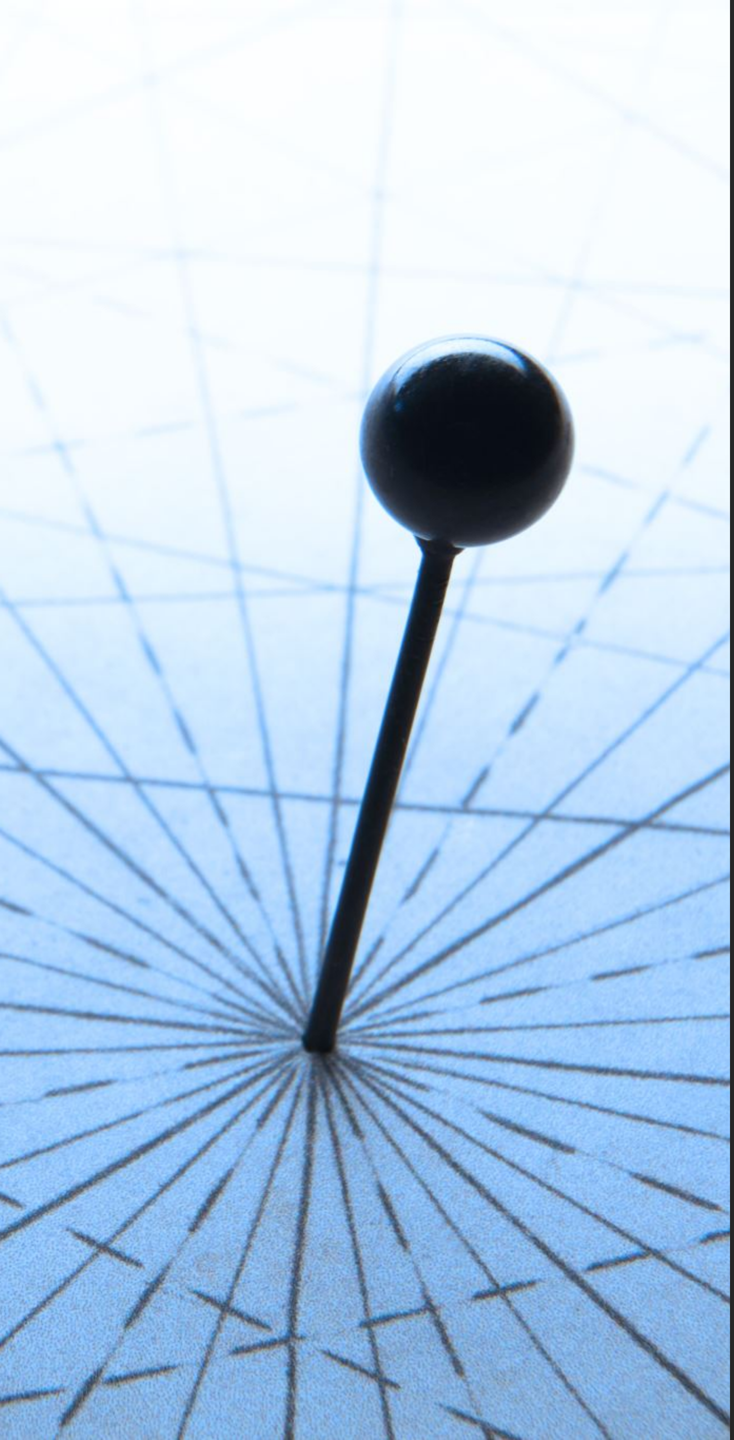


Coordinate System

Hung-yi Lee

Coordinate System

- Each coordinate system is a "*viewpoint*" for vector representation.
  - The same vector is represented differently in different coordinate systems.
  - Different vectors can have the same representation in different coordinate systems.
- A vector set B can be considered as a coordinate system for R^n if:
  - 1. The vector set B spans the R^n
  - 2. The vector set B is independent

B is a basis of R^n

Coordinate System

- Let vector set B=$\{u_1, u_2, \cdots, u_n\}$ be a basis for a subspace R^n

⇒ B is a coordinate system

- For any v in R^n, there are unique scalars $c_1, c_2, \cdots, c_n$ such that $v = c_1u_1 + c_2u_2 + \cdots + c_nu_n$

