency response of eq. (6.52) for a first-order system: (a) plots for several values of $a > 0$; (b) plots for several values of $a < 0$.

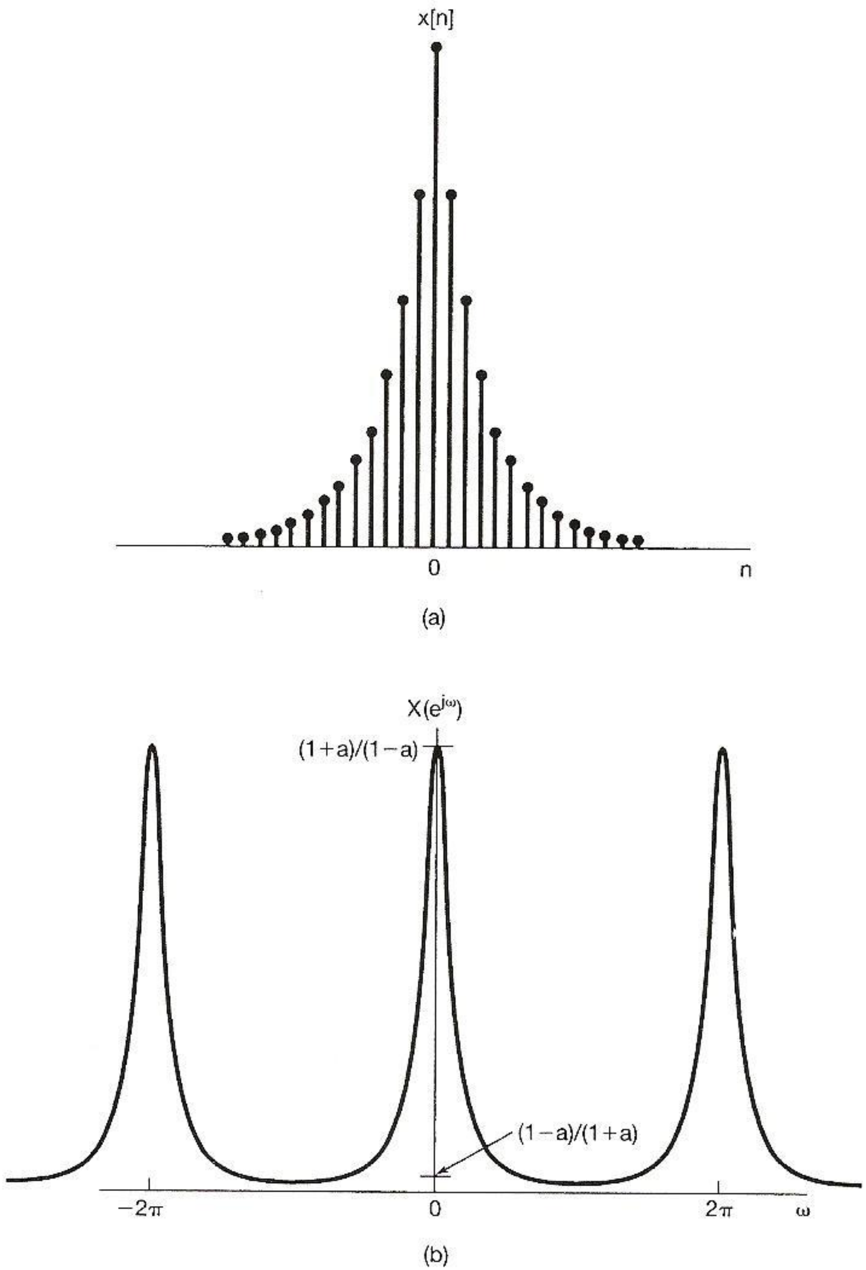
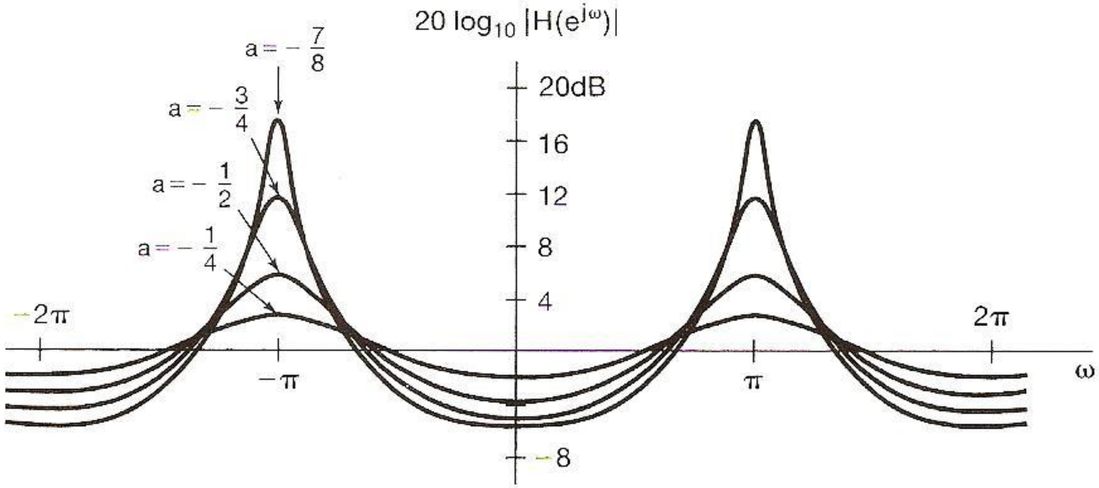
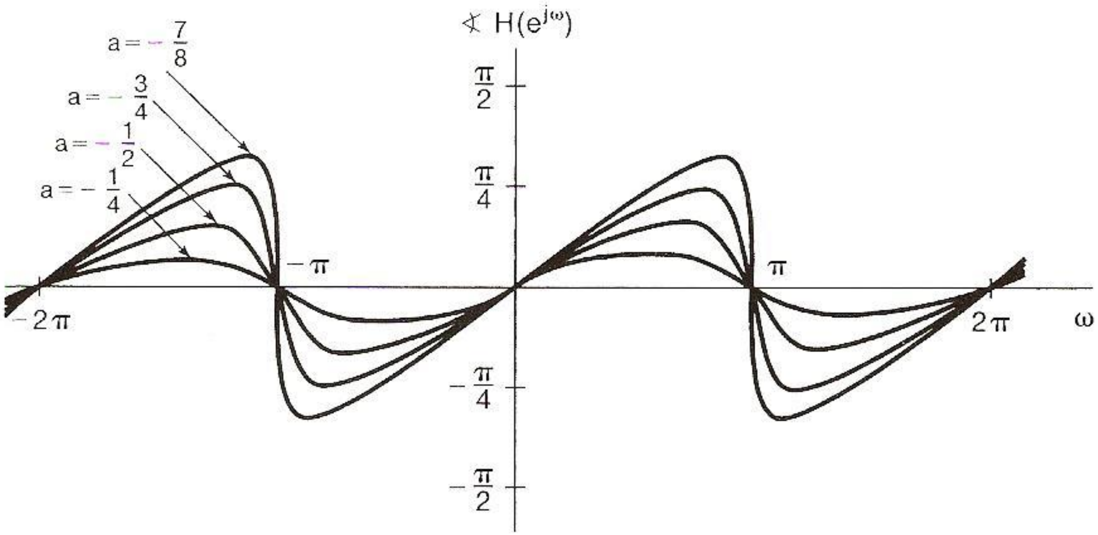


