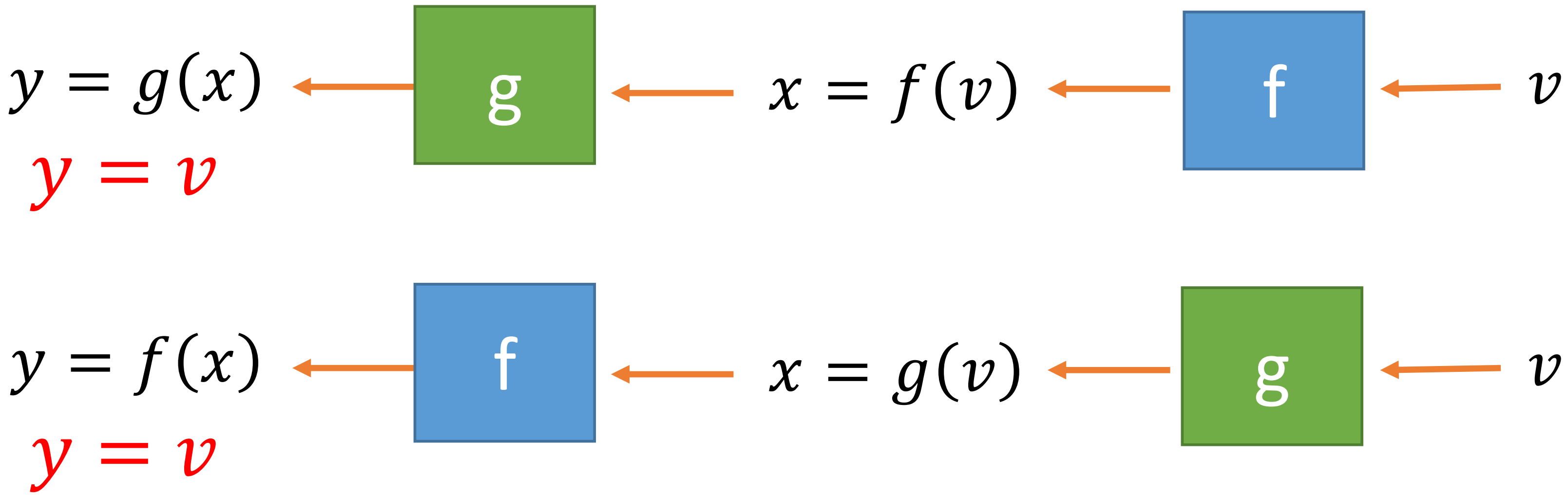
Matrix Inverse

Inverse of Function

- Two functions f and g are inverse of each other

$f = g^{-1}, \; g = f^{-1}$

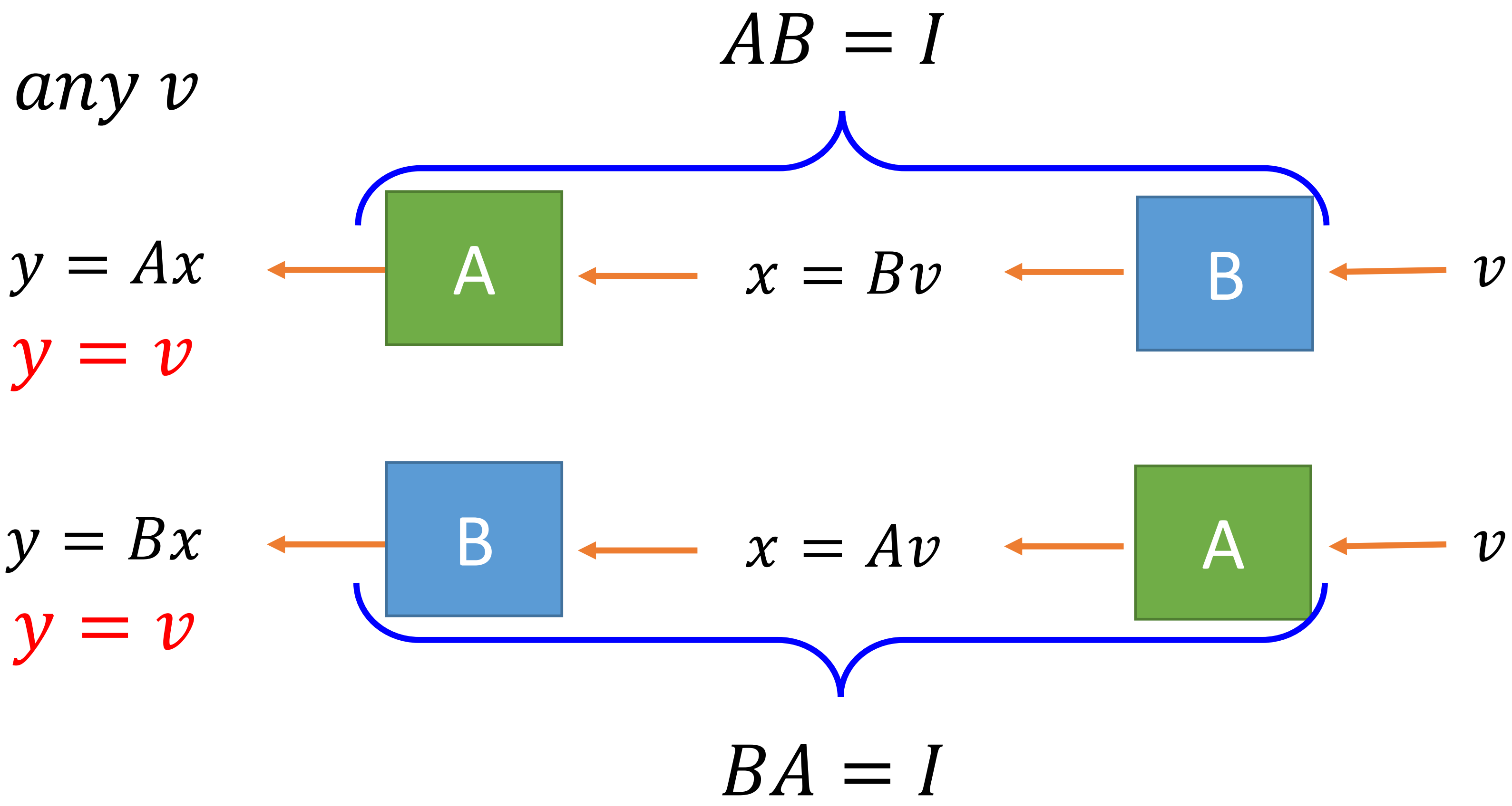
For $any\ v$



Inverse of Matrix

- A and B are inverses to each other

For $any\ v$



Inverse of Matrix

Invertible = Non-singular
Not Invertible = Singular

A is called invertible if there is a matrix B such that $AB = I$ and $BA = I$

B is an inverse of A

$$B = A^{-1} \quad A = B^{-1}$$

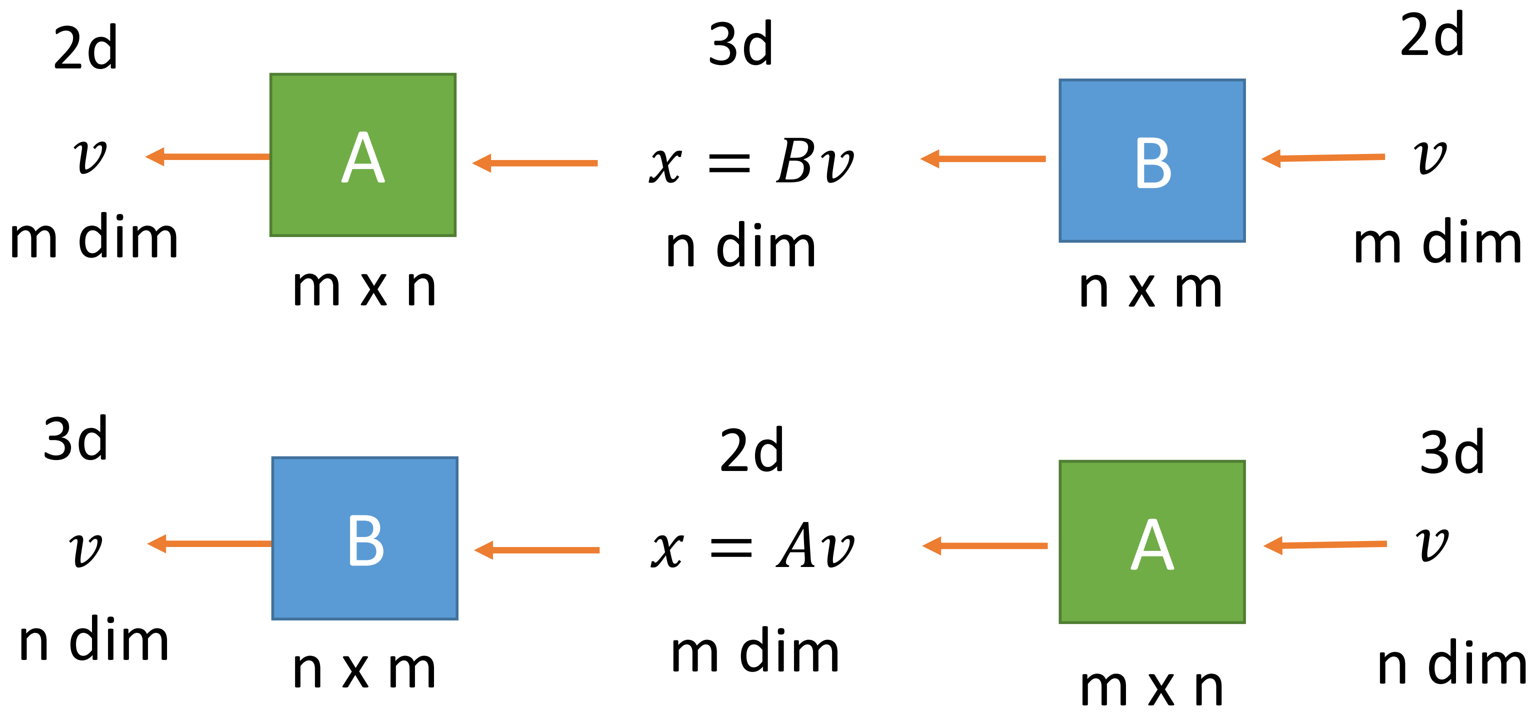