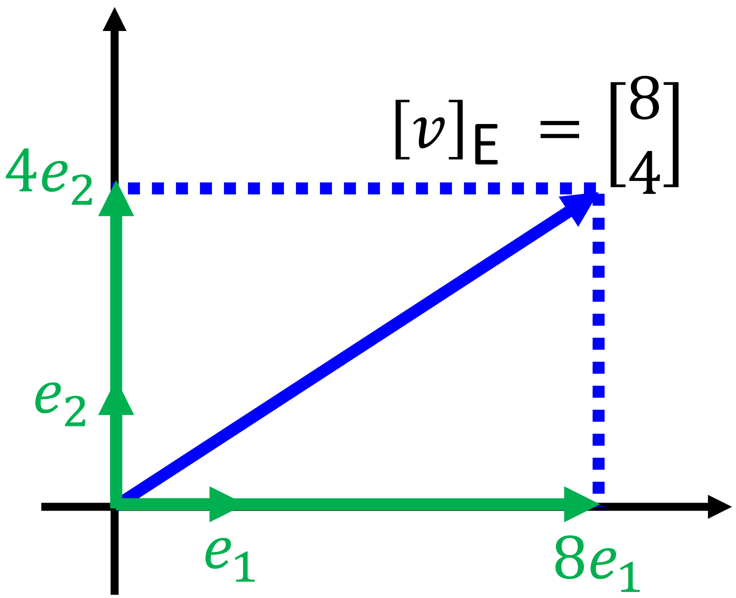
B -coordinate vector of v:

$$[v]_B = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} \in \mathcal{R}^n$$

(用 B 的觀點來看 v)

