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58. Let $A = A_{180^\circ}$, and let B be the matrix that reflects R^2 about the x-axis; that is,

$$B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Compute BA, and describe geometrically how a vector v is affected by multiplication by BA.

60. Let A be an $n \times n$ matrix. If A and B are $n \times n$ diagonal matrices, then AB is a diagonal matrix whose jth columns is $a_{jj}b_{jj}\boldsymbol{e}_j$.

59. A square matrix A is called **lower triangular** if the (i, j)-entry of A is zero whenever $i < j$. Prove that if A and B are both $n \times n$ lower triangular matrices, then AB is also a lower triangular matrix.

61. A square matrix A is called **upper triangular** if the (i, j)-entry of A is zero whenever $i > j$. Prove that if A and B are both $n \times n$ upper triangular matrices, then AB is also an upper triangular matrix.

62. Let $A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ -2 & 1 & -1 & 3 \\ -1 & -1 & 4 & 3 \\ -5 & 3 & -4 & 7 \end{bmatrix}$. Find a nonzero 4×2 matrix B with rank 2 such that $AB = O$

63. Find an example of $n \times n$ matrices A and B such that $AB = O$, but $BA \neq O$.

64. Let A and B be $n \times n$ matrices. Prove and disprove that the ranks of AB and BA are equal.

65. Recall the definition of *trace* of a matrix. Prove that if A is an $m \times n$ matrix and B is an $n \times m$ matrix, then $trace(\mathrm{AB}) = trace(\mathrm{BA})$.

66. Let $1 \leq r, s \leq n$ be integers, and let E be the $n \times n$ matrix with 1 as the (r, s)-entry and 0s elsewhere. Let B be any $n \times n$ matrix. Describe EB in terms of the entries of B.

68. (a) Let A and B be symmetric matrices of the same size. Prove that AB is symmetric if and only if $AB = BA$.

(b) Find symmetric 2×2 matrices A and B such that $AB \neq BA$.