1-7

86

Let $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \dots, \boldsymbol{u}_k\}$ be a linearly independent set of vectors in R^n, and let $\boldsymbol{v}$ be a vector in R^n such that $\boldsymbol{v} = c_1\boldsymbol{u}_1 + c_2\boldsymbol{u}_2 + \cdots + c_k\boldsymbol{u}_k$ for some scalars $c_1, c_2, \dots, c_k$ with $c_1 \neq 0$. Prove that $\{\boldsymbol{v}, \boldsymbol{u}_2, \dots, \boldsymbol{u}_k\}$ is linear independent.

87

Let $\boldsymbol{u}$ and $\boldsymbol{v}$ be distinct vectors in R^n. Prove that the set $\{\boldsymbol{u}, \boldsymbol{v}\}$ is linearly independent if and only if the set $\{\boldsymbol{u} + \boldsymbol{v}, \boldsymbol{u} - \boldsymbol{v}\}$ is linearly independent.

89

Prove that if $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_k\}$ is a linearly independent subset of R^n and $c_1, c_2, \ldots, c_k$ are nonzero scalars, then $\{c_1\boldsymbol{u}_1, c_2\boldsymbol{u}_2, \ldots, c_k\boldsymbol{u}_k\}$ is also linearly independent.

$$c_1u_1 + c_2u_2 + \cdots + c_ku_k = 0 \Rightarrow c_1 = c_2 = \cdots c_k = 0$$

$$a_1(c_1u_1) + a_2(c_2u_2) + \cdots a_k(c_ku_k) = 0$$

$$\Rightarrow (a_1c_1)u_1 + (a_2c_2)u_2 + \cdots + (a_kc_k)u_k = 0$$

$$\Rightarrow a_1c_1 = a_2c_2 = \cdots = a_kc_k = 0$$

$$a_1 = 0,\ c_1 \neq 0,\quad a_2 = 0,\ c_2 \neq 0,\quad a_k = 0$$

$$\Rightarrow a_1 = a_2 = \cdots = a_k = 0$$

Ind.

93

Let S = $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_k\}$ be a nonempty set of vectors from R^n. Prove that if S is linearly independent, then every vector v in $Span\ S$ can be written as $c_1\boldsymbol{u}_1 + c_2\boldsymbol{u}_2 + \cdots + c_k\boldsymbol{u}_k$ for unique scalars $c_1, c_2, \ldots, c_k$.

$v =$

$V = c_1u_1 + c_2u_2 + \cdots\ c_ku_k$ $\quad c_1 \cdots c_k$

$V = b_1u_1 + b_2u_2 + \cdots\ b_ku_k$ $\quad b_1 \cdots b_k$ > different

$\Rightarrow 0 = (c_1-b_1)u_1 + (c_2-b_2)u_2 + \cdots + (c_k-b_k)u_k$

$\Rightarrow c_1-b_1 = c_2-b_2 = \cdots = c_k-b_k = 0$

$\Rightarrow c_1=b_1,\ c_2=b_2, \cdots\ c_k=b_k$ 矛盾

94

Let S $= \{\boldsymbol{u}_1, \boldsymbol{u}_2, \dots, \boldsymbol{u}_k\}$ be a nonempty set of vectors from R^n. Prove that if S is linearly independent, then every vector in $Span\ S$ can be written as $c_1\boldsymbol{u}_1 + c_2\boldsymbol{u}_2 + \cdots + c_k\boldsymbol{u}_k$ for *unique* scalars $c_1, c_2, \dots, c_k$.

State and prove the converse of Exercise 93.

$v \in \text{Span } S$

$v = c_1u_1 + \cdots + c_ku_k$

$c_1 \cdots c_k$ unique

$\Rightarrow S$ ind

$0 \in \text{Span } S$

$0 = c_1u_1 + c_2u_2 + \cdots c_ku_k$

$\Rightarrow c_1 = c_2 = \cdots = c_k = 0$

95

Let S $= \{\boldsymbol{u}_1, \boldsymbol{u}_2, \dots, \boldsymbol{u}_k\}$ be a nonempty subset of R^n and A be an $m \times n$ matrix. Prove that if S is linearly dependent, and $S' = \{A\boldsymbol{u}_1, A\boldsymbol{u}_2, \dots, A\boldsymbol{u}_k\}$ contains k distinct vectors, then S' is linearly dependent.

Exist $c_1, c_2 \cdots c_k$ not all zero $\quad c_1u_1 + c_2u_2 + \cdots + c_ku_k = 0$

$\Rightarrow A(c_1u_1) + A(c_2u_2) + \cdots + A(c_ku_k) = A0 = 0$

$\Rightarrow c_1(Au_1) + c_2(Au_2) + \cdots + c_k(Au_k) = 0$ dependent

Exist $b_1 \cdots b_k$ not all zero $\quad b_1(Au_1) + b_2(Au_2) + \cdots + b_k(Au_k) = 0$

96

Let S $= \{\boldsymbol{u}_1, \boldsymbol{u}_2, \dots, \boldsymbol{u}_k\}$ be a nonempty subset of R^n and A be an $m \times n$ matrix. Prove that if S is linearly ~~dependent~~ ind, and $S' = \{A\boldsymbol{u}_1, A\boldsymbol{u}_2, \dots, A\boldsymbol{u}_k\}$ contains k distinct vectors, then S' is linearly ~~dependent.~~ ind.

Give an example to show that the preceding exercise is false if *linearly dependent* is changed to *linearly independent*.

u_1 u_2

ind.

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

2×3

$A = I$

Au_1 Au_2

dep

de. ← $\boxed{A}$ ← de.

ind. ↖ $\boxed{A}$ ← ind.

dep. ↙