Change Coordinate

Coordinate System

- Let vector set B=$\{u_1, u_2, \cdots, u_n\}$ be a basis for a subspace R^n

⇨ B is a coordinate system

- For any v in R^n, there are unique scalars $c_1, c_2, \cdots, c_n$ such that $v = c_1 u_1 + c_2 u_2 + \cdots + c_n u_n$

B -coordinate vector of v:

$$[v]_{\mathrm{B}} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} \in \mathcal{R}^n$$

(用 B 的觀點來看 v)

Other System → Cartesian

B= $\left\{\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\2\end{bmatrix}\right\}$

$$[v]_{\mathrm{B}} = \begin{bmatrix}3\\6\\-2\end{bmatrix}$$

$$v = 3\begin{bmatrix}1\\1\\1\end{bmatrix} + 6\begin{bmatrix}1\\-1\\1\end{bmatrix} - 2\begin{bmatrix}1\\2\\2\end{bmatrix} = \begin{bmatrix}7\\-7\\5\end{bmatrix}$$

C= $\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}, \begin{bmatrix}7\\8\\9\end{bmatrix}\right\}$

$$[u]_{\mathrm{C}} = \begin{bmatrix}3\\6\\-2\end{bmatrix}$$

$$u = 3\begin{bmatrix}1\\2\\3\end{bmatrix} + 6\begin{bmatrix}4\\5\\6\end{bmatrix} - 2\begin{bmatrix}7\\8\\9\end{bmatrix} = \begin{bmatrix}13\\20\\27\end{bmatrix}$$

Other System → Cartesian

- Let vector set B=$\{u_1, u_2, \cdots, u_n\}$ be a basis for a subspace R^n
- Matrix $\mathrm{B} = [u_1 \quad u_2 \quad \cdots \quad u_n]$

Given $[v]_{\mathrm{B}}$, how to find v? $[v]_{\mathrm{B}} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$

$$v = c_1 u_1 + c_2 u_2 + \cdots + c_n u_n$$

$$= B[v]_{\mathrm{B}} \quad \text{(matrix-vector product)}$$

Cartesian → Other System

$v = \begin{bmatrix} 1 \\ -4 \\ 4 \end{bmatrix}$ B= $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\}$ find $[\mathbf{v}]_B$

$$c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 4 \end{bmatrix} \quad [\mathbf{v}]_B = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 1 & 1 & 2 \end{bmatrix}$ B is invertible (?) independent

$$B[v]_B = v \Rightarrow [v]_B = B^{-1} v = \begin{bmatrix} -6 \\ 4 \\ 3 \end{bmatrix}$$

