

Matrix Multiplication

What does it mean?

Matrix Multiplication - Meaning

- Multiple Input $C = AB$

A green square labeled 'A' is followed by a blue vertical rectangle labeled b_1 , an equals sign, and an orange vertical rectangle labeled c_1 .

A green square labeled 'A' is followed by a blue vertical rectangle labeled b_2 , an equals sign, and an orange vertical rectangle labeled c_2 .

⋮

A green square labeled 'A' is followed by a blue vertical rectangle labeled b_p , an equals sign, and an orange vertical rectangle labeled c_p .

A green square labeled 'A' is followed by a row of blue vertical rectangles labeled b_1 , b_2 , and b_p , with an ellipsis between b_2 and b_p .

An equals sign is followed by a row of orange vertical rectangles labeled c_1 , c_2 , and c_p , with an ellipsis between c_2 and c_p .

$$AB = A[b_1 \quad b_2 \quad \cdots \quad b_p]$$

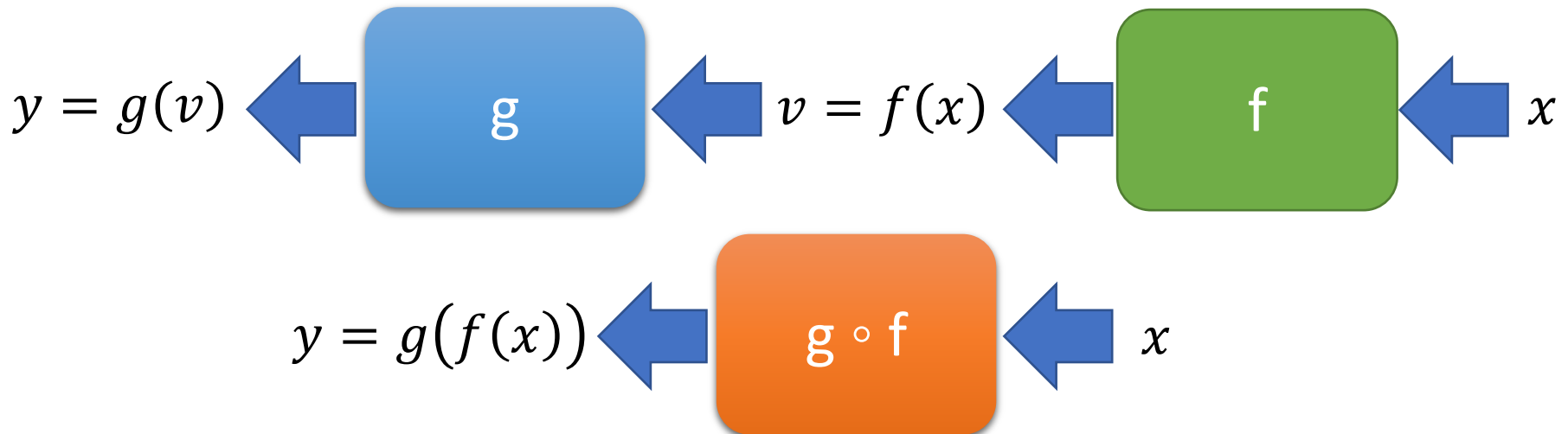
$$= [Ab_1 \quad Ab_2 \quad \cdots \quad Ab_p]$$

Matrix Multiplication - Meaning

The notation $g \circ f$ is read as "g circle f", "g round f", "g about f", "g composed with f", "g after f", "g following f", "g of f", "f then g", or "g on f".

