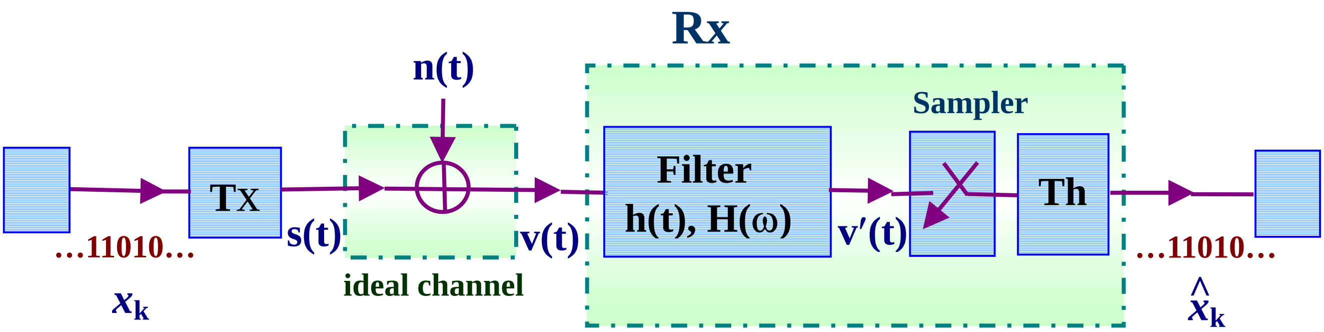
3.3 Matched Filtering

A Simplified Model



$$s(t) = \sum_k a_k\, p(t-kT)\ ,\ v(t) = s(t) + n(t)$$

$a_k = \pm 1$ for 1 and 0

$p(t)$: pulse shape

- **Considering a single pulse**

  $v(t) = p(t) + n(t)$

  bit error rate depends on $\left(\frac{A}{\sigma}\right)^2$ or $\left(\frac{E_b}{N_0}\right)$,

  or signal-to-noise ratio at pulse peak

  $$- \quad R_0 = \frac{[p(t_0)]^2}{E\{[n(t_0)]^2\}}\ ,$$

  t_0 : time instant for pulse peak

A Simplified Model

- **Considering a specially designed filter h(t), H(ω)**

  $v'(t) = p'(t) + n'(t) = p(t)*h(t) + n(t)*h(t)$

  a different pulse and a different noise

  \- $R_0' = \dfrac{[p'(t_0')]^2}{E\{[n'(t_0')]^2\}}$

  t_0' : time instant for peak of $p'(t)$

- **Matched Filtering**

  Finding an optimal filter h(t) or H(ω) , such that R_0' = max given p(t) and n(t)

  \- $R_0' = \dfrac{\left| \frac{1}{2\pi} \int_{-\infty}^{\infty} P(\omega)H(\omega)e^{j\omega t_0'} d\omega \right|^2}{\frac{1}{2\pi} \int_{-\infty}^{\infty} S_n(\omega)|H(\omega)|^2 d\omega} = \max$

  Schwartz's inequality

  $\left| \int_{-\infty}^{\infty} f_1(x)f_2^*(x)dx \right|^2 \leq \int_{-\infty}^{\infty} |f_1(x)|^2 dx \int_{-\infty}^{\infty} |f_2(x)|^2 dx$

  equality holds iff $f_1(x) = k\, f_2(x)$

A Simplified Model

- **Matched Filter Solution**

$$H(\omega) = \frac{P^*(\omega)}{S_n(\omega)} e^{-j\omega t_0'}$$

- matching in frequency domain

$$|\,H(\omega)\,| = \frac{|P(\omega)|}{S_n(\omega)}$$

- emphasizing those frequencies where signal level is relatively high with respect to noise

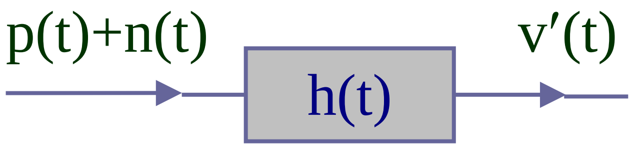
- for white noise

$$S_n(\omega) = \frac{N_0}{2}\ \text{, all } \omega$$

$$H(\omega) = P^*(\omega)\, e^{-j\omega t_0'}$$

$$h(t) = p(t_0' - t)$$

matching in time domain



$$v'(t) = \int_{-\infty}^{\infty} [\, p(\tau) + n(\tau)\,]\, h(t-\tau)d\tau$$

$$v'(t_0') = \int_{-\infty}^{\infty} [\, p(\tau) + n(\tau)\,]\, p(\tau)d\tau$$

emphasizing those time instants where signal level is relatively high with respect to noise

Ref : 4.2 of Haykin