E

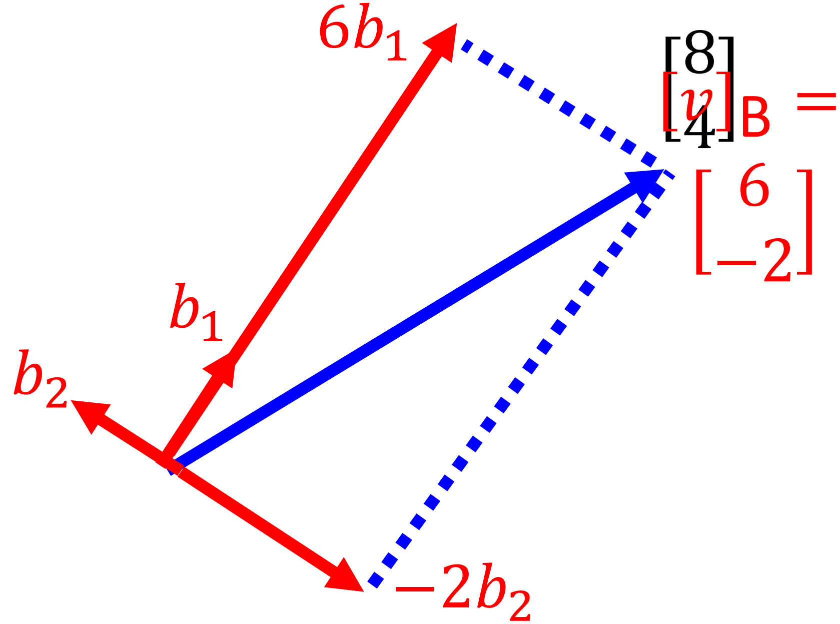
{e_1, e_2} is a coordinate system



$$\begin{bmatrix}8\\4\end{bmatrix} = 8e_1 + 4e_2$$

New Coordinate System B

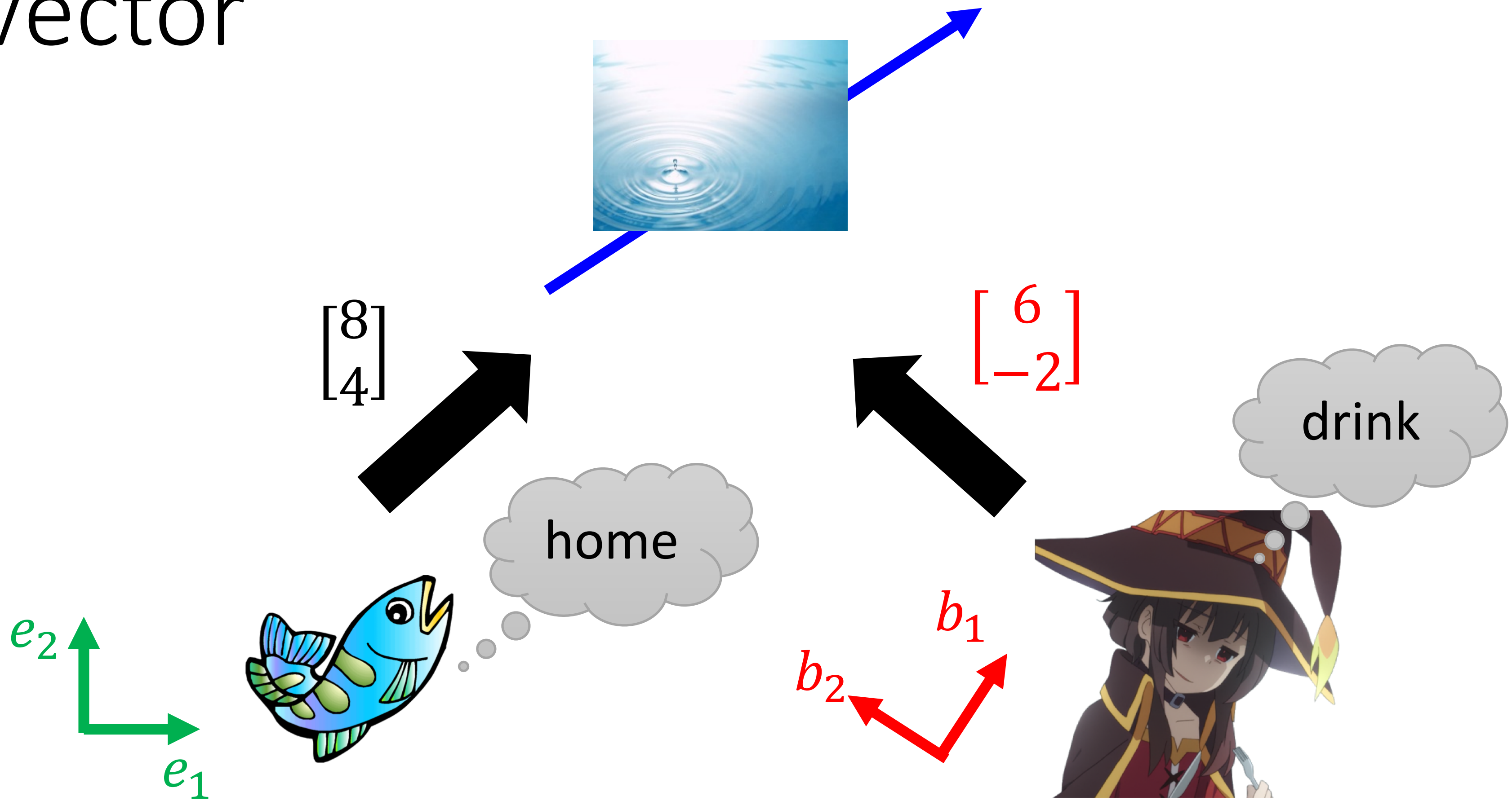
$$b_1 = \begin{bmatrix}1\\1\end{bmatrix} \quad b_2 = \begin{bmatrix}-1\\1\end{bmatrix}$$



$$\begin{bmatrix}8\\4\end{bmatrix} = 6b_1 + (-2)b_2$$

民濕寢則腰疾偏死，鰌然乎哉？木處則惴慄恂懼，猨猴然乎哉？三者孰知正處？民食芻豢，麋鹿食薦，蝍蛆甘帶，鴟鴉嗜鼠，四者孰知正味？猨猵狙以為雌，麋與鹿交，鰌與魚游。毛嬙、西施，人之所美也；魚見之深入，鳥見之高飛，麋鹿見之決驟，四者孰知天下之正色哉？《莊子·齊物論》