- **Composition**

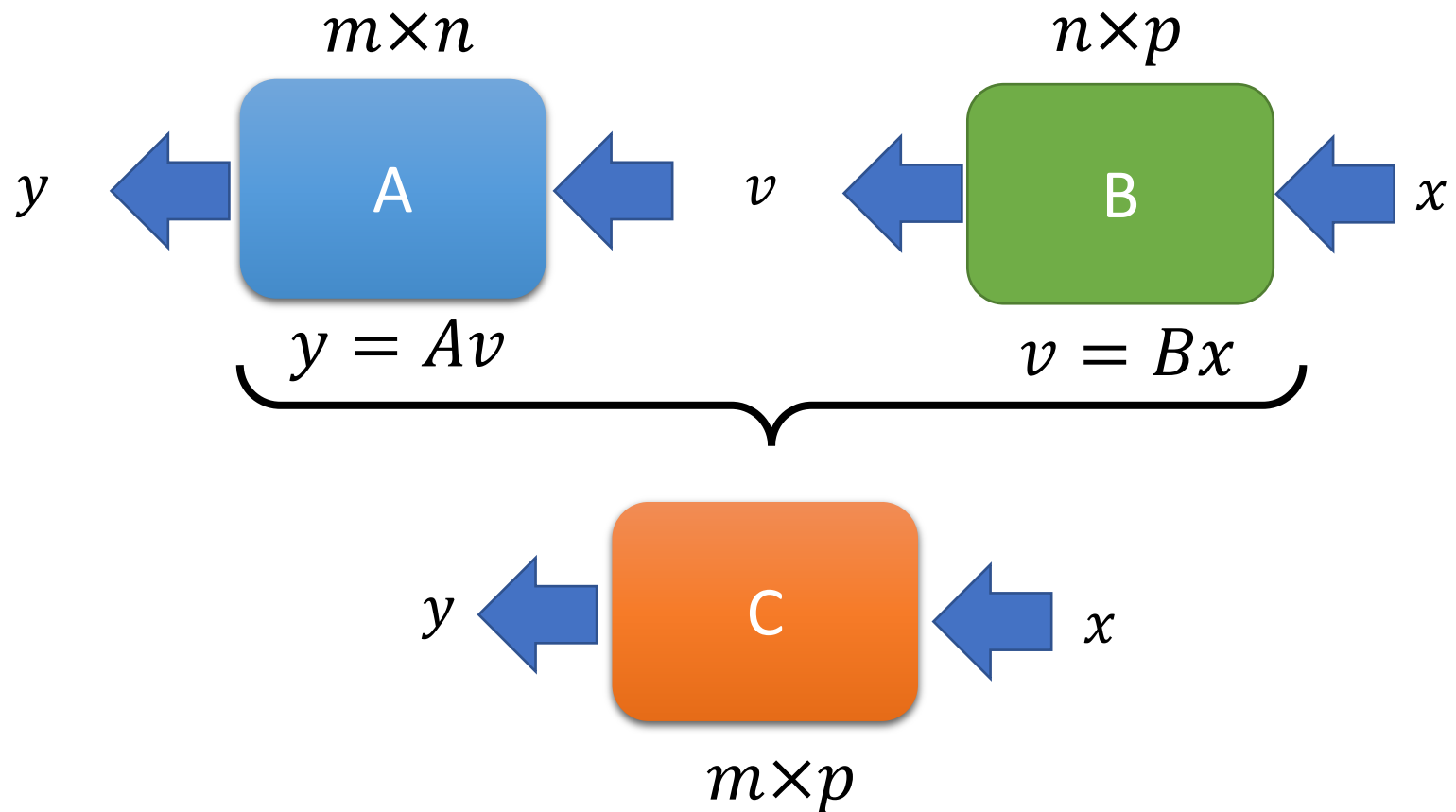
- Given two functions f and g , the function $g(f(\cdot))$ is the composition $g \circ f$.



