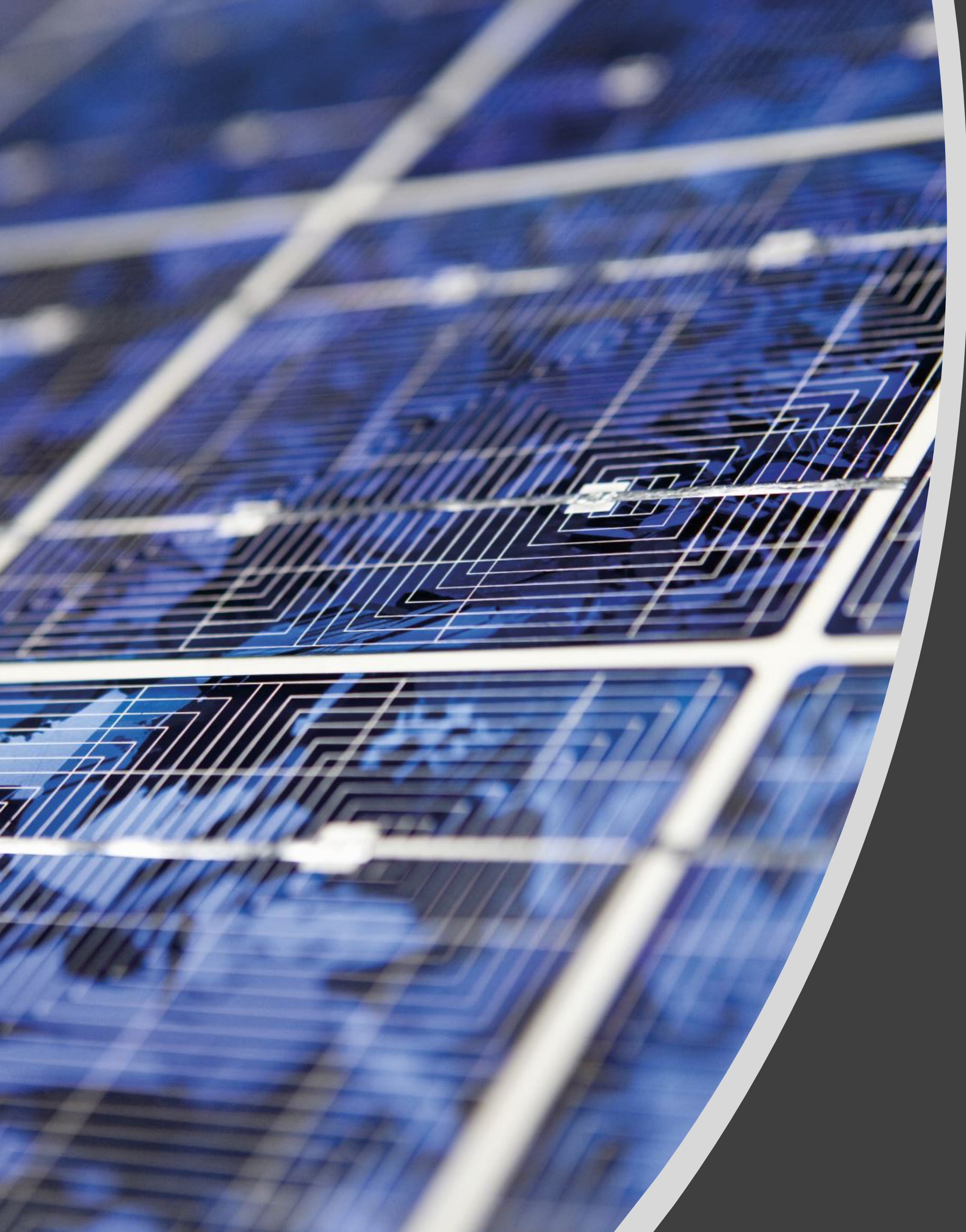


Application of Matrix Inverse

Solving Linear Equations

- The inverse can be used to solve system of linear equations.

