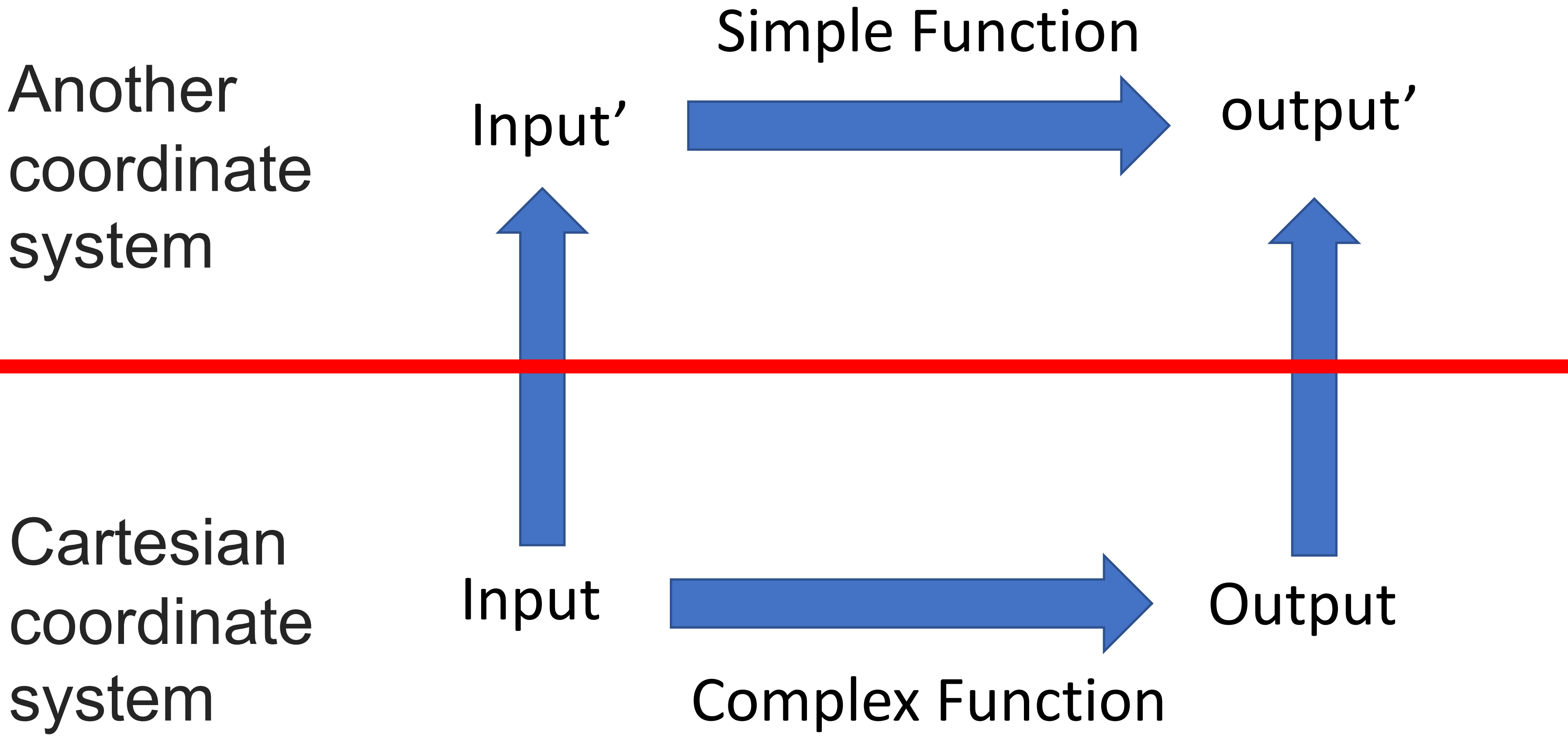
# Linear Function in Coordinate System

**A complex function in one coordinate system can be simple in other systems.**

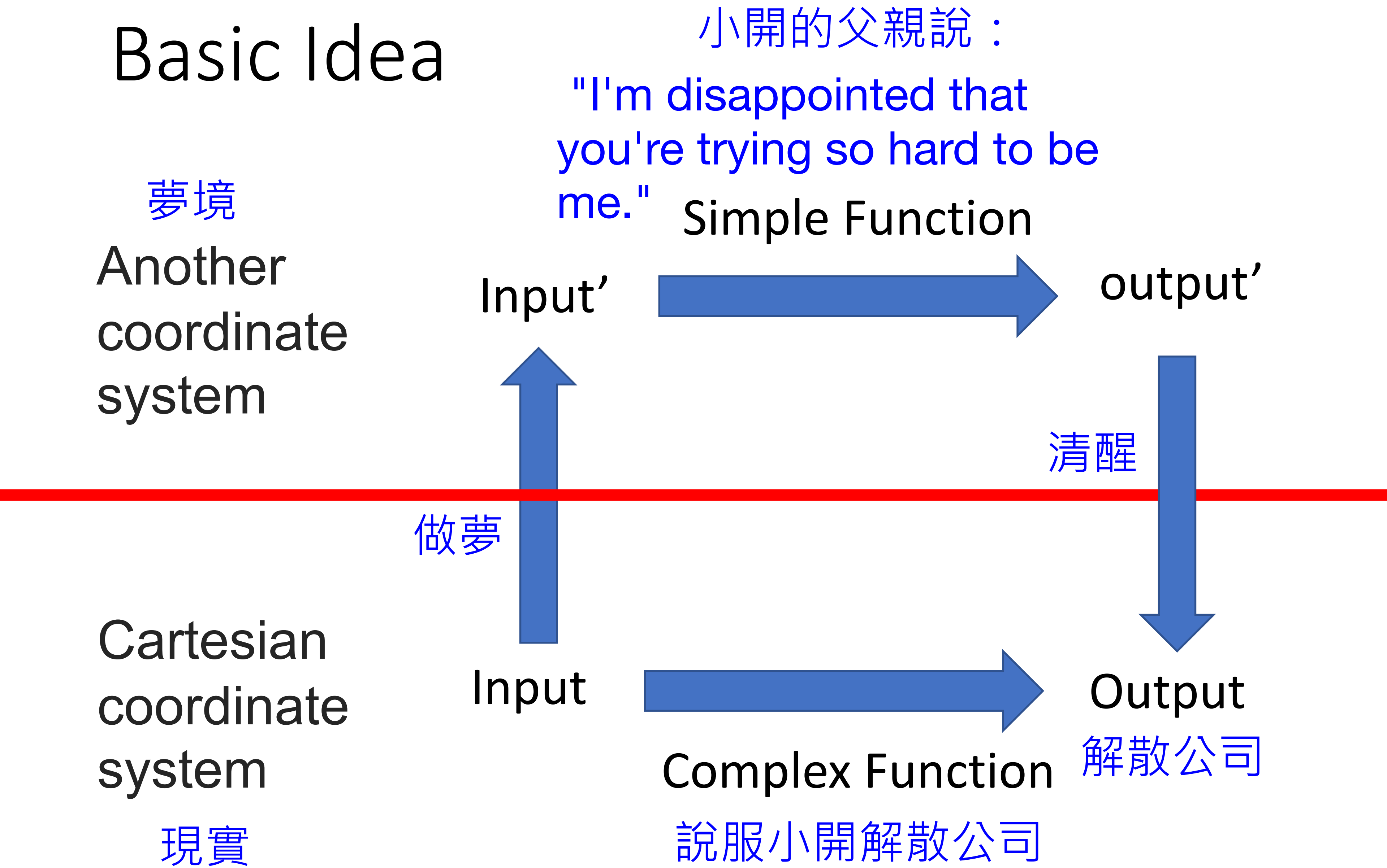
# Basic Idea



Simple Function
Another coordinate system
Input'
output'
Cartesian coordinate system
Input
Output
Complex Function


52 WEST
A FILM BY CHRISTOPHER NOLAN
INCEPTION
FROM THE DIRECTOR OF THE DARK KNIGHT

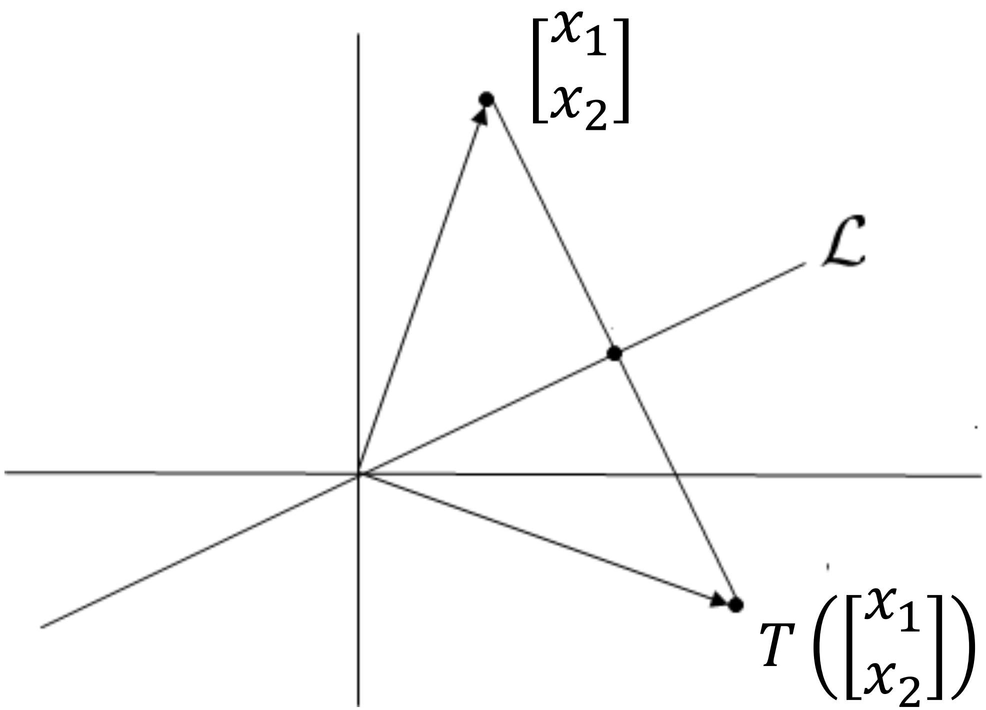