Cartesian ↔ Other System

- Let B = $\{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n\}$

$$[v]_{\mathrm{B}} = B^{-1}v$$

v → $[v]_{\mathrm{B}}$

$$v = B[v]_{\mathrm{B}}$$

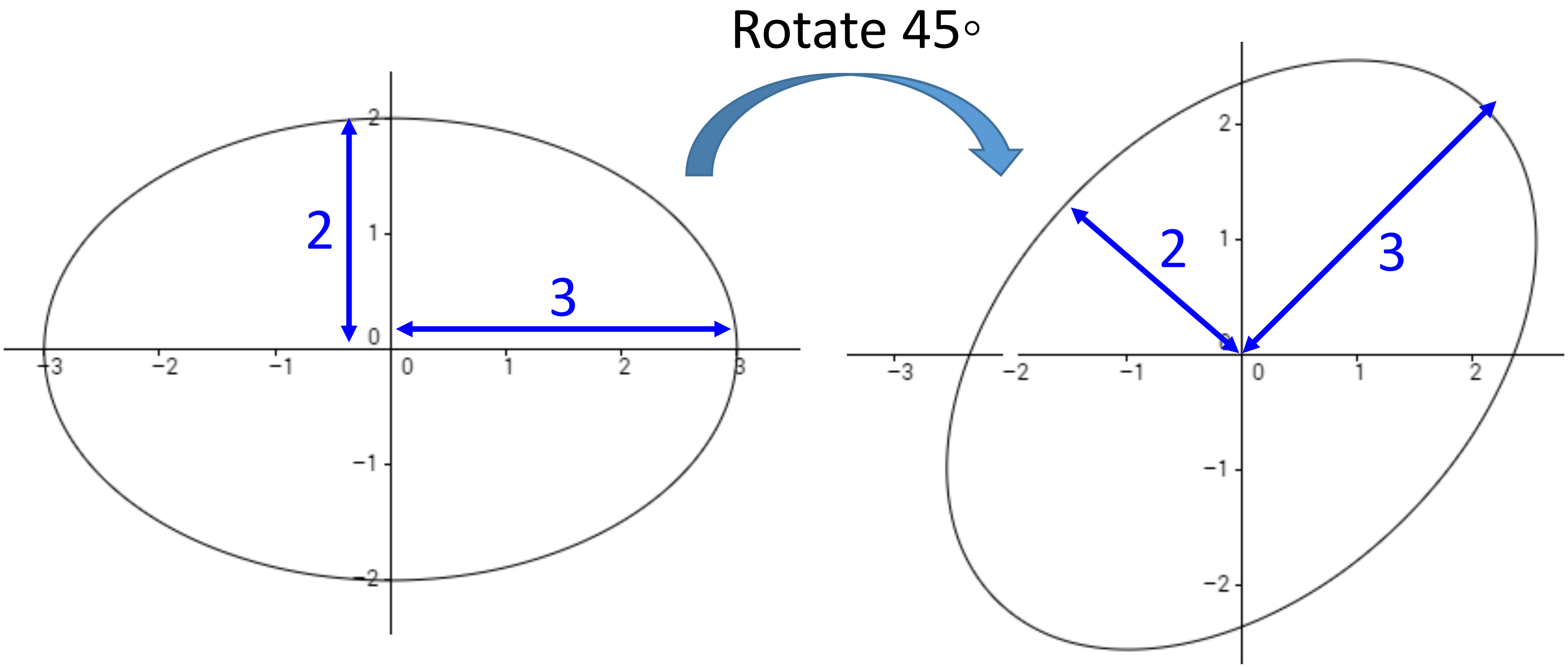
$$= c_1\mathbf{b}_1 + c_2\mathbf{b}_2 + \cdots + c_n\mathbf{b}_n$$

$$[v]_{\mathrm{B}} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

Let B=$\{b_1, b_2, \cdots, b_n\}$ be a basis of R^n. $[b_i]_{\mathrm{B}} =?$ e_i

(Standard vector)

Equation of ellipse

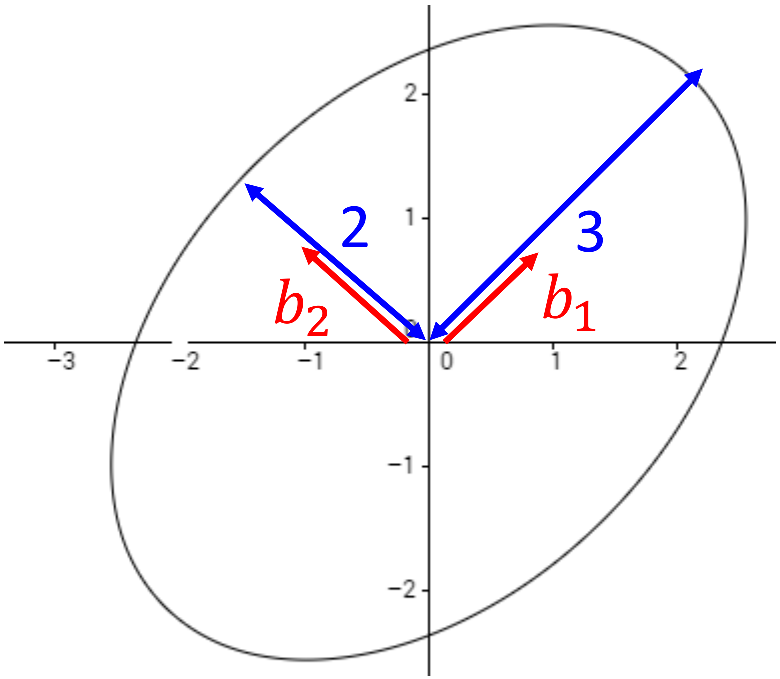


$$\frac{x^2}{3^2}+\frac{y^2}{2^2}=1$$

?

