Invertible

Summary

- Let A be an n x n matrix. A is invertible if and only if
  - The columns of A span R^n
  - For every b in R^n, the system Ax=b is consistent
  - The rank of A is n
  - The columns of A are linear independent
  - The only solution to Ax=0 is the zero vector
  - The nullity of A is zero
  - The reduced row echelon form of A is I_n
  - A is a product of elementary matrices
  - There exists an n x n matrix B such that $BA = I_n$
  - There exists an n x n matrix C such that $AC = I_n$

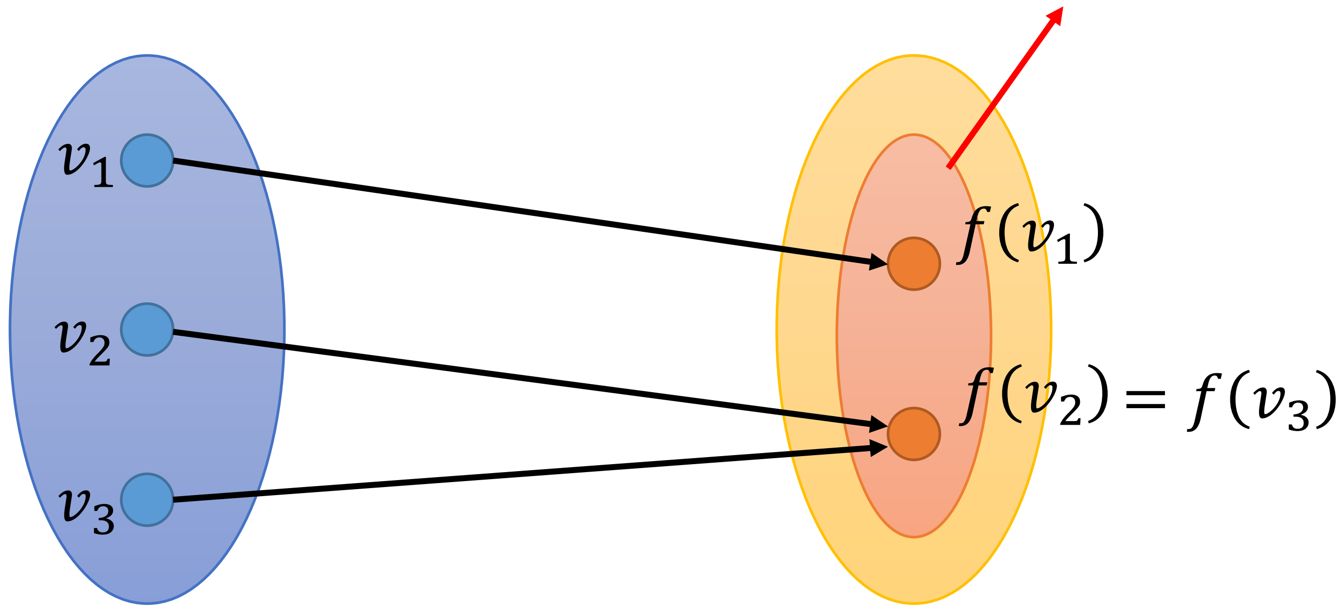
http://goo.gl/z3J5Rb

Review - Terminology

What ***actually*** come out of function f

- Given a function f

Range (值域)



Domain (定義域)

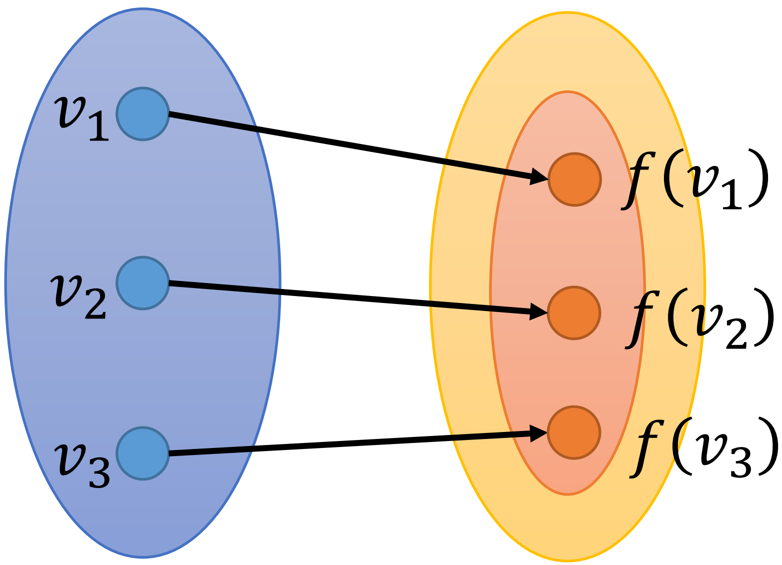
What can go into function f

Co-domain (對應域)