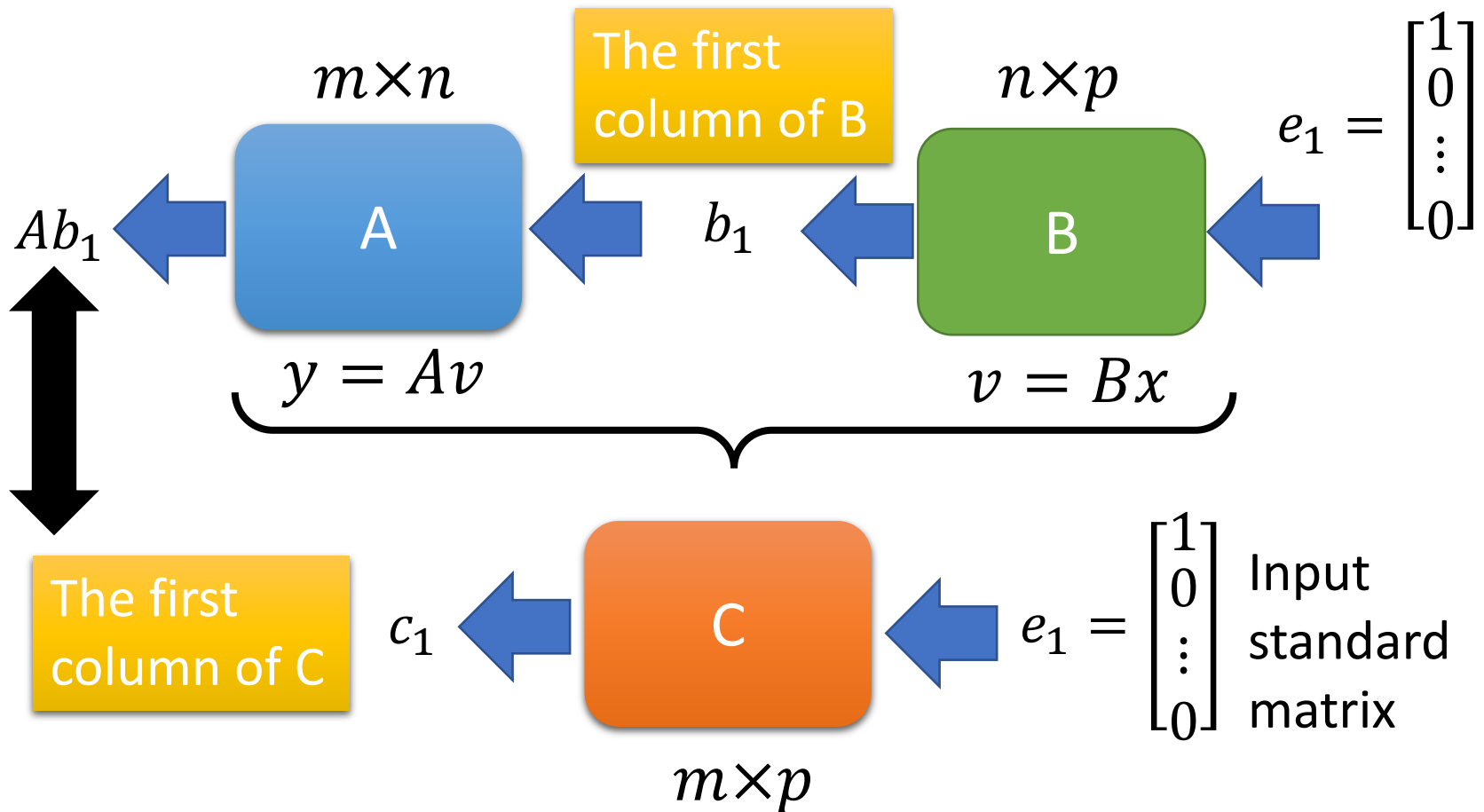
Matrix multiplication is the composition of two linear functions.

