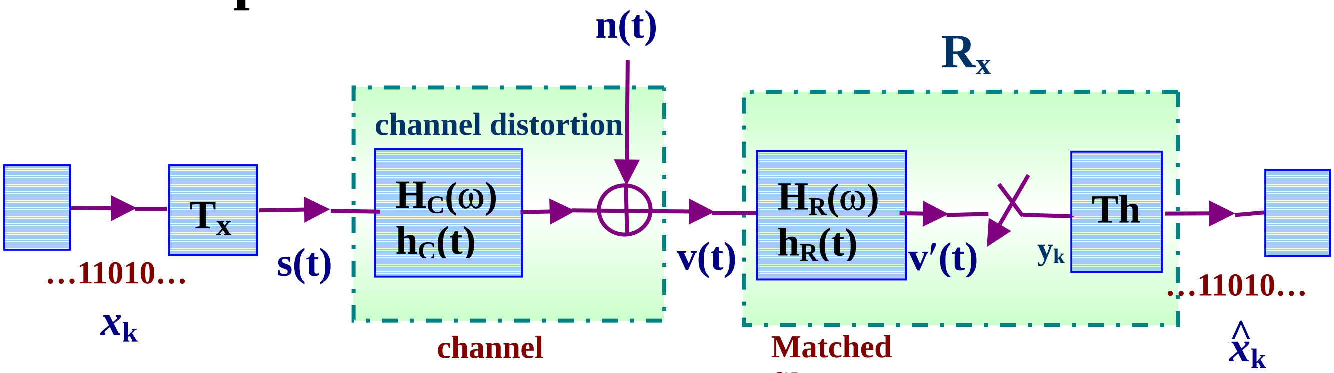
3.5 Receiver Design with Minimum Mean Square Error

- **Basic Concept**

  - to minimize the joint effects of noise plus intersymbol interference

Noise Plus Intersymbol Interference

- **A Simplified Model**



$$s(t) = \sum_k a_k\, p(t-kT)$$

$a_k = \pm 1$ for 1 and 0

p(t) : pulse shape for transmitted signal

$$v(t) = s(t)*h_c(t) + n(t)$$

$$v'(t) = \sum_k a_k\, g(t-kT) + n'(t)$$

$$g(t) = p(t)*h_C(t)*h_R(t)$$

$$n'(t) = n(t)*h_R(t)$$

$$y_k = v'(t_0' + kT)\ ,\ \text{all } k$$

$$y_j = v'(t_0' + jT) = \sum_k a_k g(t_0' + (j-k)T) + n'(t_0' + jT)$$

$$= \underbrace{a_j\, g(t_0')}_{\text{desired sample}} + \underbrace{\sum_{\substack{k \\ k\neq j}} a_k g(t_0' + (j-k)T)}_{\text{Intersymbol Interference}} + \underbrace{n'(t_0' + jT)}_{\text{channel noise}}$$

Noise Plus Intersymbol Interference

- **Optimal Receiver with Minimum Mean Square Error**

  - Error at the j-th sample

    $$e_j = \sum_{\substack{k \\ k \neq j}} a_k\, g(t_0' + (j-k)T) + n'(t_0' + jT)$$

    $$J = E[e_j^2] = \min$$

    assume n(t) white, Gaussian, zero-mean

  - Optimal Solution for the receiver filter

    $$H_C(f) = \frac{Q^*(f)}{\frac{1}{T}\sum_k \left| Q(f + \frac{k}{T}) \right|^2 + \frac{N_0}{2}}$$

    $$q(t) \overset{F}{\longleftrightarrow} Q(f)\,, \quad q(t) = p(t) * h_C(t)$$

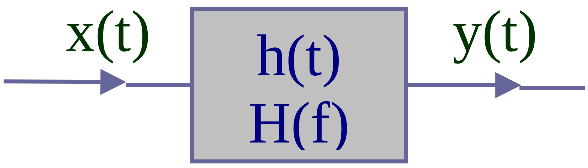
    A matched filter $Q^*(f)$ but modified to take care of the intersymbol interference and noise jointly

Adaptive Equalizer

the above solution is in fact inadequate, because very often the channel characteristics vary with time

- **Adaptive Equalizer**

See Fig. 4.28 , p. 287 of Haykin



$x(nT) = x[n]$, $y(nT) = y[n]$

w_k adaptively adjusted in real-time

equalizer initialized by a training sequence, then switched to tracking mode when data is received, in which the decoded signal serves as the desired response

See Fig. 4.30 , p. 290 of Haykin

- $y[n] = \sum_{k=0}^{N} w_k x[n-k]$

$Y(z) = X(z) \left[\sum_{k=0}^{N} w_k z^{-k} \right]$

$Y(f) = X(f) \left[\sum_{k=0}^{N} w_k e^{-j2\pi fkT} \right]$, $z = e^{j2\pi fT}$

$H(f) = \sum_{k=0}^{N} w_k e^{-j2\pi fkT}$

by properly choosing the parameters w_k , $H(f)$ can in principle approximate any desired frequency response if N is large

Ref : 4.9 , 4.10 of Haykin

Adaptive Equalizer

- **Steepest descent principle**

  $J = E[e^2_j]$ is a real function defined on a hyperplane of $(w_0, w_1, w_2, \ldots\ldots, w_N)$

  $$\hat{w}_k[n+1] = \hat{w}_k[n] - \frac{1}{2}\mu g_k[n]$$

  iterative adjustment of $\hat{w}_k[n]$

  μ : step-size parameter

  $$g_k[n] = \frac{\partial J}{\partial \hat{w}_k[n]} \quad , k = 0, 1, 2, \ldots, N$$

  - $(g_0[n], g_1[n], g_2[n], \ldots\ldots, g_N[n])$ leads to a direction opposite to the gradient vector towards the bottom of the bowl-shaped function of $J=E[e^2_j]$ defined on the hyperplane

- **Least Mean Square Algorithm**

  the above estimates of $\hat{w}_k[n+1]$ lead to the following relationship (an approximation)

  $$\hat{w}_k[n+1] = \hat{w}_k[n] + \mu x[n-k]e[n], \; k=0, 1, 2, \ldots, N$$

  $$e[n] = d[n] - \sum_{k=0}^{N} \hat{w}_k[n]x[n-k]$$

  $d[n]$: desired response

- **Linear Feedback Equalization**

  $$y[n] = \sum_{k=0}^{N} w_k x[n-k] + \sum_{k=-N}^{-1} w_k x[n-k]$$

- **Decision Feedback Equalization**

  feedback section using decision results instead of received signal samples

  See Fig. 4.32, P.292 of Haykin

  Ref : 3.13 , 4.9, 4.10 of Haykin