Figure 5.5 (a) Signal $x[n] = a^{|n|}$ of Example 5.2 and (b) its Fourier transform $(0 < a < 1)$.



$$h[n](a < 0) = h[n](a > 0)(-1)^n$$



(b)

Figure 6.28 *Continued*

Examples (p.114 of 2.0)

- Example 2.15, p.123 of text

$$y[n]-\tfrac{1}{2}y[n-1]=x[n]$$
$$y[n]=x[n]+\tfrac{1}{2}y[n-1]$$

  - initial rest condition

$$x[n]=0,\ n\leq -1 \quad \text{imples} \quad y[n]=0,\ n\leq -1$$
$$x[n]=\delta[n]$$
$$y[0]=x[0]+\tfrac{1}{2}y[-1]=1$$
$$y[1]=x[1]+\tfrac{1}{2}y[0]=\tfrac{1}{2}$$
$$y[2]=x[2]+\tfrac{1}{2}y[1]=(\tfrac{1}{2})^2$$
$$\vdots$$
$$y[\mathrm{n}]=\left(\tfrac{1}{2}\right)^n u[n]=h[n]$$

  - infinite impulse response (IIR)

- Second-order Discrete-time system (P.465 of text)

$$y[n] - 2r\cos\theta y[n-1] + r^2 y[n-2] = x[n]$$

$$H(e^{j\omega}) = \frac{1}{1 - 2r\cos\theta e^{-j\omega} + r^2 e^{-j2\omega}}$$

$$= \frac{1}{[1-(re^{j\theta})e^{-j\omega}][1-(re^{-j\theta})e^{-j\omega}]}$$

$$h[n] = r^n \frac{\sin[(n+1)\theta]}{\sin\theta} u[n]$$

$$\theta = 0,\ H(e^{j\omega}) = \frac{1}{(1-re^{-j\omega})^2}$$

$$h[n] = (n+1)r^n u[n]$$

$$\theta = \pi, H\left(e^{j\omega}\right) = \frac{1}{(1+re^{-j\omega})^2}$$

$$h[n] = (n+1)(-r)^n u[n]$$

Problem 5.46, **p.415 of text** (p.85 of 5.0)

$$\alpha^{\mathrm{n}}\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} \frac{1}{1-\alpha e^{-j\omega}}$$

$$(n+1)\alpha^{\mathrm{n}}\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} \frac{1}{(1-\alpha e^{-j\omega})^2}$$

$$nx[n] \xleftrightarrow{\mathcal{F}} j\frac{dX(e^{j\omega})}{d\omega}, \text{ example 5.13, P.385 of text}$$

$$\frac{(n+r-1)!}{n!(r-1)!}\alpha^{\mathrm{n}}\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} \frac{1}{(1-\alpha e^{-j\omega})^r}$$

true for $r=1,\ 2$

when $r=k$ is true

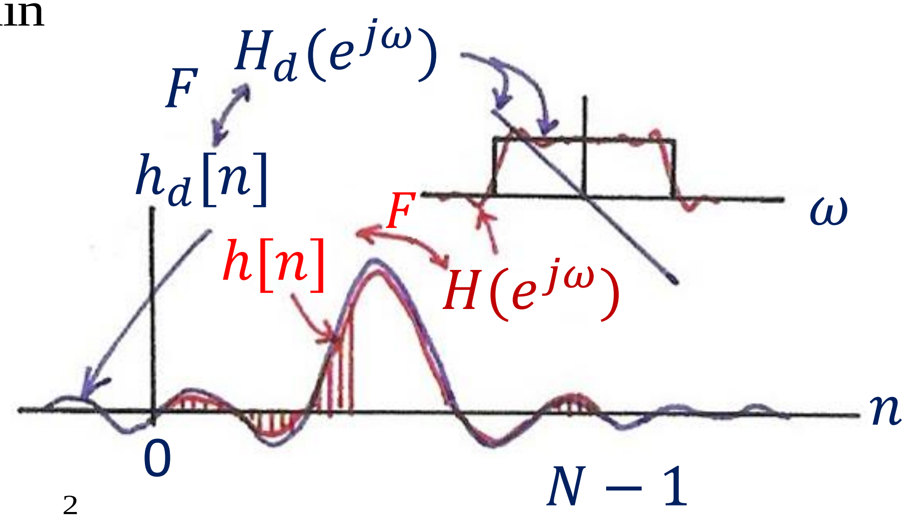
show $r=k+1$ is also true

Problem 6.59, p.508 of text

$h_{\mathrm{d}}[n] \xleftrightarrow{\mathrm{F}} H_d(e^{j\omega})$: a desired ideal system

$h[n] \xleftrightarrow{\mathrm{F}} H(e^{j\omega})$: a practical causal FIR with duration N

$$\varepsilon^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|H_d(e^{j\omega}) - H(e^{j\omega})\right|^2 d\omega = \min$$