Matrix Multiplication - Meaning

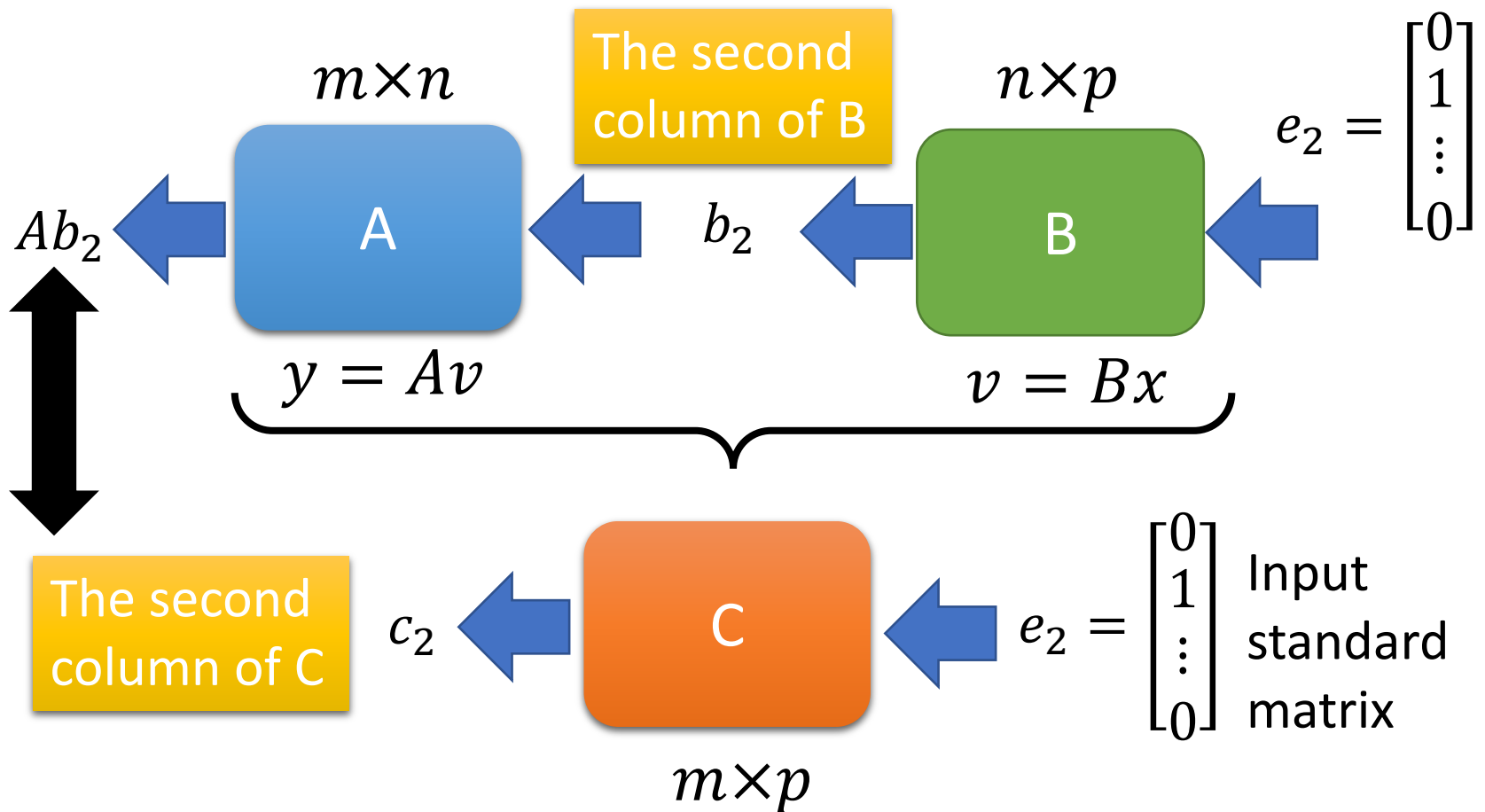
- Composition

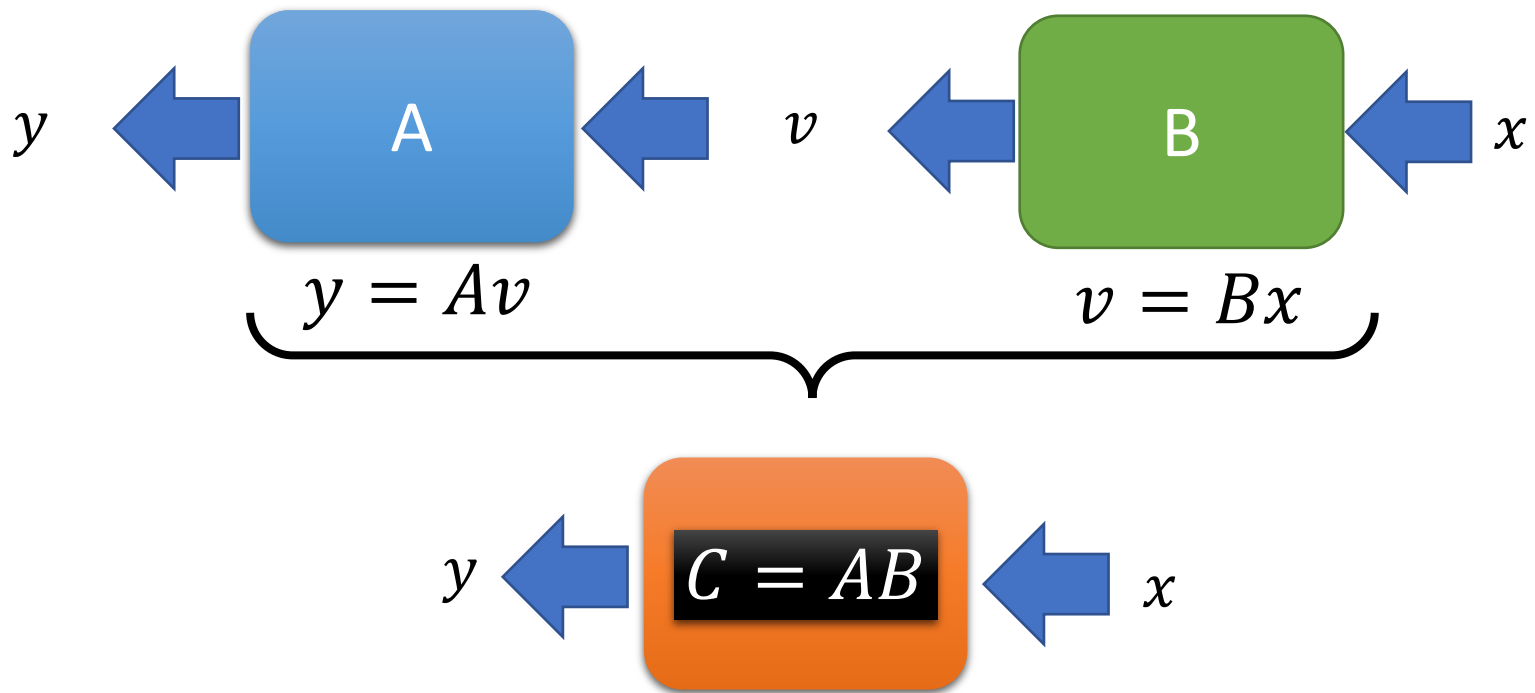


Matrix Multiplication - Meaning



Matrix Multiplication - Meaning



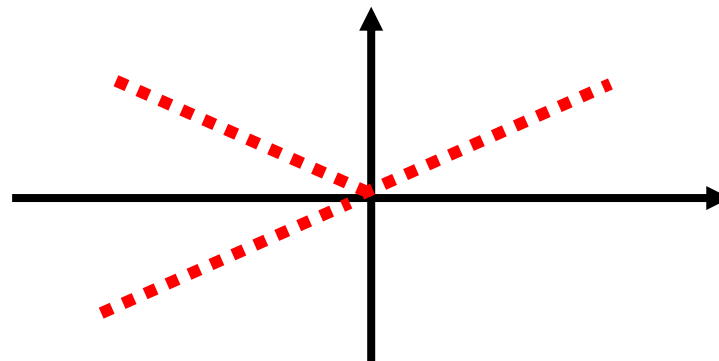