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \quad B = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} \quad AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Non-square matrix cannot be invertible

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 1 \\ -1 & -1 \\ 0 & 2 \end{bmatrix}$$

Inverse of Matrix

- Non-square matrix cannot be invertible?



Inverse of Matrix

- Not all the square matrix is invertible

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

- Unique

$$AB = I \qquad BA = I \qquad AC = I \qquad CA = I$$

$$B = BI = B(AC) = (BA)C = IC = C$$

Inverse for matrix product

- A and B are invertible nxn matrices, is AB invertible? yes

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$B^{-1}A^{-1}(AB) \;\; = B^{-1}\,(A^{-1}A)B \;\; = B^{-1}\,B \;\; = I$$

$$(AB)B^{-1}A^{-1} \;\; = A(BB^{-1})A^{-1} \;\; = A\,A^{-1} \;\; = I$$

- Let $A_1, A_2, \cdots, A_k$ be nxn invertible matrices. The product $A_1 A_2 \cdots A_k$ is invertible, and

$$(A_1 A_2 \cdots A_k)^{-1} = (A_k)^{-1}(A_{k-1})^{-1} \cdots (A_1)^{-1}$$

Inverse for matrix transpose

- If A is invertible, is A^T invertible?

$$(A^T)^{-1} =? \quad (A^{-1})^T$$

$$(AB)^T = B^T A^T$$

$$A^{-1}A = I \Rightarrow (A^{-1}A)^T = I \Rightarrow A^T(A^{-1})^T = I$$

$$AA^{-1} = I \Rightarrow (AA^{-1})^T = I \Rightarrow (A^{-1})^T A^T = I$$