# Basic Idea



小開的父親說：
"I'm disappointed that you're trying so hard to be me."
Simple Function
夢境
Another coordinate system
Input'
output'
清醒
做夢
Cartesian coordinate system
Input
Output
解散公司
Complex Function
說服小開解散公司
現實

# Sometimes a function can be complex ……

- T: reflection about a line L through the origin in $R^2$




$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = ?$$

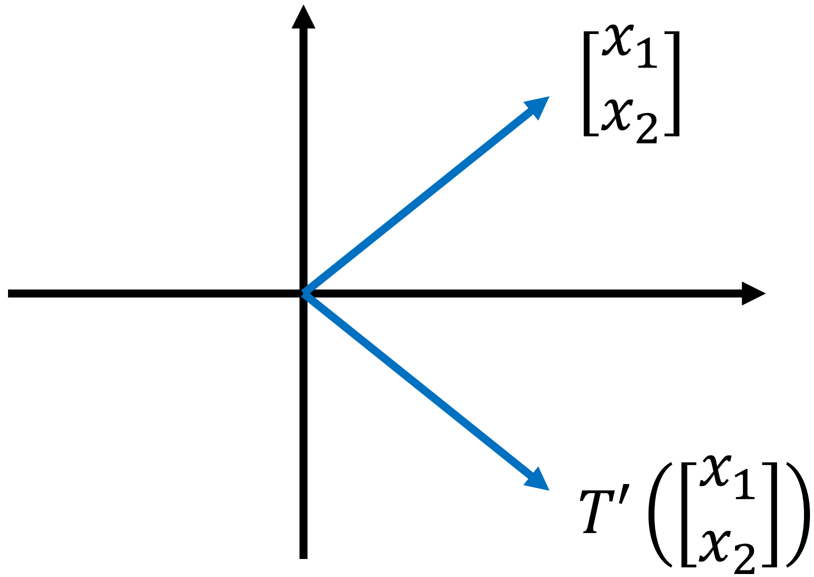
$$[\mathrm{T}] = [T(e_1) \quad T(e_2)\,]$$

