## 1-7

Let  $\{u_1, u_2, ..., u_k\}$  be a linearly independent set of vectors in  $\mathbb{R}^n$ , and let v be a vector in  $\mathbb{R}^n$  such that  $v = c_1 u_1 + c_2 u_2 + \cdots + c_k u_k$  for some scalars  $c_1, c_2, ..., c_k$  with  $c_1 \neq 0$ . Prove that  $\{v, u_2, ..., u_k\}$  is linear independent.

Let u and v be distinct vectors in  $\mathbb{R}^n$ . Prove that the set  $\{u, v\}$  is linearly independent if and only if the set  $\{u + v, u - v\}$  is linearly independent.

Prove that if 
$$\{u_1, u_2, ..., u_k\}$$
 is a linearly independent subset of  $\mathbb{R}^n$   
and  $c_1, c_2, ..., c_k$  are nonzero scalars, then  $\{c_1 u_1, c_2 u_2, ..., c_k u_k\}$  is  
also linearly independent.  
$$C_1 \mathcal{U}_1 + (\mathcal{U}_2 \mathcal{U}_2 + \cdots + \mathcal{L}_k \mathcal{U}_k = 0 \implies \mathcal{L}_1 = (\mathcal{U}_2 = \cdots + \mathcal{L}_k = 0)$$
$$\mathcal{O}_1 (c_1 \mathcal{U}_1) + \mathcal{O}_2 (c_2 \mathcal{U}_2) + \cdots + \mathcal{O}_k (c_k \mathcal{U}_k) = 0$$
$$\implies (\mathcal{A}, c_1) \mathcal{U}_1 + (\mathcal{O}_2 c_2) \mathcal{U}_2 + \cdots + (\mathcal{O}_k c_k) \mathcal{U}_k = 0$$
$$\implies \mathcal{O}_1 C_1 = \mathcal{O}_2 (\mathcal{O}_2 = \cdots = \mathcal{O}_k (\mathcal{U}_k = 0)$$
$$\implies \mathcal{O}_1 = \mathcal{O}_2 = \cdots = \mathcal{O}_k (\mathcal{U}_k = 0)$$

Let  $S = \{u_1, u_2, ..., u_k\}$  be a nonempty set of vectors from  $\mathbb{R}^n$ . Prove that if S is linearly independent, then every vector in *Span S* can be written as  $c_1u_1 + c_2u_2 + \cdots + c_ku_k$  for *unique* scalars  $c_1, c_2, ..., c_k$ .

$$V = C_1 M_1 + (2M_2 + \cdots + C_k M_k) \qquad C_1 - C_k M_k \qquad C_1 - C_k M_k \qquad b_1 - b_k \end{pmatrix} c_1 \text{ flevent}$$

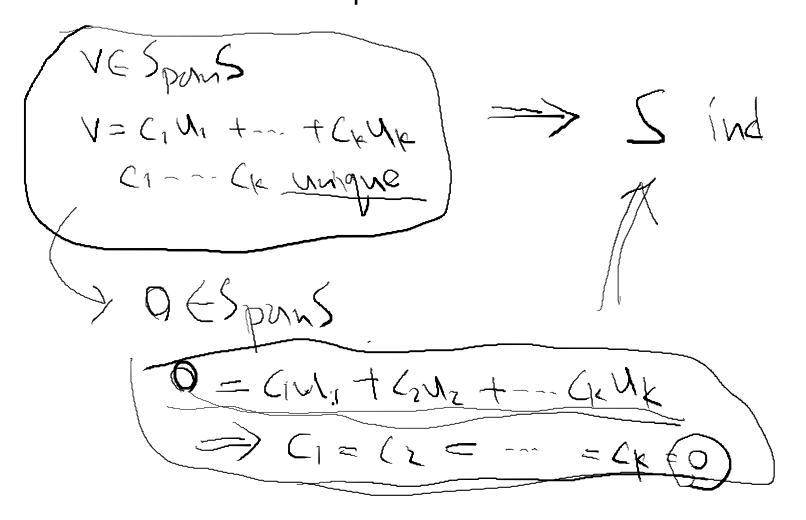
$$\Rightarrow O = (C_1 - b_1) M_1 + (C_2 - b_2) M_2 + \cdots + (C_k - b_k) M_k$$

$$\Rightarrow C_1 - b_1 = C_2 - b_2 = \cdots = C_k - b_k = 0$$

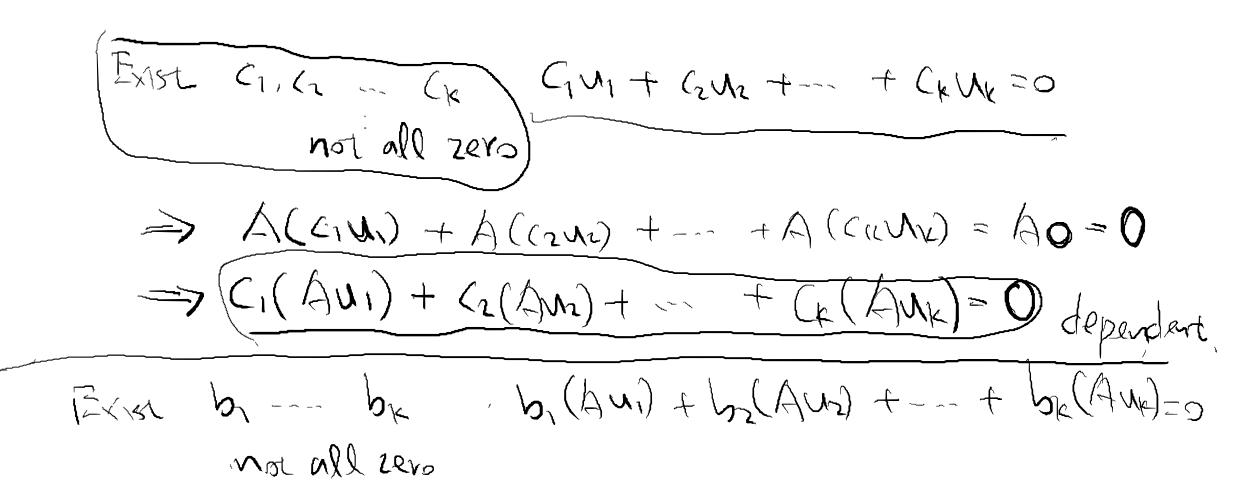
今 Ci=h, Ci=hz:~~ Ck=hk 矛盾

93

Let  $S = \{u_1, u_2, ..., u_k\}$  be a nonempty set of vectors from  $\mathbb{R}^n$ . Prove that if S is linearly independent, then every vector in Span S can be written as  $c_1u_1 + c_2u_2 + \cdots + c_ku_k$  for *unique* scalars  $c_1, c_2, ..., c_k$ . State and prove the converse of Exercise 93.



Let  $S = \{u_1, u_2, ..., u_k\}$  be a nonempty subset of  $R^n$  and A be an  $m \times n$  matrix. Prove that if S is linearly dependent, and  $S' = \{Au_1, Au_2, ..., Au_k\}$  contains k distinct vectors, then S' is linearly dependent.



Let  $S = \{u_1, u_2, \dots, u_k\}$  be a nonempty subset of  $\mathbb{R}^n$  and A be an  $m \times n$  matrix. Prove that if S is linearly dependent, and S' =96  $\{Au_1, Au_2, \dots, Au_k\}$  contains k distinct vectors, then S' is linearly dependent. Give an example to show that the preceding exercise is false if *linearly dependent* is changed to *linearly independent*.  $A = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ dep\_