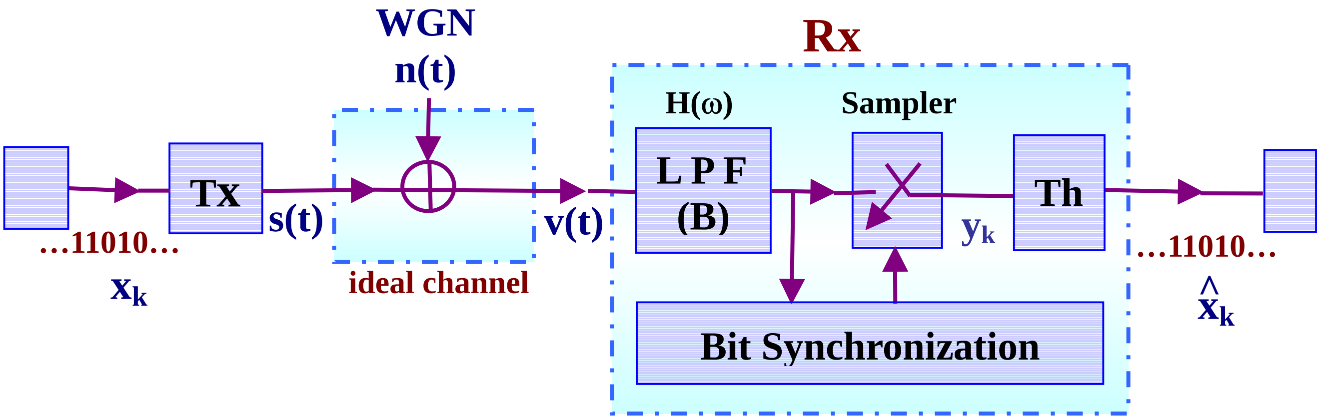
3.0 Baseband Digital Transmission

3.1 Transmission Bandwidth and Bit Error Rate

A Simplified Model



$$s(t) = \sum_k a_k\, p(t-kT)\ ,\ T : \text{bit duration}$$

$$a_k = \begin{cases} +1\ , & x_k = 1 \\ -1\ , & x_k = 0 \end{cases}$$

$p(t)$: pulse shape

$n(t)$: White, Gaussian, zero-mean noise (WGN)

$$S_n(\omega) = \frac{N_0}{2}\ ,\ \text{all } \omega$$

Low Pass Filter (LPF)

$$H(\omega) = \begin{cases} 1\ , & |\omega| \le B \\ 0\ , & \text{else} \end{cases}$$

Sampler : taking samples at pulse peaks

Threshold :

$$\hat{x}_k = 1\ ,\ y_k > 0$$

$$\hat{x}_k = 0\ ,\ y_k < 0$$

A Simplified Model

- **Transmission Bandwidth**

$$p(t) \xleftrightarrow{F} P(\omega) \text{ : bandwidth B}$$

- scaling property of Fourier Transform

$$p(t) \xleftrightarrow{F} P(\omega)$$

$$p(at) \xleftrightarrow{F} \frac{1}{|a|} P(\frac{\omega}{a})$$

See 4.3.5, p. 308 of Oppenheim for proof

$\therefore$ pulse widths in the two domains are reciprocals of each other

- If p(t) has a width τ in time domain, $\tau \leq T$

$$B \; \alpha \; \frac{1}{\tau} \geq \frac{1}{T} = r \, , \qquad \eta = \frac{r}{B}$$

r : bit rate (speed), bits/sec (bps)

η : bandwidth efficiency, bps/Hz

$\therefore$ transmission bandwidth is proportional to bit rate or speed

A Simplified Model

- **Bit Error Rate**

$$P_e = P_{e1} P_1 + P_{e0} P_0$$

$$P_{e1} = \text{Prob}[\text{error}|x_k=1], \quad P_{e0} = \text{Prob}[\text{error}|x_k=0]$$

$$P_1 = \text{Prob}[x_k=1] = \frac{1}{2}, \; P_0 = \text{Prob}[x_k=0] = \frac{1}{2}$$

$$P_e = \frac{1}{2}(P_{e1} + P_{e0})$$

- $y_k = \begin{cases} A + n', & x_k = 1 \\ -A + n', & x_k = 0 \end{cases}$

A : peak value of p(t)

n' : zero-mean Gaussian random variable
a sample of the filtered noise $n'(t)$

$$f_{n'}(n') = \frac{1}{(2\pi\sigma^2)^{1/2}} e^{-n'^2/2\sigma^2}$$

$$P_{e1} = P_{e0} = \text{Prob}[n' > A] = \text{Prob}[n' < -A]$$

$$= \int_A^\infty f_{n'}(n')dn' = Q\left(\frac{A}{\sigma}\right)$$

$$Q(x) = \frac{1}{(2\pi)^{1/2}} \int_x^\infty e^{-\lambda^2/2} d\lambda$$

$$\therefore P_e = \frac{1}{2}(P_{e1} + P_{e0}) = Q\left(\frac{A}{\sigma}\right)$$

- optimal threshold at $y_k = 0$

See Fig. 4.5 , p. 255 of Haykin

A Simplified Model

- **E_b / N_0**

  $E_b = \int_{-\infty}^{\infty} p^2(t)\,dt = bA^2$,

  b depends on the shape of p(t)

  $\sigma^2 = E\{[\,n'(t)]^2\} = E[(n'(t))(n'(t))] = R_{n'}(0)$

  $= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{n'}(\omega)d\omega$

  $R_{n'}(\tau)$: autocorrelation function of n′(t)

  $S_{n'}(\tau)$: power spectral density of n′(t)

  $$S_{n'}(\omega) = S_n(\omega) \bullet |\,H(\omega)\,|^2 = \begin{cases} \frac{N_0}{2}, & |\omega| \le B \\ 0, & \text{else} \end{cases}$$

  $\sigma^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{n'}(\omega)d\omega = \frac{1}{2\pi}\frac{N_0}{2} \bullet 2B = b'N_0$

  b′ depends on the shape of p(t) , $b' \alpha B$

  $\therefore h^2 \frac{E_b}{N_0} = (\frac{A}{\sigma})^2$, h depends on the shape of p(t)

  $\therefore P_e = Q[\,h(\frac{E_b}{N_0})^{1/2}\,]$

  - System Design can be performed based on this function

    See Fig. 4.6 , p. 258 of Haykin

  - Bit error rate can be arbitrarily suppressed as long as higher E_b/N_0 can be used

  - It is easier to distinguish between two values only : 0 and 1

A Simplified Model

- **Influence of P_e**
  - assume a bit rate of 10^5 bits/sec (100 kbps or 0.1 Mbps)

P_e	Roughly one error every
10^{-2}	10^{-3}sec
10^{-4}	10^{-1}sec
10^{-6}	10 sec
10^{-8}	20 min
10^{-10}	1 day
10^{-12}	3 months