$$= ?$$

# Sometimes a function can be complex ……

- T: reflection about a line L through the origin in $R^2$

special case: L is the *horizontal axis*




$$T'\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) =? \begin{bmatrix} x_1 \\ -x_2 \end{bmatrix}$$

$$[T'] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

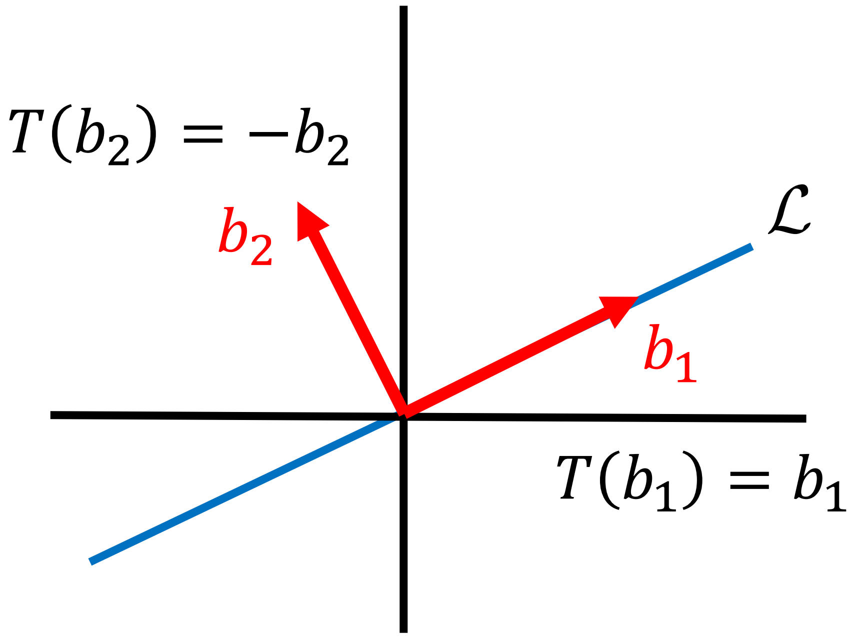
$T'(e_1) = e_1$

$T'(e_2) = -e_2$

# Describing the function in another coordinate system

- T: reflection about a line L through the origin in $R^2$

In another coordinate system B ....




B = $\{b_1, b_2\}$

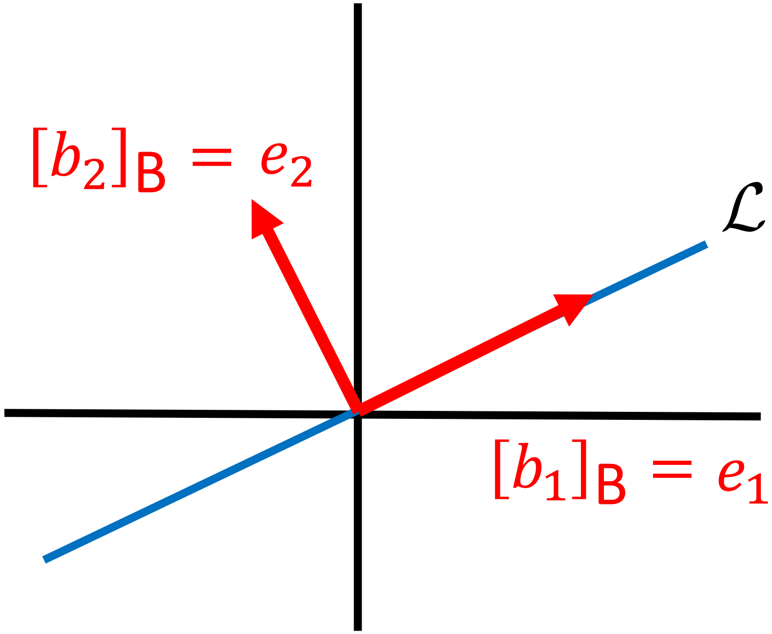
# Describing the function in another coordinate system

- T: reflection about a line L through the origin in $R^2$

In another coordinate system B ....