Equation of ellipse

Use another coordinate system



$$B = \left\{ \overset{b_1}{\begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}}, \overset{b_2}{\begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}} \right\}$$

What is the equation of the ellipse in the new coordinate system?

$$\frac{(x')^2}{3^2} + \frac{(y')^2}{2^2} = 1$$

Equation of ellipse

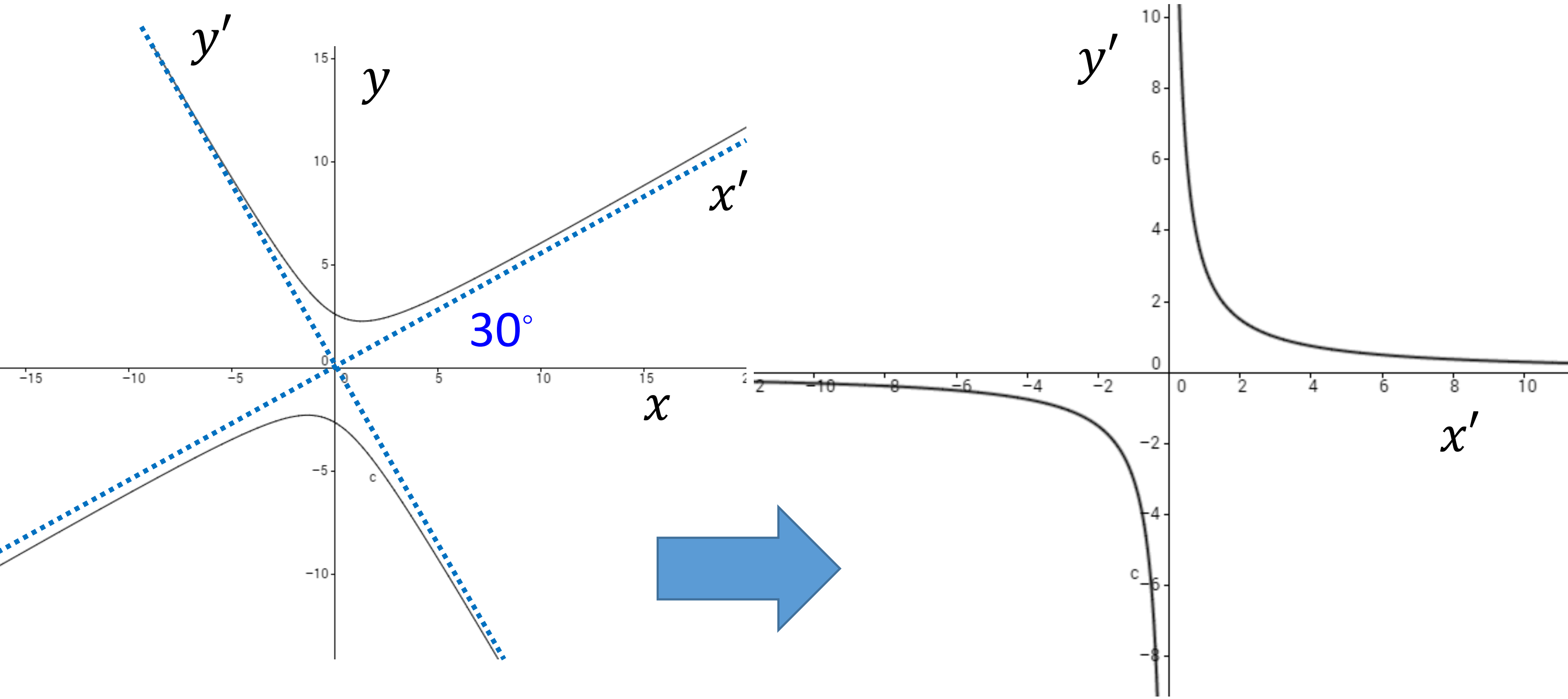
$$B = \left\{ \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}, \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix} \right\} \qquad \mathbf{v} = \begin{bmatrix} x \\ y \end{bmatrix}, \; [\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\frac{(x')^2}{3^2} + \frac{(y')^2}{2^2} = 1 \Rightarrow \frac{\left(\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y\right)^2}{3^2} + \frac{\left(-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y\right)^2}{2^2} = 1$$

$$[\mathbf{v}]_{\mathcal{B}} = B^{-1}\mathbf{v}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = B^{-1} \begin{bmatrix} x \\ y \end{bmatrix}$$