Vector



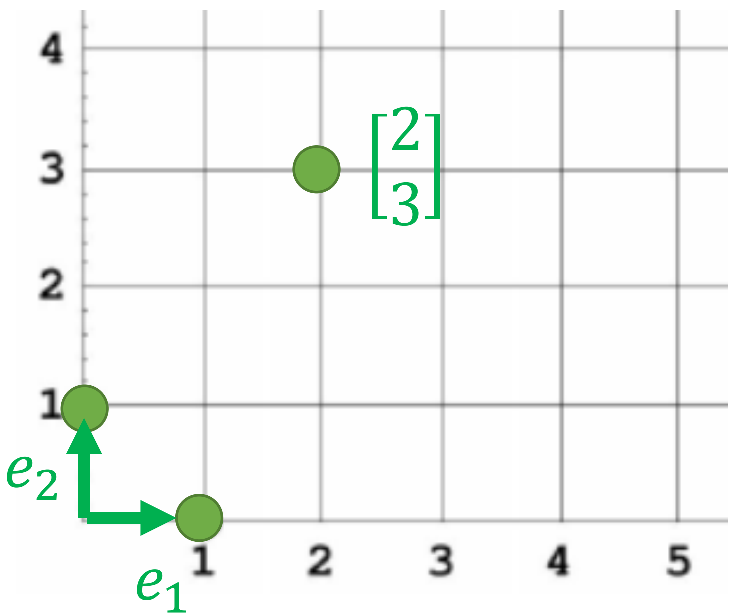
$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

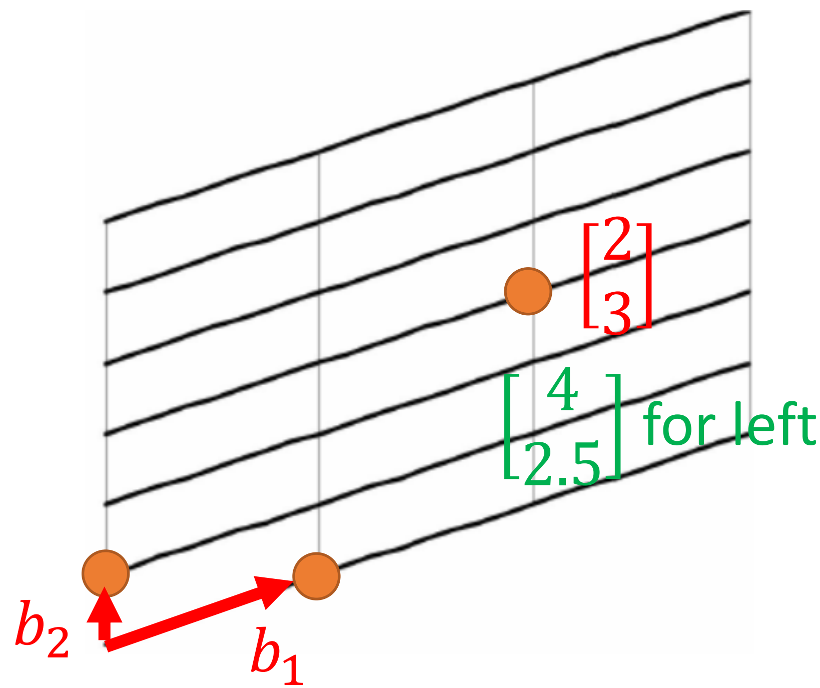
$b_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad b_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Vector

$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$b_1 = \begin{bmatrix} 2 \\ 0.5 \end{bmatrix}$ $b_2 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$