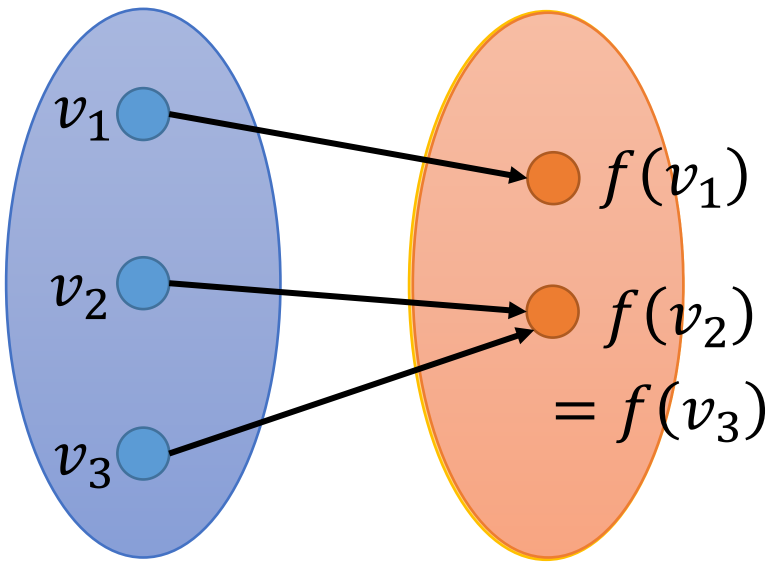
What ***may possible*** come out of function f

Review - Terminology

- one-to-one (一對一)



- Onto (映成)

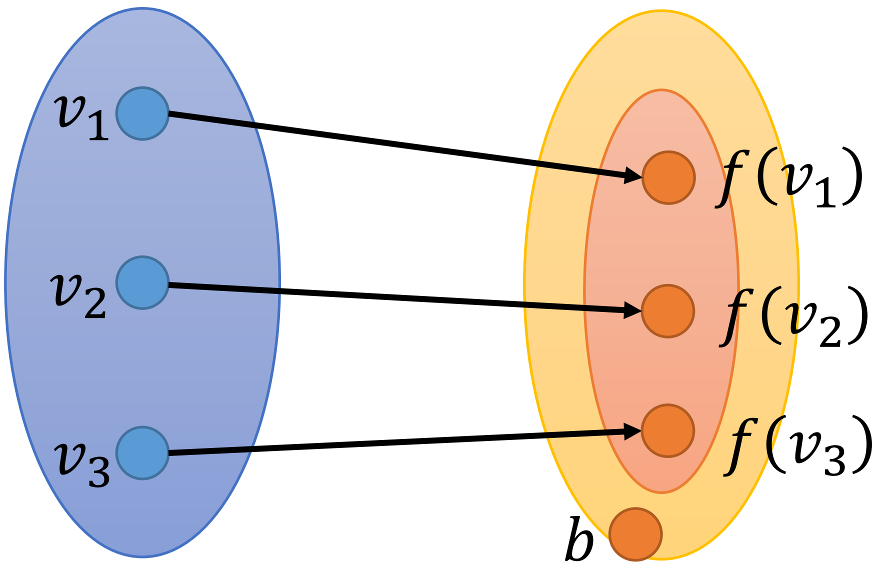


Co-domain = range

Review: One-to-one

2 x 3

- A function f is one-to-one



~~$f(x) = b$ has one solution~~

$f(x) = b$ has at most one solution

If co-domain is “smaller” than the domain, f cannot be one-to-one.

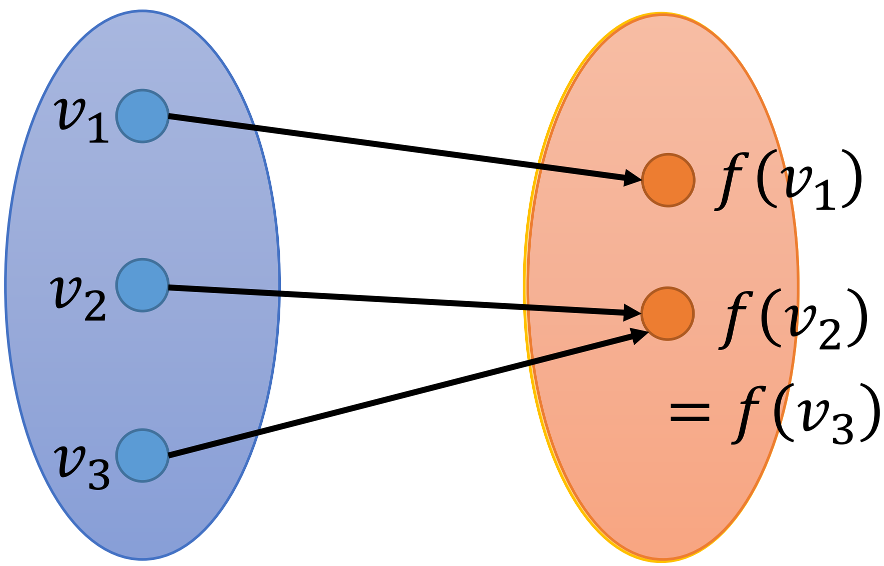
If a matrix A is 矮胖, it cannot be one-to-one.