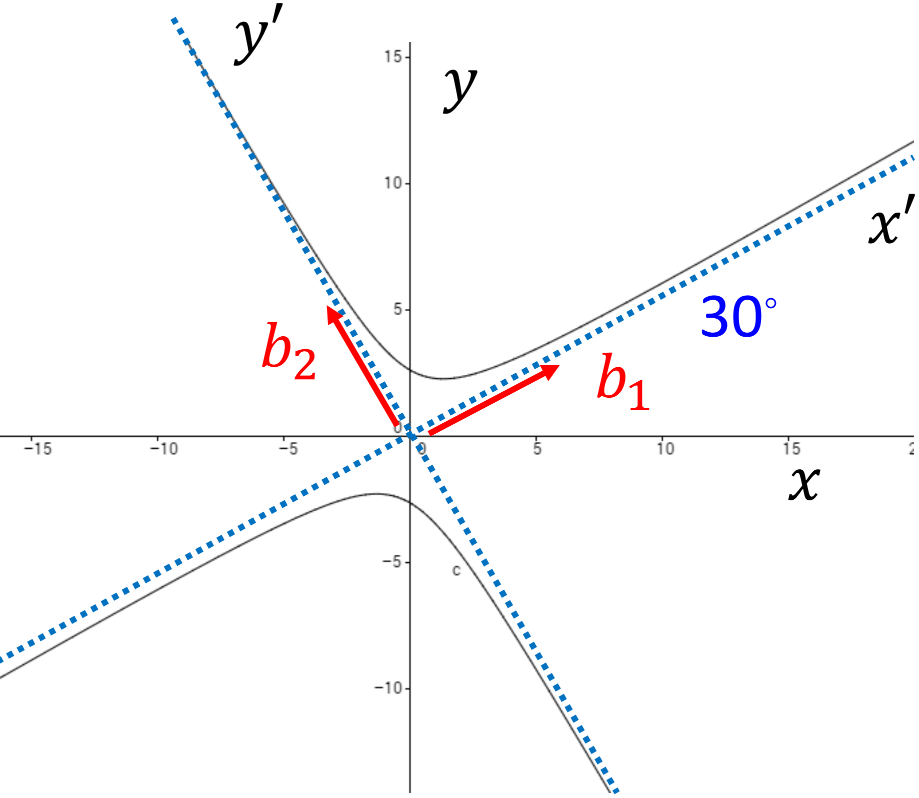
Equation of hyperbola



$$-\sqrt{3}x^2 + 2xy + \sqrt{3}y^2 = 12$$

$$equation?$$

Equation of hyperbola



$$-\sqrt{3}x^2 + 2xy + \sqrt{3}y^2 = 12$$

$$B = \begin{bmatrix} b_1 & b_2 \end{bmatrix}$$

$$b_1 = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} \quad b_2 = \begin{bmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$v = \begin{bmatrix} x \\ y \end{bmatrix} \quad [v]_{\mathrm{B}} = \begin{bmatrix} x' \\ y' \end{bmatrix} \quad v = B[v]_{\mathrm{B}}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Equation of hyperbola

$$B = [b_1 \quad b_2]$$

$$-\sqrt{3}x^2 + 2xy + \sqrt{3}y^2 = 12$$

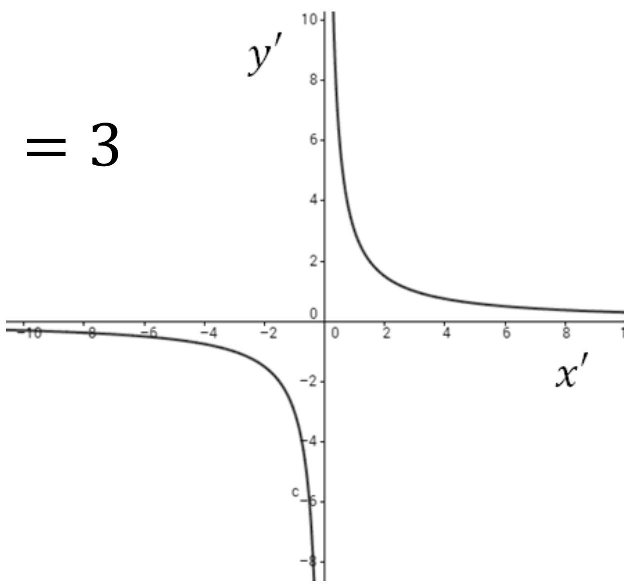
$$b_1 = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} \quad b_2 = \begin{bmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$x = \frac{\sqrt{3}}{2}x' - \frac{1}{2}y'$$

$$y = \frac{1}{2}x' + \frac{\sqrt{3}}{2}y'$$

$$v = \begin{bmatrix} x \\ y \end{bmatrix} \quad [v]_{\mathrm{B}} = \begin{bmatrix} x' \\ y' \end{bmatrix} \quad v = B[v]_{\mathrm{B}}$$

$$x'y' = 3$$



$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$