(a) $$E(e^{j\omega}) = H_d(e^{j\omega}) - H(e^{j\omega})$$

$$= \sum_{n=-\infty}^{\infty}\left(h_{\mathrm{d}}[n] - h[n]\right)e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} e[n]\, e^{-j\omega n}$$

(b) $$\varepsilon^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|E(e^{j\omega})\right|^2 d\omega = \sum_{n=-\infty}^{\infty}\left|e[n]\right|^2$$

$$= \sum_{n=0}^{N-1}\left|h_{\mathrm{d}}[n] - h[n]\right|^2 + \sum_{n=-\infty}^{0}\left|h_{\mathrm{d}}[n]\right|^2 + \sum_{n=\mathrm{N}}^{\infty}\left|h_{\mathrm{d}}[n]\right|^2$$

$$\varepsilon^2 = \min \text{ when } h[n] = \begin{cases} h_{\mathrm{d}}[n], & \mathrm{n} = 0,1,\ldots N\text{-}1 \\ 0, & \text{else} \end{cases}$$

Truncated Dimensions (p.21 of 3.0)

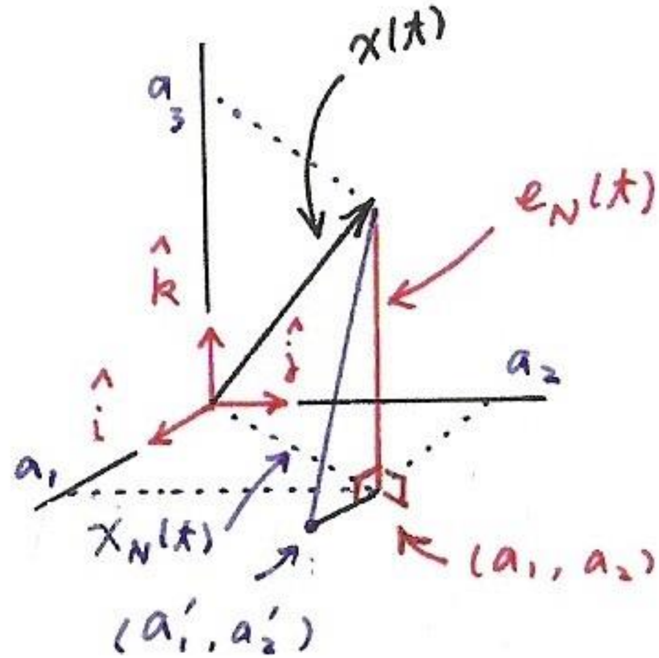
$$x_N(t) \Leftarrow \quad \Rightarrow e_N(t)$$

$$x(t) = a_0 + a_1 e^{j\omega_0 t} + a_2 e^{j2\omega_0 t} + \cdots + a_N e^{jN\omega_0 t} \quad \Big| \quad \cdots\cdots$$

$$+ a_{-1} e^{-j\omega_0 t} + a_{-2} e^{-j2\omega_0 t} + \cdots + a_{-N} e^{-jN\omega_0 t} \quad \Big|_{N} \quad \cdots\cdots$$

$$x(t) = \overbrace{a_1 \hat{i} + a_2 \hat{j}}^{x_N(t)} + \overbrace{a_3 \hat{k}}^{e_N(t)}$$

$$x_N(t) = a_1' \hat{i} + a_2' \hat{j} \quad \Big|_{N=2}$$



$\Rightarrow$ $a_1' = a_1, a_2' = a_2,$

for orthogonal $\hat{i}, \hat{j}, \hat{k}$

- All truncated dimensions are orthogonal to the subspace of dimensions kept.

Problem 3.66, p.275 of text (p.62 of 3.0)

- $\{\phi_i(t), i = 0, \pm 1, \pm 2, \ldots\}$ a set of orthonormal functions over $[a, b]$

  $\int_a^b \phi_i(t)\phi_j^*(t)dt = \delta_{ij}$

  for a signal $x(t)$ over $[a, b]$, $\hat{x}_N(t) = \sum_{i=-N}^{N} a_i\phi_i(t)$, $e_N(t) = x(t) - \hat{x}_N(t)$