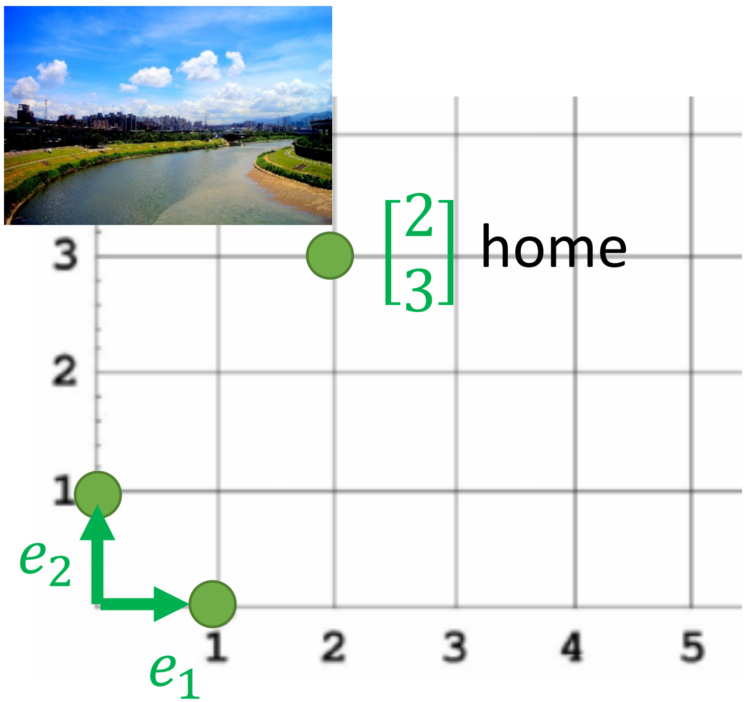


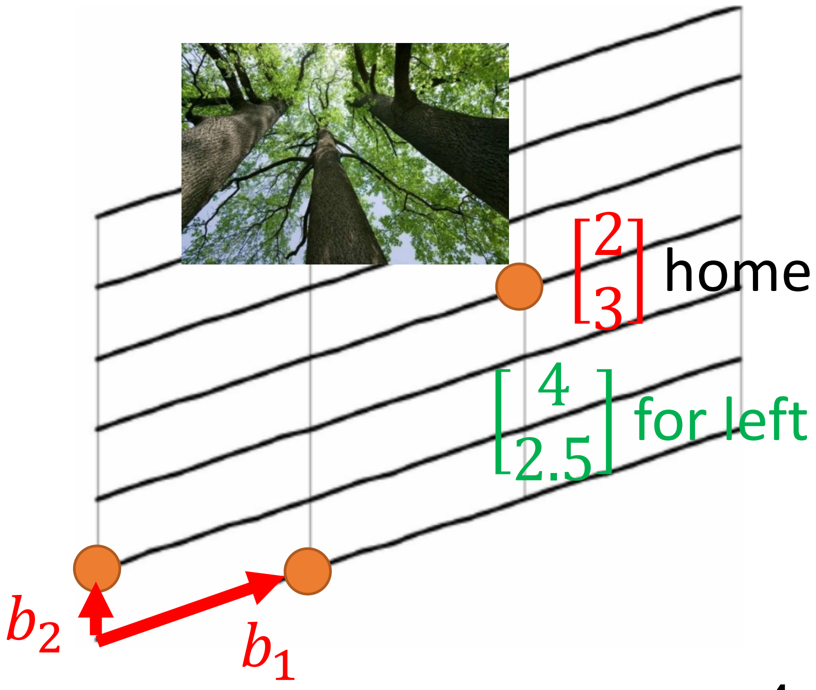
$$2b_1 + 3b_2 = \begin{bmatrix} 4 \\ 2.5 \end{bmatrix}$$

Vector

$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$b_1 = \begin{bmatrix} 2 \\ 0.5 \end{bmatrix}$ $b_2 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$





