Matrix Multiplication

Properties

Not Communicative

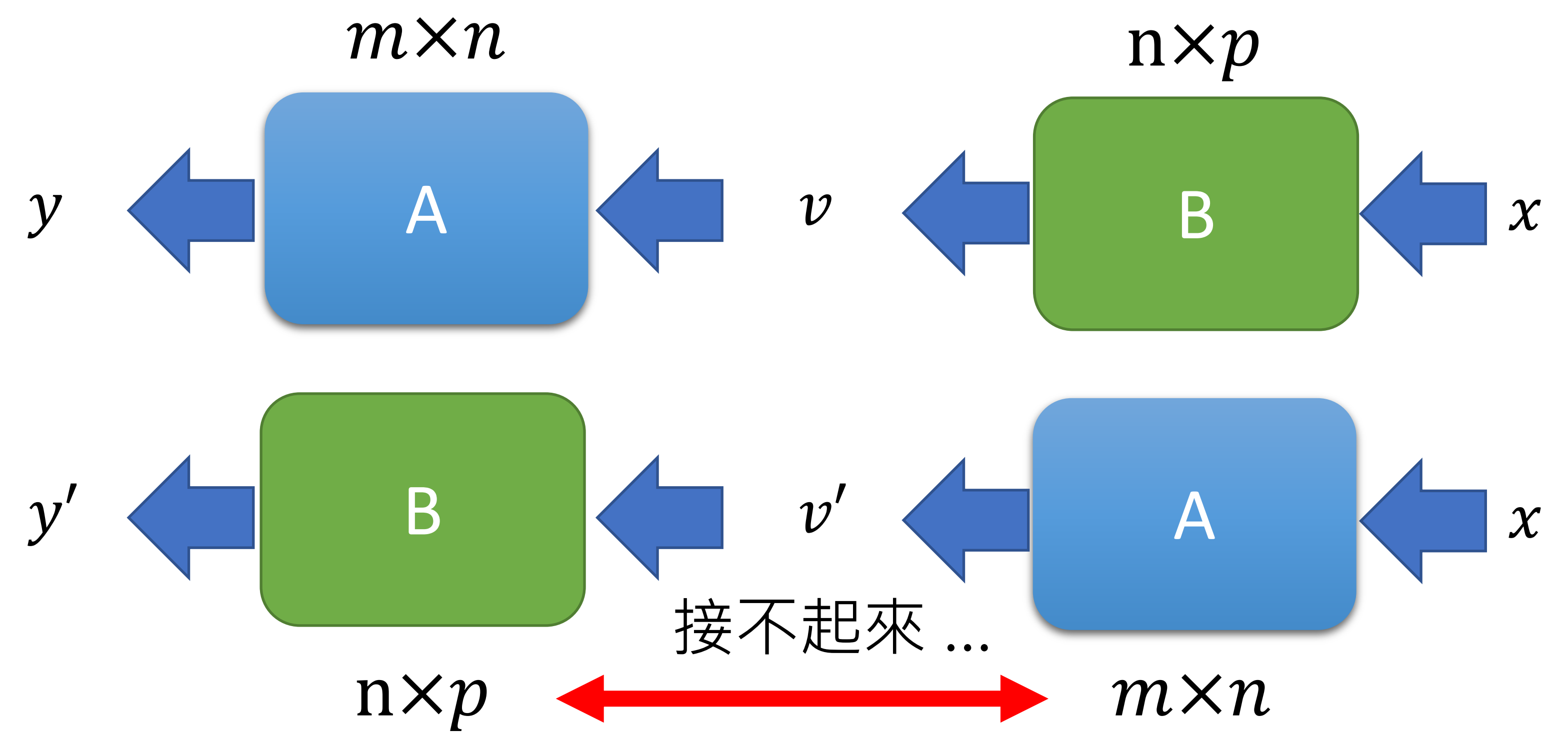
- $AB \neq BA$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

$$\neq BA = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

Not Communicative



m x m

n x n

If A and B are matrices, then both AB and BA are defined if and only if A and B are square matrices?

