# Reduced Row Echelon Form (RREF)

# Solving system of linear equation

A ***complex*** system of linear equations

A ***simple*** system of linear equations

$A\mathbf{x} = \mathbf{b}$ ⟷ equivalent ⟷ $R'\mathbf{x} = \mathbf{b}'$

$A' = [\, A \;\; \mathbf{b} \,]$ → $A''$ → $A'''$ → … → $R = [\, R' \; \mathbf{b}' \,]$

…

Reduced Row Echelon Form (RREF)

elementary row operations:

1. Interchange any two rows of the matrix
2. Multiply every entry of some row by the same nonzero scalar
3. Add a multiple of one row of the matrix to another row

階層

# Reduced Row Echelon Form

- A system of linear equations is easily solvable if its augmented matrix is in ***<u>reduced row echelon form</u>***
- ***<u>Row Echelon Form (REF)</u>***

1. Each nonzero row lies above every zero row

2. The leading entries are in echelon form

$$\begin{bmatrix} 1 & 7 & 2 & -3 & 9 & 4 \\ 0 & 0 & 1 & 4 & 6 & 8 \\ 0 & 0 & 0 & 2 & 3 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

# Reduced Row Echelon Form

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- ***Row Echelon Form (REF)***

1. Each nonzero row lies above every zero row

2. The leading entries are in echelon form

**NO**

$$\begin{bmatrix} 1 & 0 & 0 & 6 & 3 & 0 \\ 0 & 0 & 1 & 5 & 7 & 0 \\ 0 & 1 & 0 & 2 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

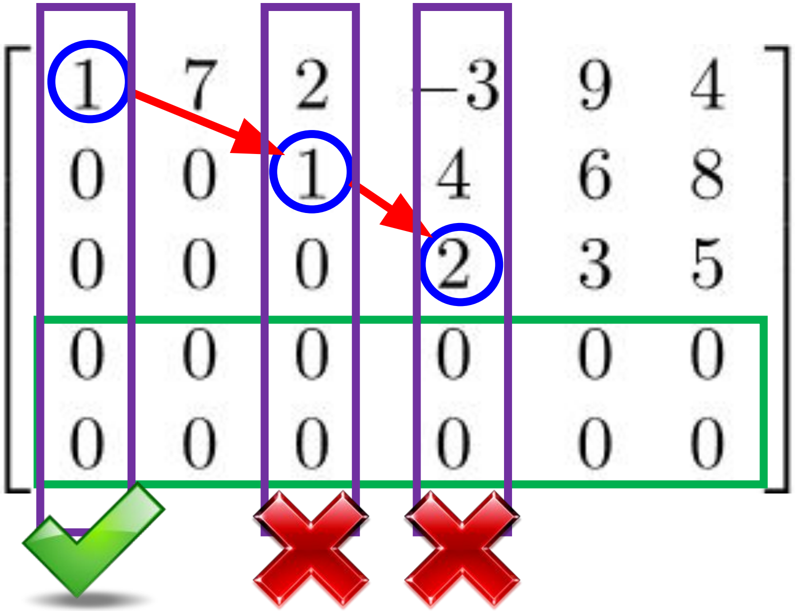
No zero rows

# Reduced Row Echelon Form

- A system of linear equations is easily solvable if its augmented matrix is in ***reduced row echelon form***
- ***Reduced Row Echelon Form (RREF)***

1-2 The matrix is in row echelon form

3. The columns containing the leading entries are standard vectors.



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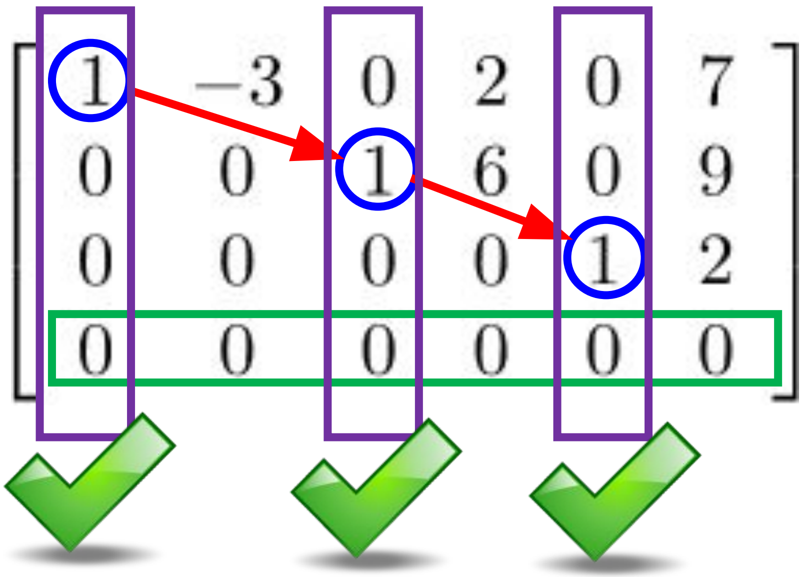

Pivot 中樞

# Reduced Row Echelon Form

A

$$\begin{bmatrix} 1 & 2 & -1 & 2 & 1 & 2 \\ -1 & -2 & 1 & 2 & 3 & 6 \\ 2 & 4 & -3 & 2 & 0 & 3 \\ -3 & -6 & 2 & 0 & 3 & 9 \end{bmatrix}$$

R

$$\begin{bmatrix} 1 & 2 & 0 & 0 & -1 & -5 \\ 0 & 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Leading Entry

The pivot positions of A are (1,1), (2,3) and (3,4).

The pivot columns of A are 1st, 3rd and 4th columns.

Not going to proof

# RREF is unique!

- A matrix can be transformed into multiple REF by row operation, but only one RREF

$$\begin{bmatrix} 1 & -2 & -1 & 3 \\ 3 & -6 & -5 & 3 \\ 2 & -1 & 1 & 0 \end{bmatrix}$$

REF

$$\begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 3 & 3 & -6 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

REF

$$\begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 3 & 3 & -6 \\ 0 & 0 & 3 & 9 \end{bmatrix}$$

REF

$$\begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 3 & 0 & -15 \\ 0 & 0 & 3 & 9 \end{bmatrix}$$

RREF

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$