Invertible (Proof)

Summary

- Let A be an n x n matrix. A is invertible if and only if
  - The columns of A span R^n
  - For every b in R^n, the system Ax=b is consistent
  - The rank of A is n
  - The columns of A are linear independent
  - The only solution to Ax=0 is the zero vector
  - The nullity of A is zero
  - The reduced row echelon form of A is I_n
  - A is a product of elementary matrices
  - There exists an n x n matrix B such that BA = I_n
  - There exists an n x n matrix C such that AC = I_n

Invertible

- Let A be an n x n matrix.
  - Onto → One-to-one → invertible
    - The columns of A span R^n
    - For every b in R^n, the system Ax=b is consistent
    - The rank of A is the number of rows
  - One-to-one → Onto → invertible
    - The columns of A are linear independent
    - The rank of A is the number of columns
    - The nullity of A is zero
    - The only solution to Ax=0 is the zero vector
    - The reduced row echelon form of A is I_n

Rank A = n

Is A Invertible?

- Let A be an n x n matrix. A is invertible if and only if
  - The reduced row echelon form of A is I_n

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & 4 & 8 \end{bmatrix} \xrightarrow{\text{RREF}} I_n$$

Invertible

$$B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

Not Invertible

Summary

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onto

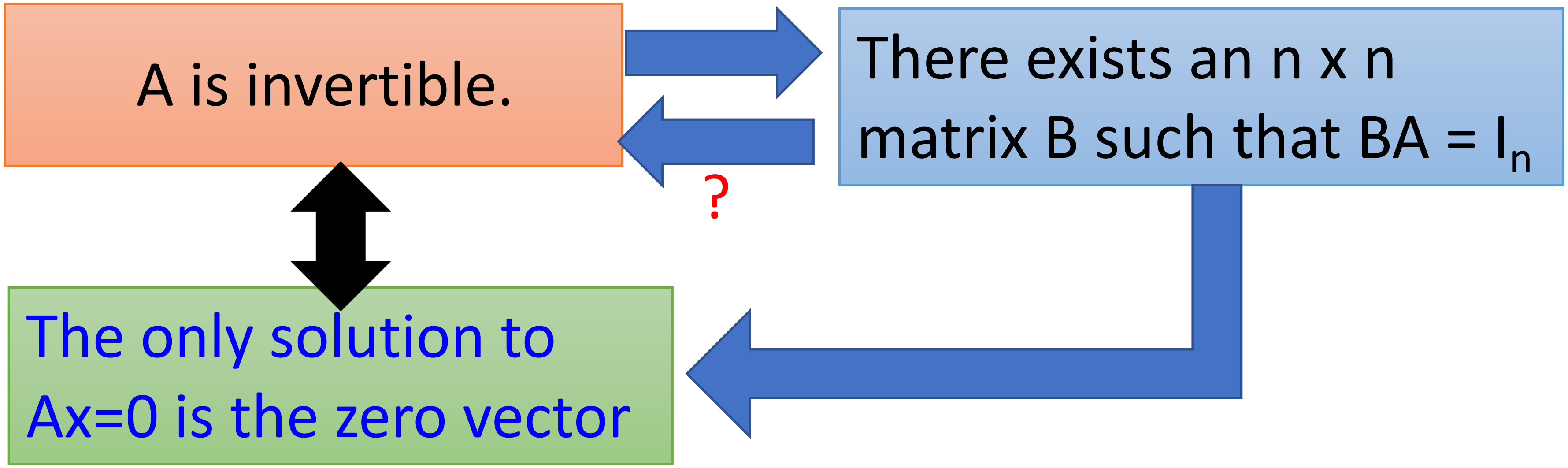
One-to-one

=

square matrix

Invertible

A is n x n



If $Av = 0$, then ....

$$BA = I_n$$

$$BAv = 0 \qquad I_n v = v$$

$$\Rightarrow \quad v = 0$$

Invertible

A is n x n

A is invertible.

There exists an n x n matrix C such that $AC = I_n$

?

For every b in R^n, Ax=b is consistent

For any vector b,

$$AC = I_n$$

$$ACb \qquad I_n b = \mathrm{b}$$

Cb is always a solution for b

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