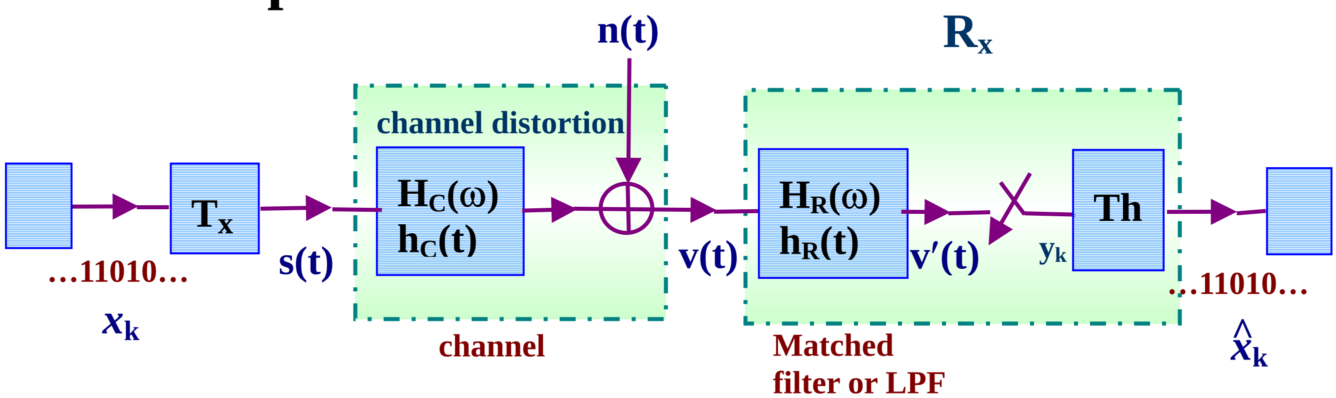
3.4 Intersymbol Interference and Raised Cosine Pulse Shape

Intersymbol Interference

- **A Simplified Model**



$$s(t) = \sum_k a_k\, p(t-kT)$$

$a_k = \pm 1$ for 1 and 0

p(t) : pulse shape for transmitted signal

$$v(t) = s(t) * h_c(t) + n(t)$$

$$v'(t) = \sum_k a_k\, g(t-kT) + n'(t)$$

$$g(t) = p(t) * h_C(t) * h_R(t)$$

$$n'(t) = n(t) * h_R(t)$$

$$y_k = v'(t_0' + kT)\ , \text{ all } k$$

$$y_j = v'(t_0' + jT) = \sum_k a_k g(t_0' + (j-k)T) + n'(t_0' + jT)$$

$$= \underbrace{a_j\, g(t_0')}_{\text{desired sample}} + \underbrace{\sum_{\substack{k \\ k\neq j}} a_k g(t_0' + (j-k)T)}_{\text{Intersymbol Interference}} + \underbrace{n'(t_0' + jT)}_{\text{channel noise}}$$

- the tail of $g(t) = p(t) * h_c(t) * h_R(t)$ extends across one or more bit durations, thus causing interference to other symbols

Intersymbol Interference

- **Ideal Nyquist Channel**

  - Conditions for zero intersymbol interference can be derived

$$g(t_0'+mT) = \begin{cases} 1 & \text{if} \quad m = 0 \\ 0 & \text{if} \quad m = \pm1, \pm2, \pm3, \ldots \end{cases}$$

  - Optimal g(t) with minimum bandwidth

$$g(t) = \frac{\sin(2\pi Wt)}{2\pi Wt} = \text{sinc}(2Wt)$$

$$G(f) = \begin{cases} \frac{1}{2W} \quad , & |f| < W = \frac{1}{2T}, \ f = \frac{\omega}{2\pi} \\ 0 \quad , & \text{else} \end{cases}$$

transmission bandwidth

$$B = W = \frac{1}{2T} = \frac{r}{2}$$

Ideal Nyquist Channel

See Figs. 4.8 , 4.9 , p. 263 of Haykin

  - Ideal Nyquist Channel is not practical

g(t) extends to $\pm\infty$

G(W) has abrupt transition

g(t) decreases at $\frac{1}{|t|}$, which gives serious errors when sampling instant is not precise

Raised Cosine Pulse Shape

- **Zero Interference Condition satisfied at wider transmission bandwidth**

$$G(f) = \begin{cases} \dfrac{1}{2W} & , 0 \le |f| \le f_1 \\ \dfrac{1}{4W}\left\{1-\sin\left[\dfrac{\pi(|f|-W)}{2(W-f_1)}\right]\right\}, & f_1 \le |f| \le 2W-f_1 \\ 0 & , |f| > 2W-f_1 \end{cases}$$

$$W = \frac{1}{2T} = \frac{r}{2}$$

$$g(t) = \text{sinc}(2Wt)\left[\frac{\cos(2\pi\alpha Wt)}{1-16\alpha^2W^2t^2}\right]$$

ideal Nyquist pulse

decreases at $\dfrac{1}{|t|^2}$

$\alpha = 1 - \dfrac{f_1}{W}$ rolloff factor

transmission bandwidth

$B = 2W - f_1 = W(1+\alpha)$

See Fig. 4.10 , p. 265 of Haykin

- Full-cosine rolloff

$\alpha = 1$, $B = 2W$, $f_1 = 0$

Ref : 4.4, 4.5 of Haykin