Find Inverse of Matrix

2 X 2 Matrix

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad A^{-1} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$ $\quad$ Find e, f, g, h

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

If $ad - bc = 0$, A is not invertible.

Algorithm for Matrix Inversion

- Let A be an n x n matrix. A is invertible if and only if the reduced row echelon form of A is I_n

$$\underbrace{E_k \cdots E_2 E_1}_{A^{-1}} A = R = I_n$$

$$A^{-1} = E_k \cdots E_2 E_1$$

Algorithm for Matrix Inversion

- Let A be an n x n matrix. Transform [A I_n] into its RREF [R B]
  - R is the RREF of A
  - B is a nxn matrix (not RREF)
- If R = I_n, then A is invertible
  - B = A^{-1}

$$E_k \cdots E_2 E_1 [A \quad I_n]$$

$$= [\underbrace{R}_{I_n} \quad \underbrace{E_k \cdots E_2 E_1}_{A^{-1}}]$$

Algorithm for Matrix Inversion

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & 4 & 8 \end{bmatrix} \xrightarrow{\text{RREF}} I_n$$

Invertible

$$\begin{bmatrix} A & I_3 \end{bmatrix} = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 6 & 0 & 1 & 0 \\ 3 & 4 & 8 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & -2 & -1 & -3 & 0 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & -1 & -7 & 2 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 7 & -2 & -1 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -20 & 6 & 3 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 7 & -2 & -1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -16 & 4 & 3 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 7 & -2 & -1 \end{array}\right]$$

A^{-1}

Algorithm for Matrix Inversion

- Let A be an n x n matrix. Transform [A I_n] into its RREF [R B]
  - R is the RREF of A
  - B is a nxn matrix (not RREF)
- If R = I_n, then A is invertible
  - B = A^{-1}
- To find $A^{-1}C$, transform [A C] into its RREF [R C']
  - C' = $A^{-1}C$

$$E_k \cdots E_2 E_1 [A \quad C] = [\underbrace{R}_{I_n} \quad \overbrace{\underbrace{E_k \cdots E_2 E_1}_{A^{-1}} C}^{A^{-1}C}]$$