More about **Rank**

Rank

Maximum number of Independent Columns

||

Number of Pivot Column

||

Number of Non-zero rows

Rank R = Rank A

$$A = \begin{bmatrix} 1 & 2 & -1 & 2 & 1 & 2 \\ -1 & -2 & 1 & 2 & 3 & 6 \\ 2 & 4 & -3 & 2 & 0 & 3 \\ -3 & -6 & 2 & 0 & 3 & 9 \end{bmatrix}$$

Rank = ? **3**

$$R = \begin{bmatrix} 1 & 2 & 0 & 0 & -1 & -5 \\ 0 & 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank = ? **3**

Rank

Maximum number of Independent Columns $\Rightarrow$ Rank A $\leq$ Number of columns

$\Downarrow$

Rank A $\leq$ Min(Number of columns, Number of rows)

$\Uparrow$

Number of Non-zero rows $\Rightarrow$ Rank A $\leq$ Number of rows

Maximum number of Independent Columns

$=$

Number of Pivot Column

$=$

Number of Non-zero rows

Rank

Matrix A is ***full rank***
if Rank A = min(m,n)

Matrix A is ***rank deficient***
if Rank A < min(m,n)

- Given a mxn matrix A:
  - Rank A $\leq$ min(m, n)
  - Because "the columns of A are independent" is equivalent to "rank A = n"
    - If m < n, the columns of A is dependent.

$$\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$$

3 X 4

Rank A $\leq$ 3

$$\left\{ \begin{bmatrix} * \\ * \\ * \end{bmatrix}, \begin{bmatrix} * \\ * \\ * \end{bmatrix}, \begin{bmatrix} * \\ * \\ * \end{bmatrix}, \begin{bmatrix} * \\ * \\ * \end{bmatrix} \right\}$$

A matrix set has 4 vectors belonging to R^3 is dependent

In R^m, you cannot find more than m vectors that are independent.

Independent

All columns are independent

↓

Every column is a pivot column

↓

Every column in RREF(A) is standard vector.

3X4

$$\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$$

Columns are linear independent

RREF

Cannot be a pivot column

$$\begin{bmatrix} 1 & 0 & 0 & * \\ 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \end{bmatrix}$$

這個發現已經提過，現在只是用

Rank A ≤ min(m, n) 再說一次

Basic, Free Variables v.s. Rank

3 useful equations

$$Ax = b$$

$$\begin{aligned} x_1 + 2x_2 - x_3 + 2x_4 + 5x_5 &= 2 \\ -x_1 - 2x_2 + x_3 + 2x_4 + 3x_5 &= 6 \\ 2x_1 + 4x_2 - 3x_3 + 2x_4 &= 3 \\ -3x_1 - 6x_2 + 2x_3 + 3x_5 &= 9 \end{aligned}$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -1 & 2 & 5 & 2 \\ -1 & -2 & 1 & 2 & 3 & 6 \\ 2 & 4 & -3 & 2 & 0 & 3 \\ -3 & -6 & 2 & 0 & 3 & 9 \end{array}\right]$$

A b

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & -1 & -5 \\ 0 & 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

RREF(A) b'

$$\begin{aligned} x_1 + 2x_2 - x_5 &= -5 \\ x_3 &= -3 \\ x_4 + x_5 &= 2 \\ \cancel{0} &= \cancel{0} \end{aligned}$$

$$x_1 = -5 - 2x_2 + x_5$$

$$x_3 = -3$$

$$x_4 = 2 - x_5$$

rank = non-zero row = 3 basic variables

nullity = No. column – non-zero row = 2 free variables

Rank

Rank

- Maximum number of Independent Columns
- Number of Pivot Column
- Number of Non-zero rows of RREF
- Number of Basic Variables

Nullity = no. column - rank

- Number of zero rows of RREF
- Number of Free Variables