$$[T]_B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

B matrix of T: Input and output are both in B




$$[T]b_1 = b_1$$

$$\Rightarrow [T]_B([b_1]_B) = [b_1]_B$$

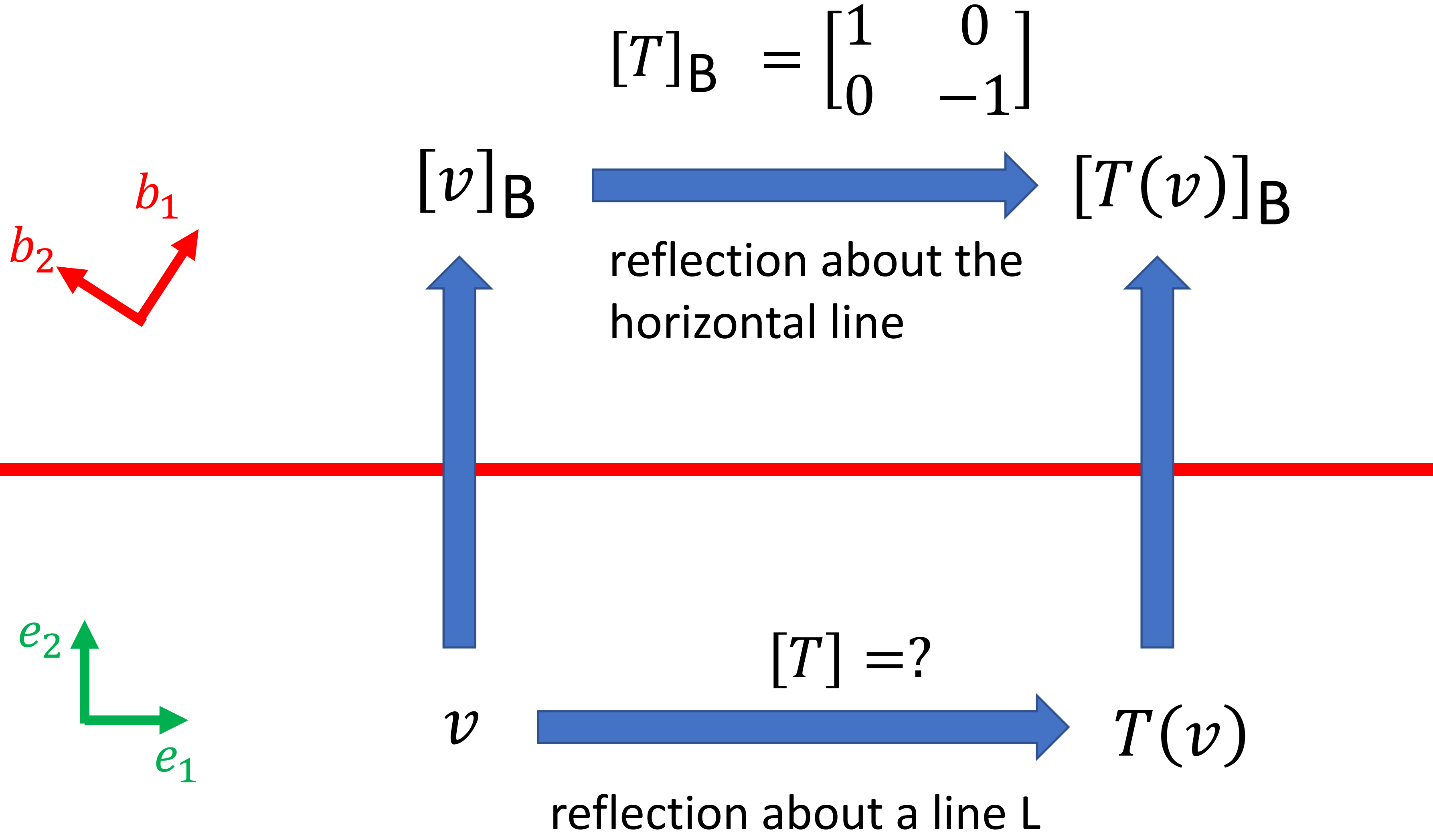
$$\Rightarrow [T]_B(e_1) = e_1$$

$$[T]b_2 = -b_2$$

$$\Rightarrow [T]_B([b_2]_B) = [-b_2]_B$$

$$\Rightarrow [T]_B(e_2) = -e_2$$

# Flowchart



$[T]_B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
$[v]_B$
$[T(v)]_B$
reflection about the horizontal line
$b_1$
$b_2$
$e_2$
$e_1$
$[T] = ?$
$v$
$T(v)$
reflection about a line L

$$[T]_B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$[v]_B \rightarrow [T(v)]_B$