  $E_N = \int_a^b |e_N(t)|^2 dt$

- It can be shown $E_N = \min$ when $a_i = \int_a^b x(t)\phi_i^*(t)dt$

  $a_i = b_i + \mathrm{j}c_i$

  $\frac{\partial E_N}{\partial b_i} = 0$, $\frac{\partial E_N}{\partial c_i} = 0$, $i = 0, \pm 1, \pm 2 \ldots$

- For basis functions not normalized

  $\int_a^b \phi_i(t)\phi_j^*(t)dt = A\delta_{ij}$

  $a_i = \frac{1}{A}\int_a^b x(t)\phi_i^*(t)dt$

Problem 6.64, p.511 of text

$$H(e^{j\omega}) = H_r(e^{j\omega})e^{-jM\omega}, \quad -\pi < \omega < \pi$$

$H_r(e^{j\omega})$: real and even, M : positive integer

$h[n] \xleftrightarrow{F} H(e^{j\omega}), \quad h_r[n] \xleftrightarrow{F} H_r(e^{j\omega})$

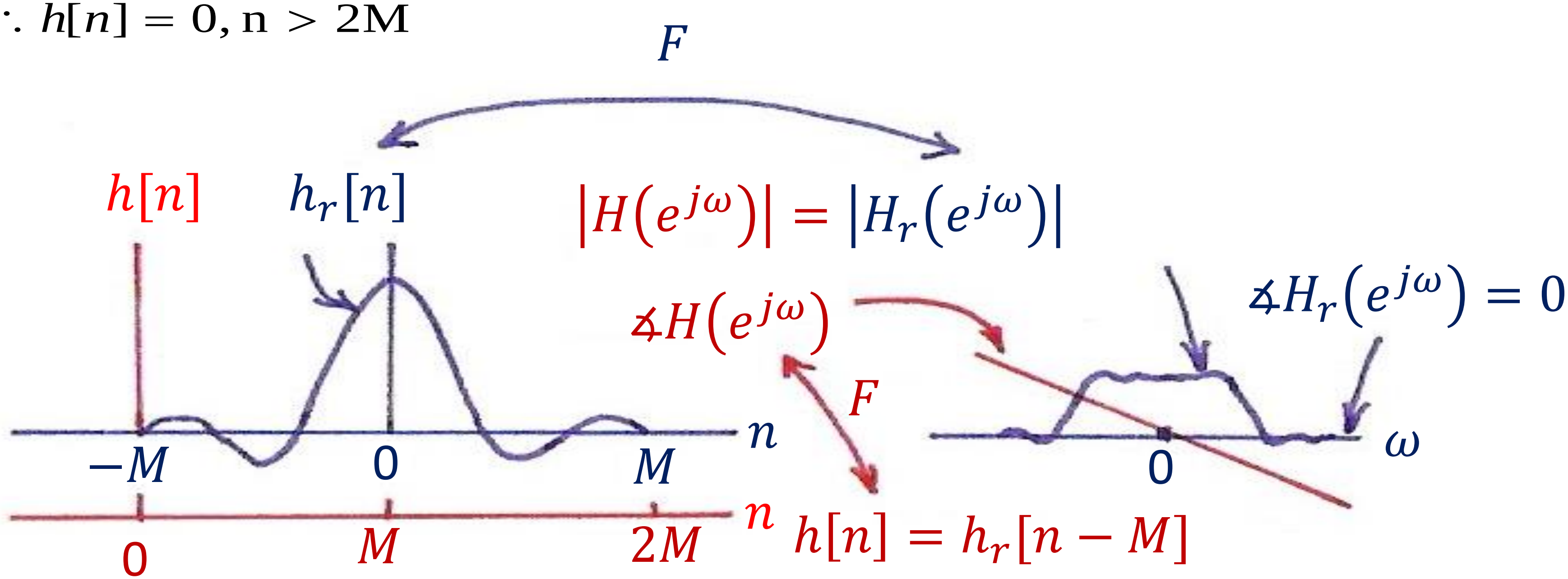
(a) $H_r(e^{j\omega})$ real and even $\longrightarrow$ $h_r[n]$ real and even (4.3.3 of Table 4.1)