The reverse is not true.

If a matrix A is one-to-one, its columns are independent.

Review: Onto

- A function f is onto



Co-domain = range

$f(x) = b$ always have solution

If co-domain is “larger” than the domain, f cannot be onto.

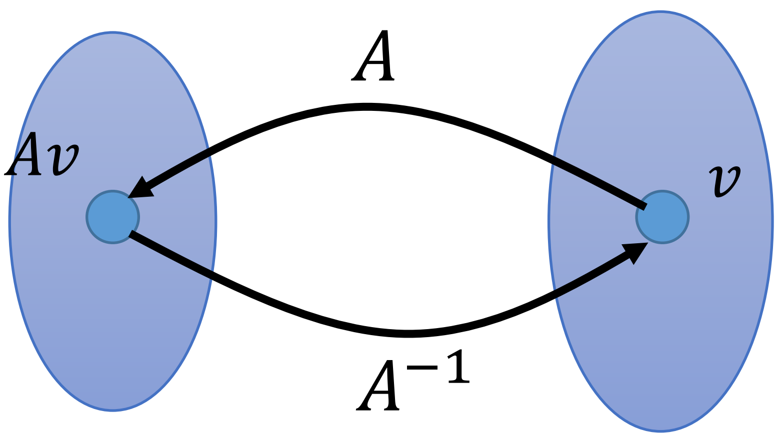
If a matrix A is 高瘦, it cannot be onto.

The reverse is not true.

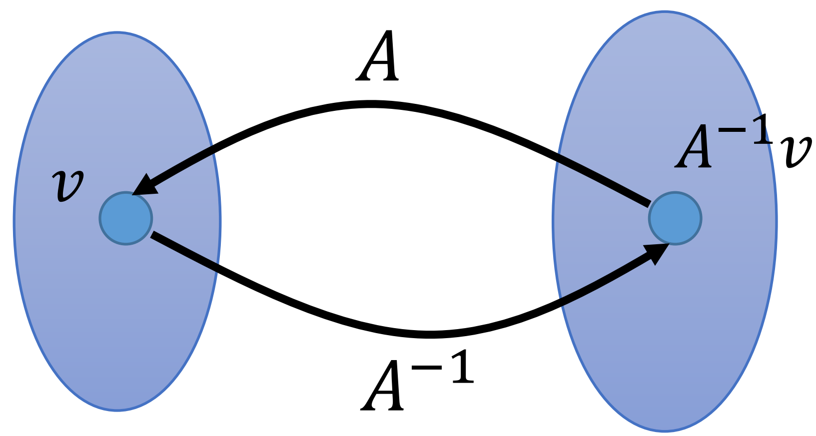
If a matrix A is onto, rank A = no. of rows.

Invertible

- A is called invertible if there is a matrix B such that $AB = I$ and $BA = I$ ($B = A^{-1}$)



A must be one-to-one



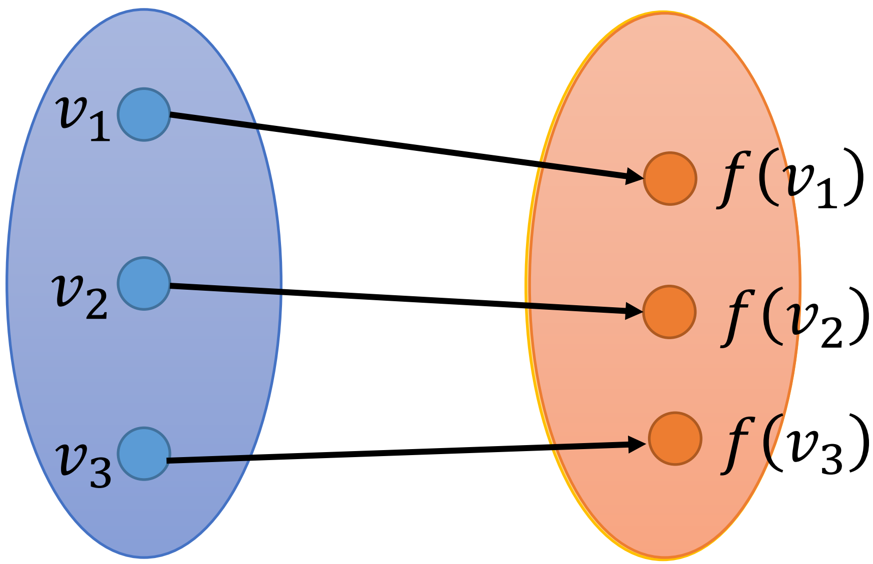
A must be onto

(不然 A^{-1} 的 input 就會有限制)

One-to-one and onto

An invertible matrix A is always square.

- A function f is one-to-one and onto



The domain and co-domain must have "the same size".

The corresponding matrix A is square.

One-to-one ⟺ Onto

在 Square 的前提下，要就都成立，要就都不成立

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