A: m x n, B: n x m

Properties

- Let A and B be k x m matrices, C be an m x n matrix, and P and Q be n x p matrices
  - For any scalar s, s(AC) = (sA)C = A(sC)
  - (A + B)C = AC + BC
  - C(P+Q)=CP+CQ
  - $I_k A = A = A I_m$
  - The product of any matrix and a zero matrix is a zero matrix
- Power of square matrices: A is n x n, $A^k = A\,A \cdots A$ (k times), and by convention, $A^1 = A$, $A^0 = I_n$

Properties

AC: k X n $\quad (AC)^T$: n X k

- Let A be kxm matrices, C be an mxn matrix,
  - $(AC)^T =? \quad C^T A^T$

$A^T C^T$

m X k $\quad$ n X m

$C^T A^T$

n X m $\quad$ m X k

n X k

Special Matrix

- Diagonal Matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad AB = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

- Symmetric Matrix $A^T = A$

AA^T and A^TA are square and symmetric

Square: A is m x n

$AA^T \rightarrow$ m x m

m x n   n x m

$A^TA \rightarrow$ n x n

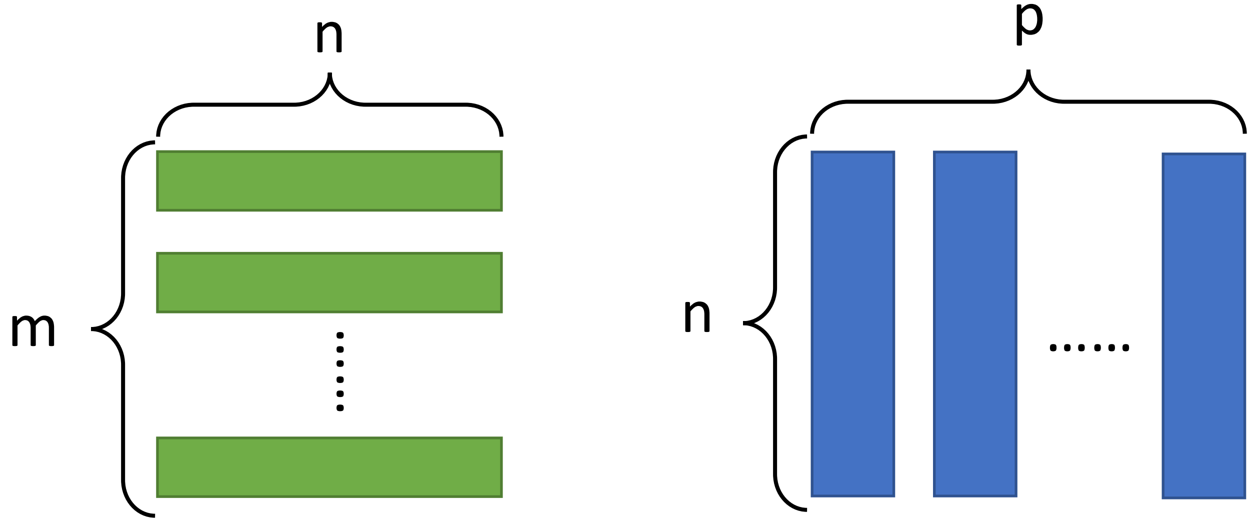
n x m   m x n

Symmetric:

$$(AA^T)^T = A^{TT}A^T = AA^T \qquad (A^TA)^T = A^TA^{TT} = A^TA$$

Practical Issue

- Let A and B be k x m matrices, C be an m x n matrix, and P and Q be n x p matrices
  - A(CP) = (AC)P



Multiplication count: m X n X p

Practical Issue

$k=1$ $m=1000$

$n=1$ $p=1000$

- Let A and B be k x m matrices, C be an m x n matrix, and P and Q be n x p matrices
  - A(CP) = (AC)P