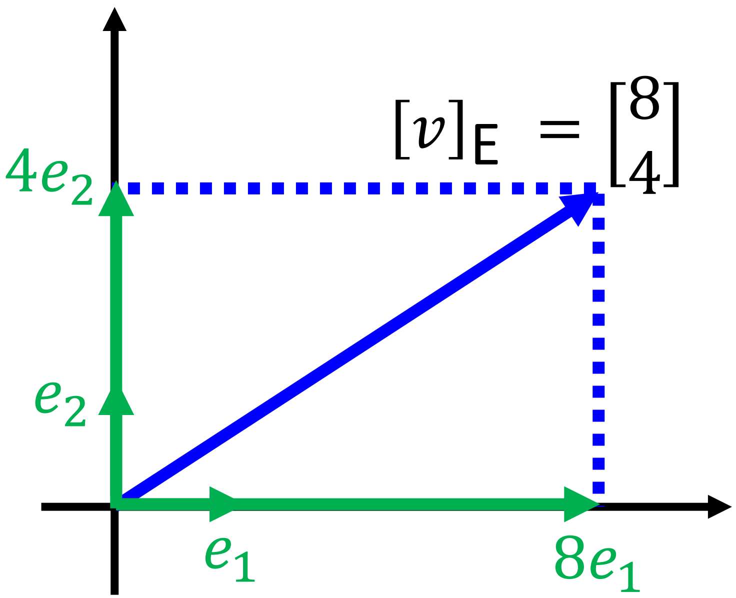
$2b_1 + 3b_2 = \begin{bmatrix} 4 \\ 2.5 \end{bmatrix}$

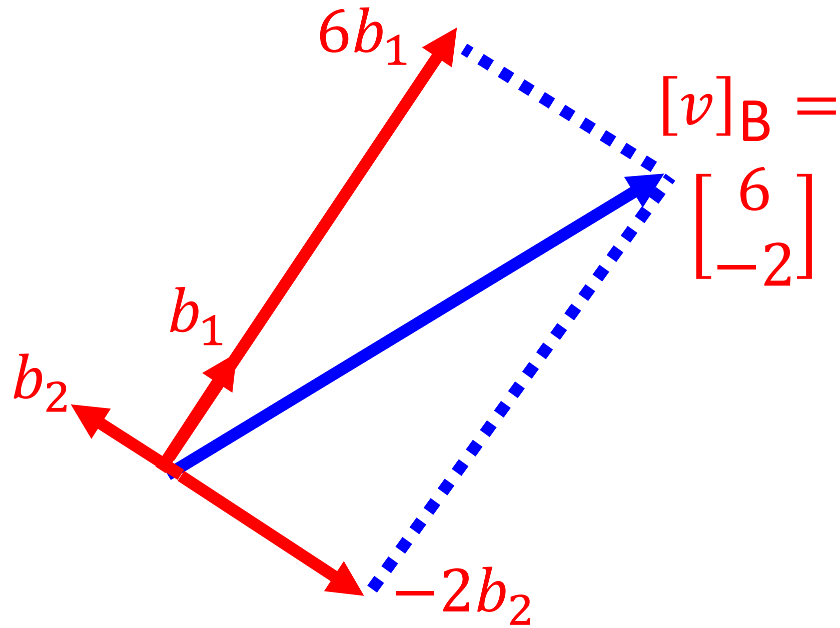
E

$\{e_1, e_2\}$ is a coordinate system

New Coordinate System B

$b_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad b_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$





$\begin{bmatrix} 8 \\ 4 \end{bmatrix} = 8e_1 + 4e_2$

$\begin{bmatrix} 8 \\ 4 \end{bmatrix} = 6b_1 + (-2)b_2$

E=$\{e_1, e_2, \cdots, e_n\}$ (standard vectors) $\quad v = [v]_E$

E is Cartesian coordinate system (直角坐標系)

Coordinate System

- A vector set B can be considered as a coordinate system for R^n if:
- 1. The vector set B spans the R^n

→ Every vector should have representation

- 2. The vector set B is independent

→ Unique representation

B is a basis of R^n

Why Basis?

- Let vector set B=$\{u_1, u_2, \cdots, u_k\}$ be independent.
- Any vector v in Span B can be uniquely represented as a linear combination of the vectors in B.
- That is, there are unique scalars $a_1, a_2, \cdots, a_k$ such that $v = a_1u_1 + a_2u_2 + \cdots + a_ku_k$
- Proof:

Unique? $v = a_1u_1 + a_2u_2 + \cdots + a_ku_k$

$$v = b_1u_1 + b_2u_2 + \cdots + b_ku_k$$

$$(a_1 - b_1)u_1 + (a_2 - b_2)u_2 + \cdots + (a_k - b_k)u_k = 0$$

B is independent ⇒ $a_1 - b_1 = a_2 - b_2 = \cdots = a_k - b_k = 0$