reflection about the horizontal line

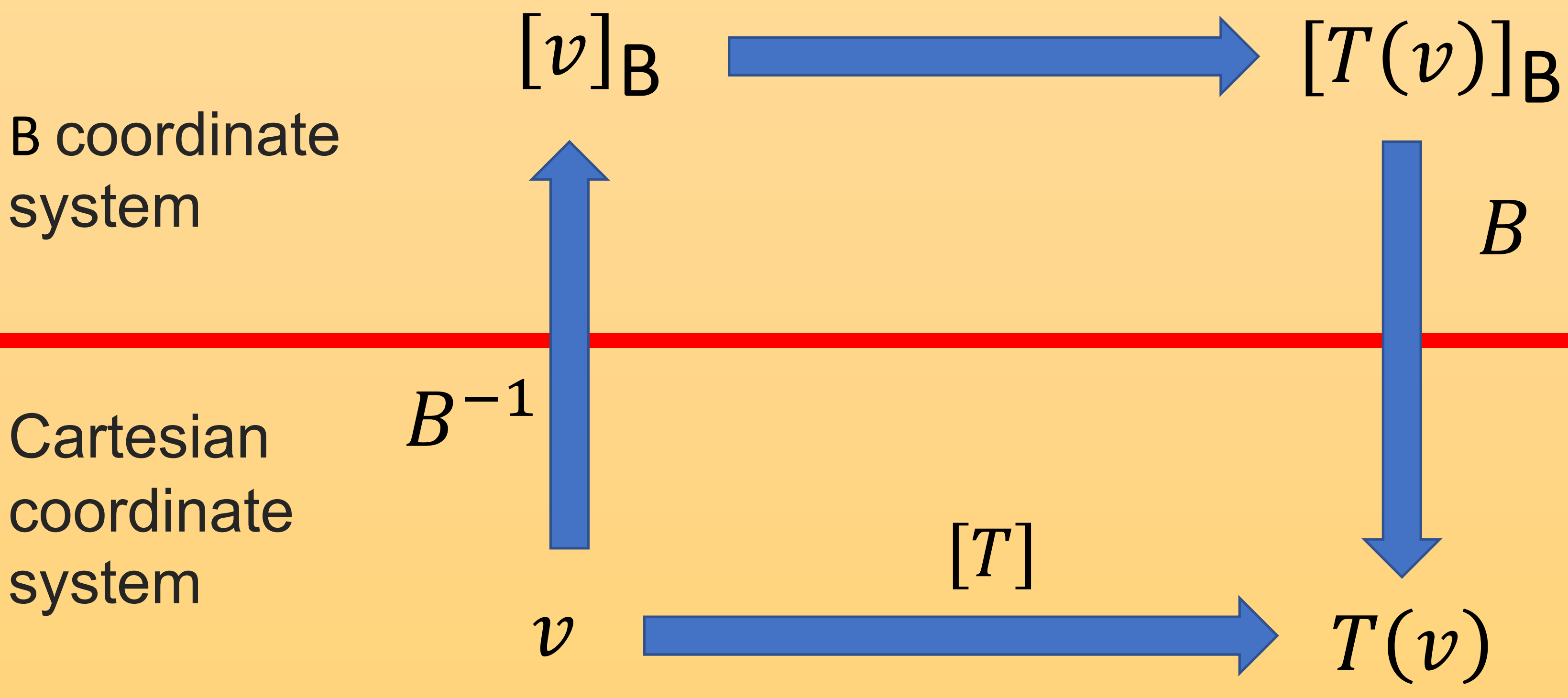
同一件事情
不同的詮釋

$[T] = ?$

$v \rightarrow T(v)$

reflection about a line L

## ***Flowchart***




$$[T] = B[T]_B B^{-1}$$

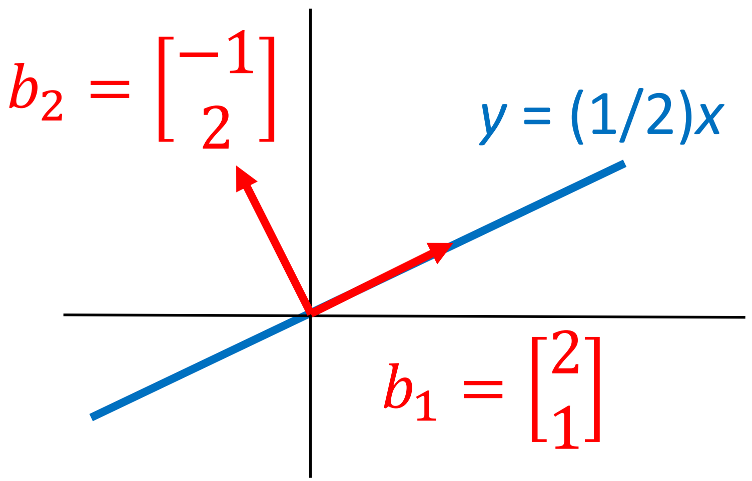
similar

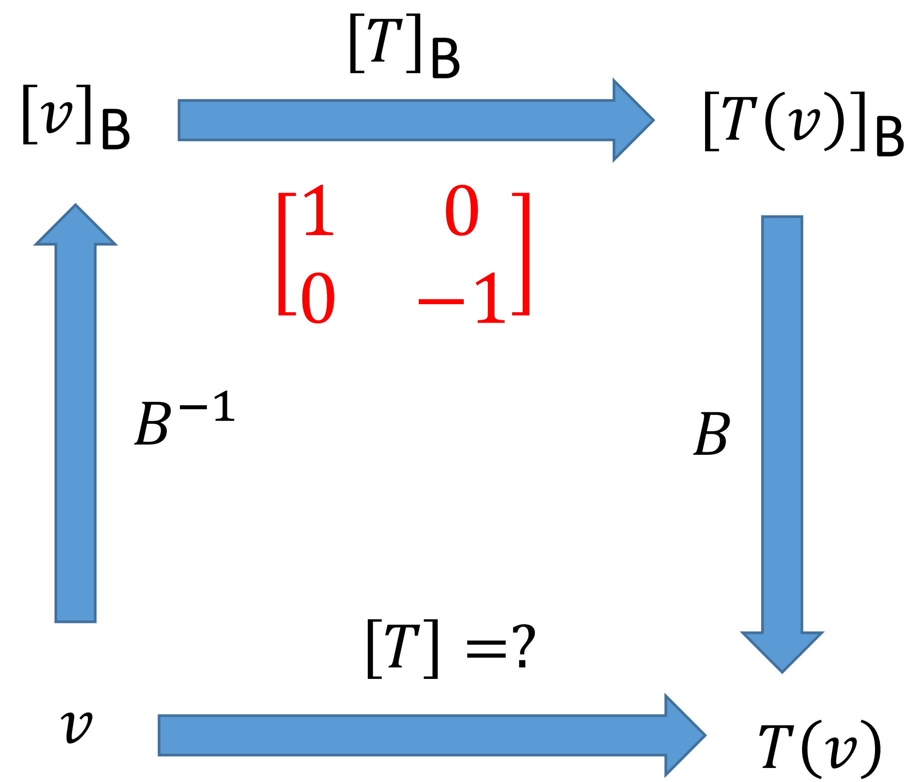
$$[T]_B = B^{-1}[T]B$$

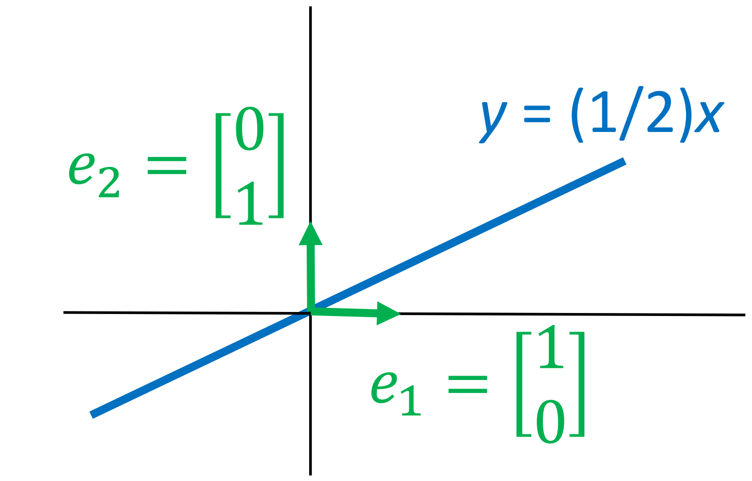
similar

- Example: reflection operator *T* about the line *y* = (1/2)*x*

$$B^{-1} = \begin{bmatrix} 0.4 & 0.2 \\ -0.2 & 0.4 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$



