1-4

Summary

$\color{blue}{A}: \color{green}{m} \times \color{red}{n}$ $\quad$ $\boldsymbol{x} \in R^n$ $\quad$ $\boldsymbol{b} \in R^m$

Is $\boldsymbol{b}$ a ***linear combination*** of columns of A?

Is $\boldsymbol{b}$ in the ***span*** of the columns of A?

NO

No solution

YES

The columns of A are ***independent***.

Rank A = n

Nullity A = 0

Unique solution

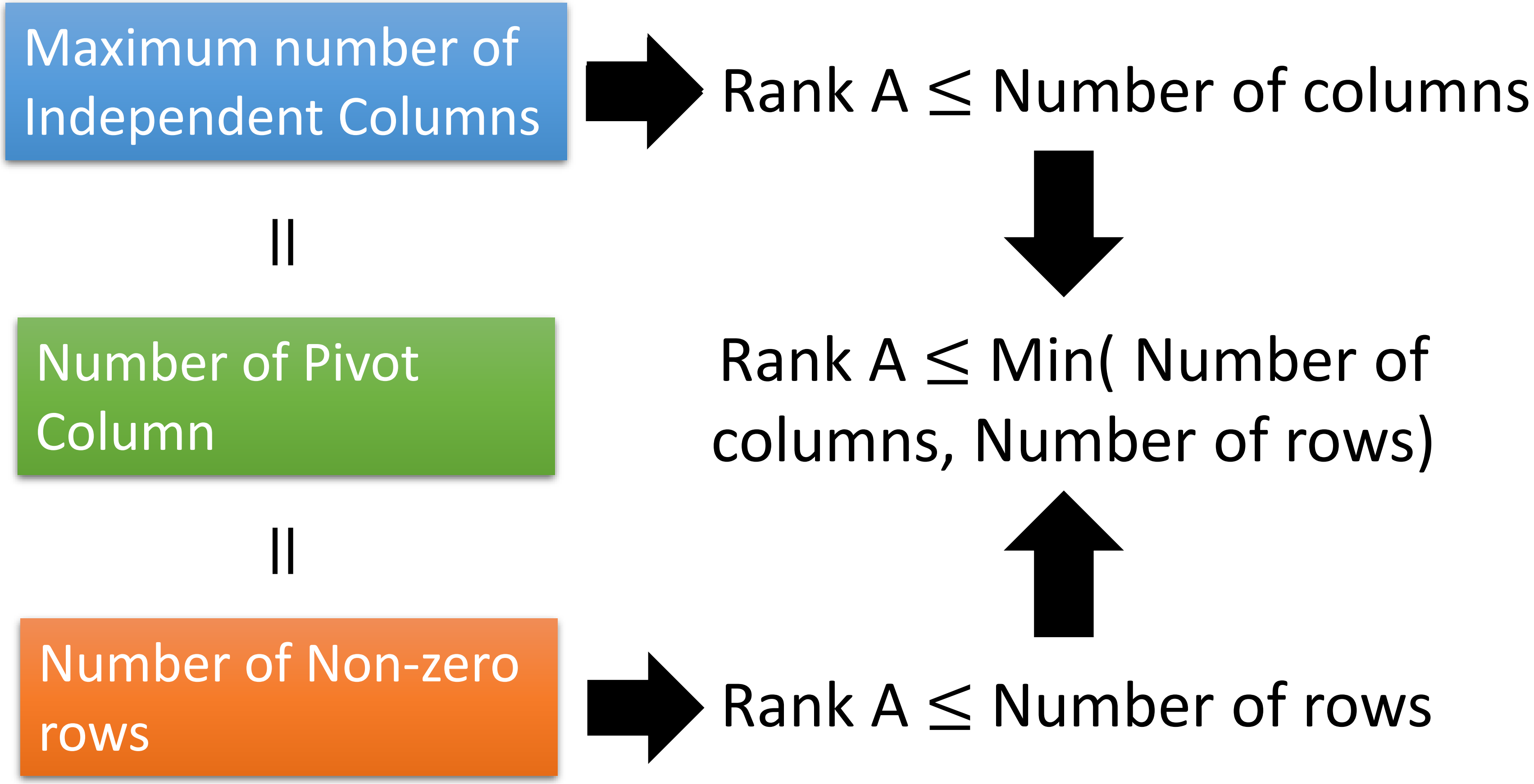
The columns of A are ***dependent***.

Rank A < n

Nullity A > 0

Infinite solution

Rank


Maximum number of Independent Columns
Number of Pivot Column
Number of Non-zero rows
Rank A ≤ Number of columns
Rank A ≤ Min(Number of columns, Number of rows)
Rank A ≤ Number of rows

73. Describe an $m \times n$ matrix with rank 0

79. What is the largest possible rank of an $m \times n$ matrix?

79. What is the smallest possible nullity of an $m \times n$ matrix?

82. Let A be an $\mathrm{m} \times n$ matrix and $\boldsymbol{b}$ be a vector in R^m. What must be true about the rank of A if $A\boldsymbol{x} = \boldsymbol{b}$ has a unique solution?

Rank $A =$?

83. A system of linear equations is called *underdetermined* if it has fewer equations than variables. What can be said about the number of solutions of an *underdetermined* system?

84. A system of linear equations is called *overdetermined* if it has fewer equations than variables. What can be said about the number of solutions of an *overdetermined* system?

Summary

$A: m \times n$ $\quad \boldsymbol{x} \in R^n$ $\quad \boldsymbol{b} \in R^m$

Is b a ***linear combination*** of columns of A?

Is b in the ***span*** of the columns of A?

NO

No solution

YES

The columns of A are ***independent***.

Rank A = n

Nullity A = 0

Unique solution

The columns of A are ***dependent***.

Rank A < n

Nullity A > 0

Infinite solution

Ax =b is consistent for ***every*** b $A: m \times n$

$\Vert$

Every b is in the span of the columns of A= $[a_1 \quad \cdots \quad a_n]$

$\Vert$

Every b belongs to $Span\{a_1, \quad \cdots \quad ,a_n\}$

$\Vert$

$Span\{a_1, \quad \cdots \quad ,a_n\} = R^m$

$\Vert$

RREF of [A b] cannot have a row whose only non-zero entry is at the last column

$\Vert$

RREF of A cannot have zero row

$\Vert$

Rank A = no. of rows

81. Let A be a 4×3 matrix. Is it possible that $A\boldsymbol{x} = \boldsymbol{b}$ is consistent for every $\boldsymbol{b}$ in R^4?

85. Prove that if A is an $m \times n$ matrix with rank m, then $A\boldsymbol{x} = \boldsymbol{b}$ is consistent for every $\boldsymbol{b}$ in R^m.

86. Prove that a matrix equation $A\boldsymbol{x} = \boldsymbol{b}$ is consistent if and only if the ranks of A and $[A \quad \boldsymbol{b}]$ are equal.

Chapter 1: Review

74

Let A be an $m \times n$ matrix with reduced row echelon form R. Describe the reduced row echelon form of each of the following matrices.

(a) $[A \quad \mathbf{0}]$

(b) $[\boldsymbol{a_1} \quad \boldsymbol{a_2} \quad \cdots \quad \boldsymbol{a_k}]$ for $k < n$

(c) cA, where c is a nonzero scalar

(d) $[I_m \quad A]$

(e) $[A \quad cA]$, where c is any scalar