$$A\mathbf{x} = \mathbf{b}$$

If A is invertible.

$$A^{-1}(A\mathbf{x}) = A^{-1}\mathbf{b}$$

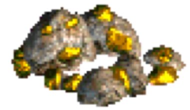
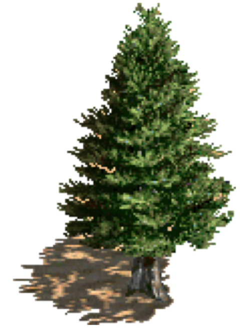
$$\underbrace{\begin{aligned} x_1 + 2x_2 &= 4 \\ 3x_1 + 5x_2 &= 7 \end{aligned}}_{Ax = b}$$

$$\mathbf{x} = A^{-1}\mathbf{b}$$

$$= \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 7 \end{bmatrix} = \begin{bmatrix} -6 \\ 5 \end{bmatrix}$$

However, this method is computationally inefficient.

Input-output Model

- 假設世界上只有食物、黃金、木材三種資源

	需要食物	需要黃金	需要木材
生產一單位食物	0.1	0.2	0.3
生產一單位黃金	0.2	0.4	0.1
生產一單位木材	0.1	0.2	0.1

Cx　　　　　C　　　　　x

$$\begin{bmatrix} 0.1x_1 + 0.2x_2 + 0.1x_3 \\ 0.2x_1 + 0.4x_2 + 0.2x_3 \\ 0.3x_1 + 0.1x_2 + 0.1x_3 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ 0.2 & 0.4 & 0.2 \\ 0.3 & 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

須投入　　　　　Consumption matrix　　　　　想生產

Input-output Model

$$\underset{\text{須投入}}{\overset{Cx}{\begin{bmatrix}48\\96\\53\end{bmatrix}}} = \underset{\text{Consumption matrix}}{\overset{C}{\begin{bmatrix}0.1 & 0.2 & 0.1\\0.2 & 0.4 & 0.2\\0.3 & 0.1 & 0.1\end{bmatrix}}} \underset{\text{想生產}}{\overset{x}{\begin{bmatrix}100\\150\\80\end{bmatrix}}}$$

須考慮成本：

淨收益

$$x - Cx = \begin{bmatrix}100\\150\\80\end{bmatrix} - \begin{bmatrix}48\\96\\53\end{bmatrix} = \begin{bmatrix}52\\54\\27\end{bmatrix}$$

Demand Vector d

Input-output Model

$$C = \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ 0.2 & 0.4 & 0.2 \\ 0.3 & 0.1 & 0.1 \end{bmatrix} \quad d = \begin{bmatrix} 90 \\ 80 \\ 60 \end{bmatrix}$$

Demand Vector d

生產目標 x 應該訂為多少?

$$x - Cx = d$$

$$Ix - Cx = d$$

$$(I - C)x = d$$

Ax=b

$$\mathrm{A} = I - \mathrm{C} = \begin{bmatrix} 0.9 & -0.2 & -0.1 \\ -0.2 & 0.6 & -0.2 \\ -0.3 & -0.1 & 0.9 \end{bmatrix}$$

$$d = \begin{bmatrix} 90 \\ 80 \\ 60 \end{bmatrix} \quad x = \begin{bmatrix} 170 \\ 240 \\ 150 \end{bmatrix}$$

Input-output Model

- 提升一單位食物的淨產值，需要多生產多少資源？

Ans: The first column of $(I-C)^{-1}$

$$(I-C)x = d \quad \Rightarrow \quad x = (I-C)^{-1}d$$

$$d \quad \Rightarrow \quad d + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = d + e_1$$

$$x' = (I-C)^{-1}(d+e_1)$$
$$= (I-C)^{-1}d + \underline{(I-C)^{-1}e_1}$$

$$(I-C)^{-1} = \begin{bmatrix} 1.3 & 0.475 & 0.25 \\ 0.6 & 1.950 & 0.50 \\ 0.5 & 0.375 & 1.25 \end{bmatrix}$$

食物　黃金　木材