(b) $h_r[n]$: symmetric about $n = 0$

$h[n] = h_r[n-M]$: symmetric about $n = M$

(c) $h[n]$ is causal, $h[n] = 0, n < 0$

$\therefore h[n] = 0, n > 2M$



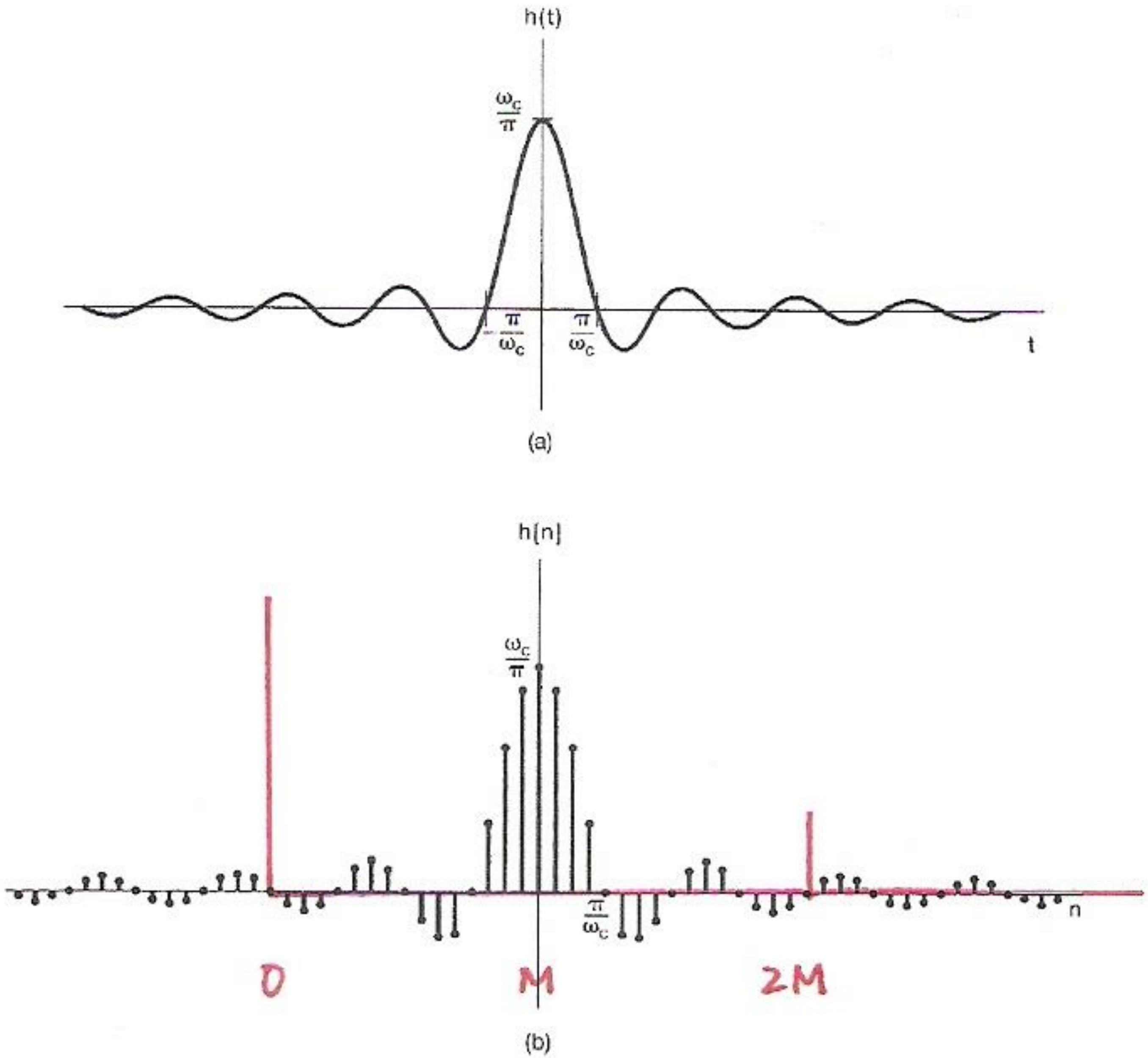


Figure 6.12 (a) The impulse response of the continuous-time ideal lowpass filter of Figure 6.10(a); (b) the impulse response of the discrete-time ideal lowpass filter of Figure 6.10(b) with $\omega_c = \pi/4$.