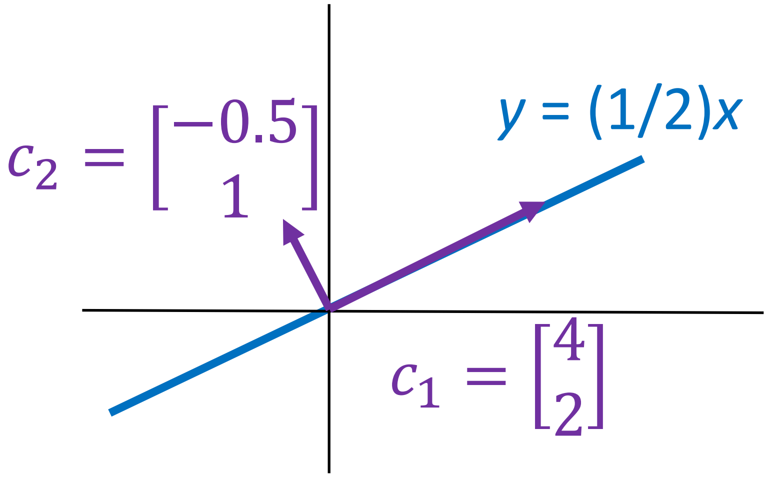



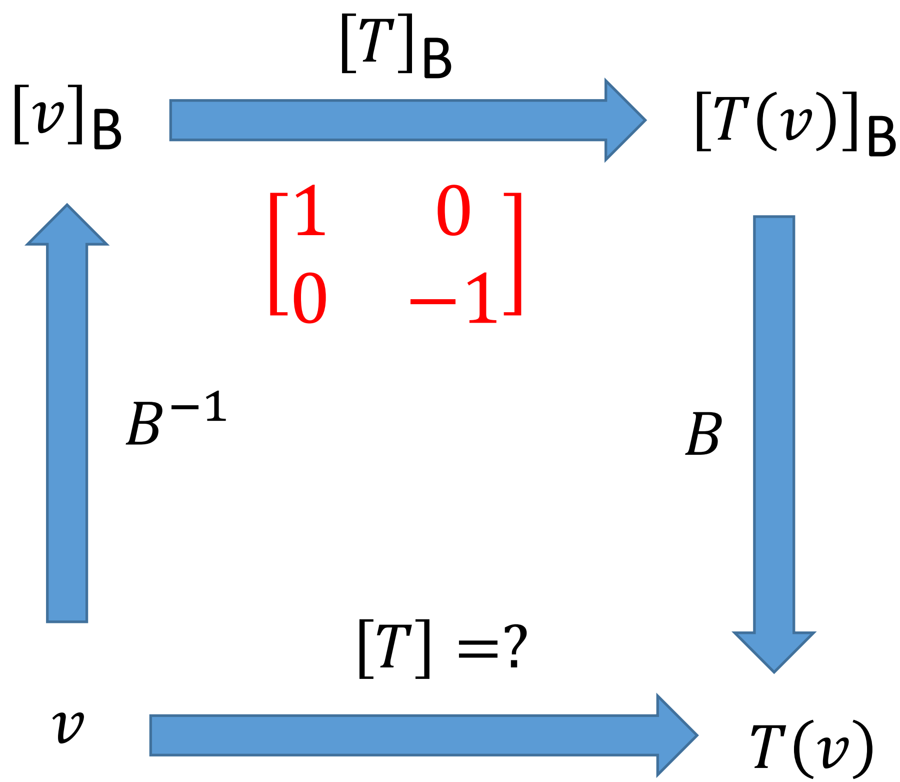

$$[T] = B[T]_B B^{-1}$$

- Example: reflection operator $T$ about the line $y = (1/2)x$

$$B^{-1} = \begin{bmatrix} 0.4 & 0.2 \\ -0.2 & 0.4 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

$$[T] = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$$




$$[T] = C[T]_C C^{-1}$$

$$= \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$$

$$[T] = B[T]_B B^{-1}$$

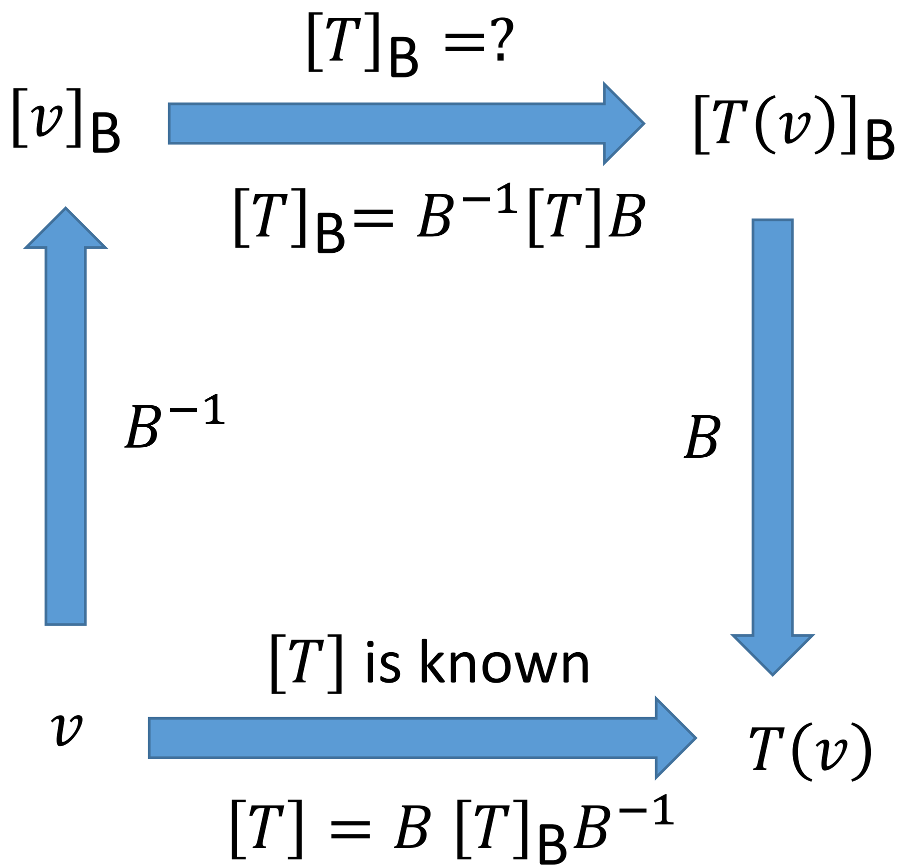
# ***Example (P279)***

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 + x_3 \\ x_1 + x_2 \\ -x_1 - x_2 + 3x_3 \end{bmatrix} \quad \mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}$$