The composition of A and B is

$$C = [Ab_1 \quad Ab_2 \quad \cdots \quad Ab_p]$$

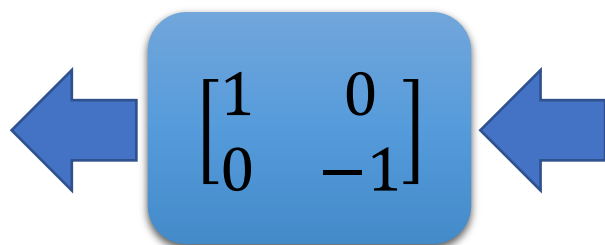
Matrix Multiplication

Example



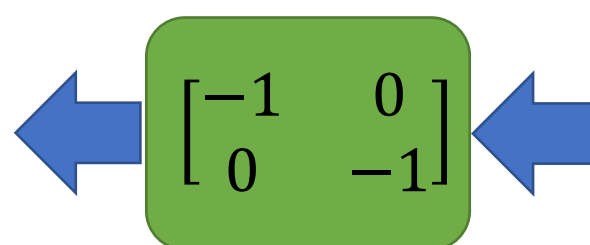
reflection about
the x-axis

\mathbb{R}^2
 y



$$y = Av$$

\mathbb{R}^2
 v



rotation by 180°

\mathbb{R}^2
 x

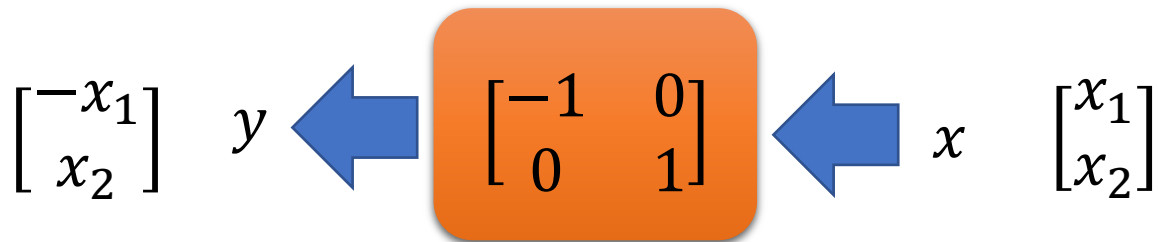
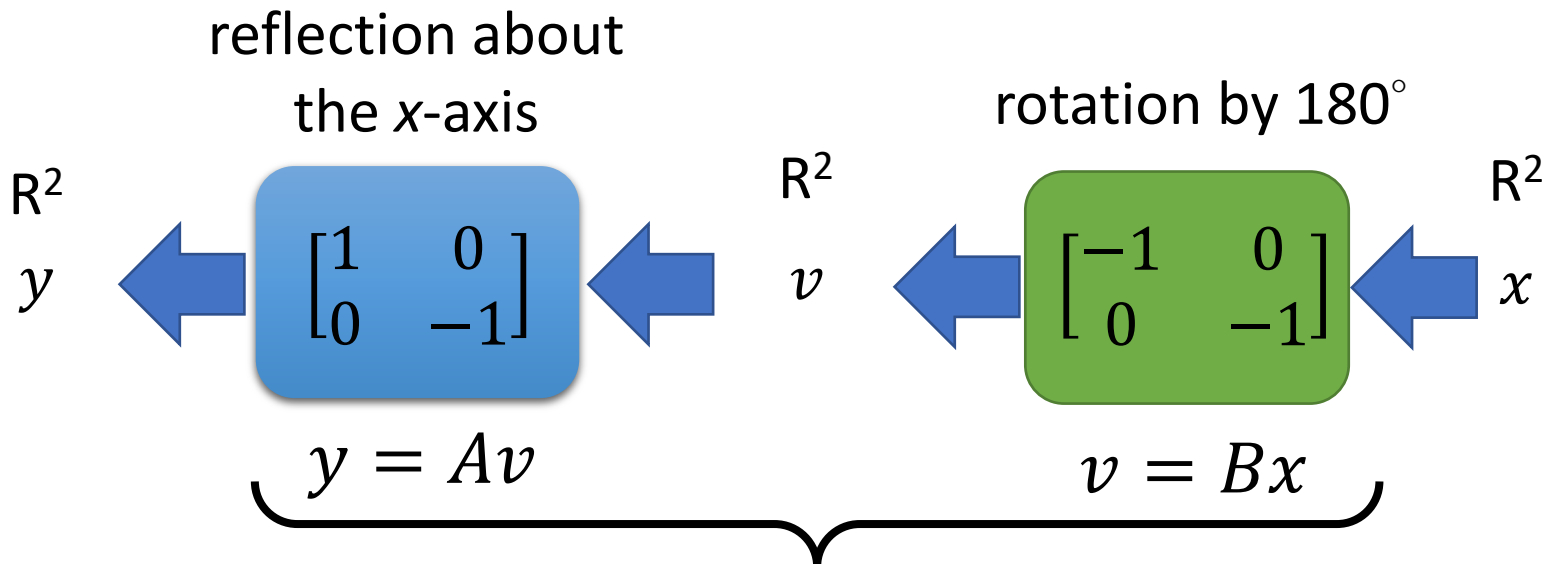
$$v = Bx$$



reflection about the y-axis

Example

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$



reflection about the y-axis