$$[T] = \begin{bmatrix} 3 & 0 & 1 \\ 1 & 1 & 0 \\ -1 & -1 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$$

$$[T]_B = \begin{bmatrix} 3 & -9 & 8 \\ -1 & 3 & -3 \\ 1 & 6 & 1 \end{bmatrix}$$



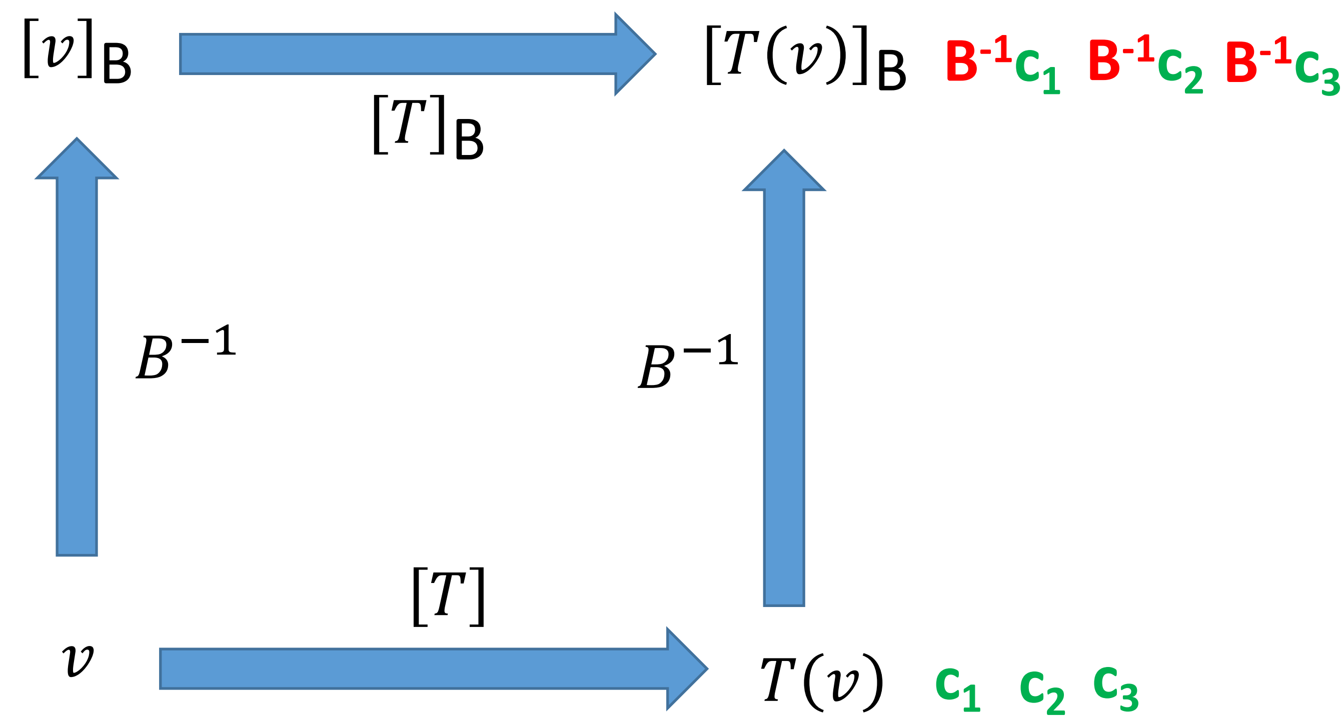
## ***Example (P279)*** Determine T

$$T\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right)=\begin{bmatrix}1\\2\\1\end{bmatrix} \qquad T\left(\begin{bmatrix}1\\0\\1\end{bmatrix}\right)=\begin{bmatrix}3\\-1\\1\end{bmatrix} \qquad T\left(\begin{bmatrix}0\\1\\1\end{bmatrix}\right)=\begin{bmatrix}2\\0\\1\end{bmatrix}$$

$\mathbf{b_1}$ $\mathbf{c_1}$ $\mathbf{b_2}$ $\mathbf{c_2}$ $\mathbf{b_3}$ $\mathbf{c_3}$

$\mathbf{b_1}$, $\mathbf{b_2}$, $\mathbf{b_3}$ as a coordinate system

{$\mathbf{b_1}$, $\mathbf{b_2}$, $\mathbf{b_3}$} is a basis of $R^3$



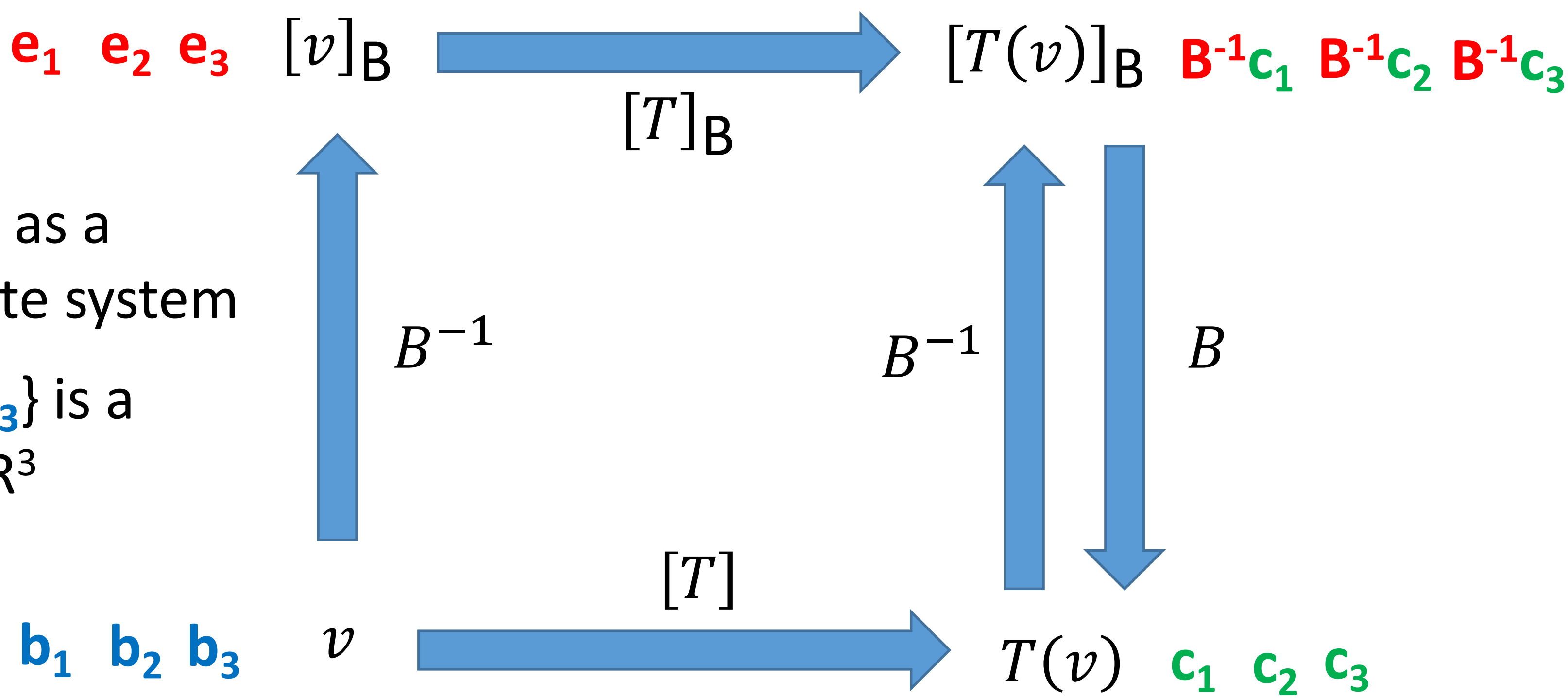
## ***Example (P279)*** Determine T

$$[T]_{\mathrm{B}} = [B^{-1}c_1 \quad B^{-1}c_2 \quad B^{-1}c_3] = B^{-1}C$$

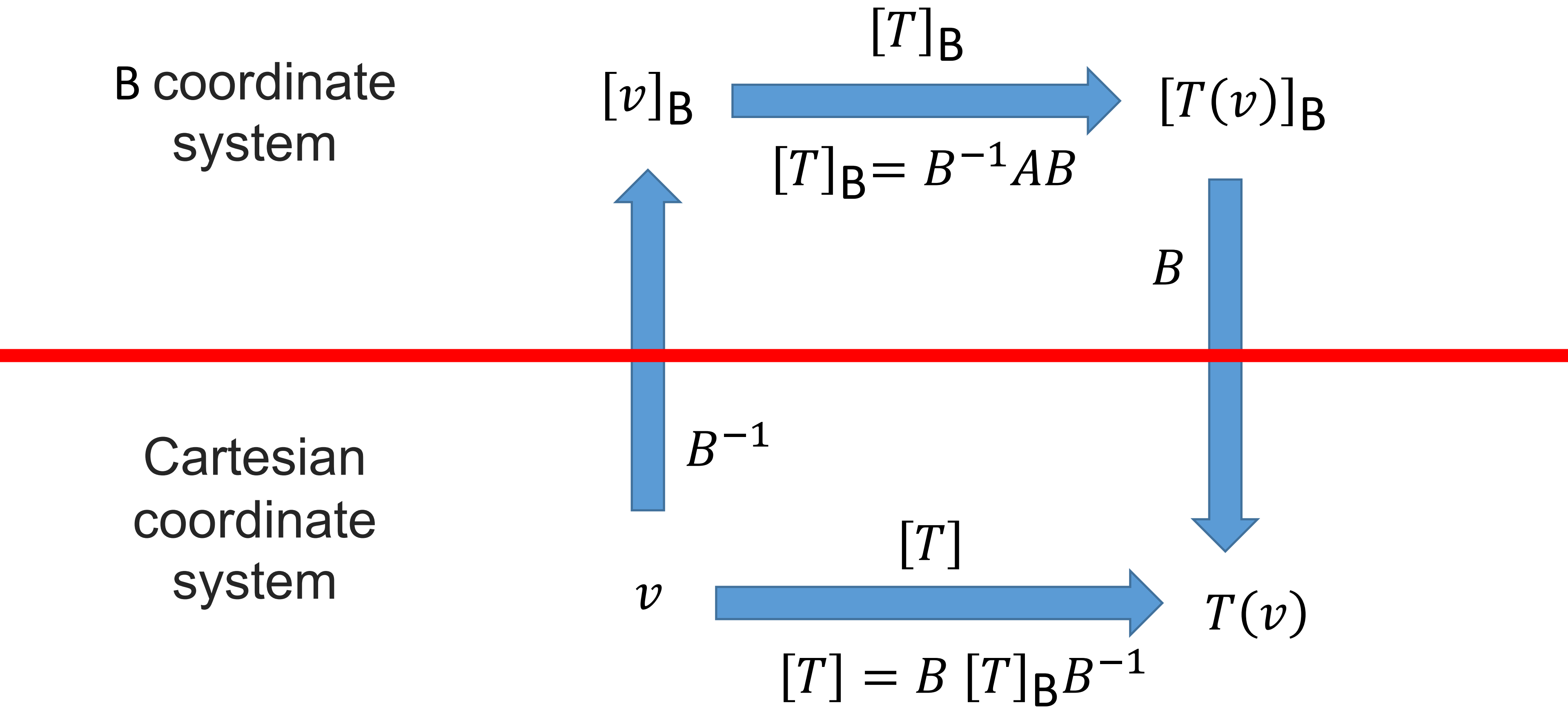
$$[T] = B[T]_{\mathrm{B}}B^{-1} = BB^{-1}CB^{-1} = CB^{-1}$$

**$b_1$**, **$b_2$**, **$b_3$** as a coordinate system

{**$b_1$**, **$b_2$**, **$b_3$**} is a basis of $R^3$



# Conclusion



B coordinate system
Cartesian coordinate system
$[T]_B$
$[v]_B$
$[T(v)]_B$
$[T]_B = B^{-1}AB$
$B$
$B^{-1}$
$[T]$
$v$
$T(v)$
$[T] = B\ [T]_B B^{-1}$