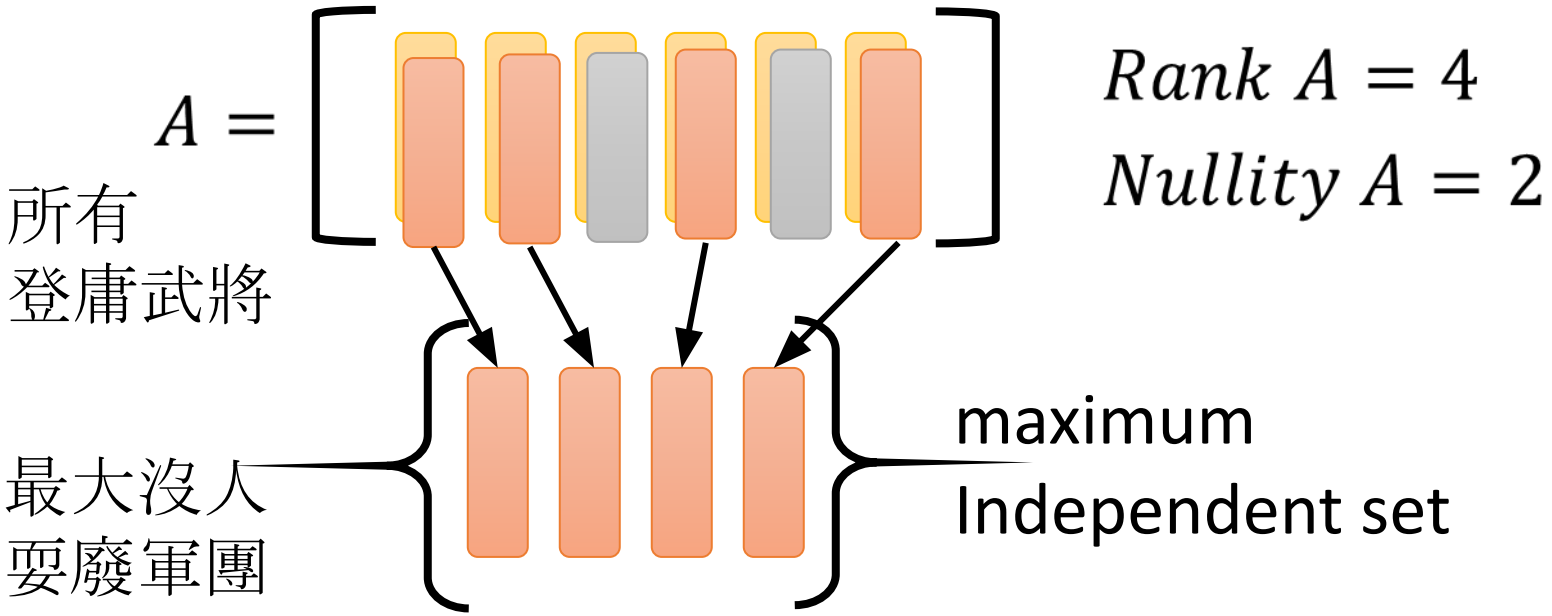
Rank and Nullity

秩　　零度

Rank and Nullity

- The **rank** of a matrix is defined as the maximum number of *linearly independent columns*
- **Nullity** = Number of columns - **rank**



Rank and Nullity

$$A = \begin{bmatrix} 1 & 3 & 10 \\ 2 & 6 & 20 \\ 3 & 9 & 30 \end{bmatrix}$$

$$Rank\ A = ?\ , Nullity\ A = ?$$

以下是最土炮的作法

$\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}3\\6\\9\end{bmatrix}, \begin{bmatrix}10\\20\\30\end{bmatrix}\right\}$ dependent

$\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}3\\6\\9\end{bmatrix}\right\}$ dependent

$\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}10\\20\\30\end{bmatrix}\right\}$ dependent

$\left\{\begin{bmatrix}3\\6\\9\end{bmatrix}, \begin{bmatrix}10\\20\\30\end{bmatrix}\right\}$ dependent

$\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}\right\}$ independent

$\left\{\begin{bmatrix}3\\6\\9\end{bmatrix}\right\}$ independent

$\left\{\begin{bmatrix}10\\20\\30\end{bmatrix}\right\}$ independent

$$Rank\ A = 1$$

$$Nullity\ A = 2$$

Rank and Nullity

$$A = \begin{bmatrix} -3 & 2 & -1 \\ 7 & 9 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$Rank\ A = ?$$

$$Nullity\ A = ?$$

Assume the three columns are independent

If A is a mxn matrix (n columns)

Columns of A are independent ⇄ Rank A = n

Columns of A are independent ⇄ Nullity A = 0

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 6 & 8 \end{bmatrix}$$

$Rank\ A = ?$

$Nullity\ A = ?$

$$\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$Rank\ A = ?$

$Nullity\ A = ?$

$$A = \begin{bmatrix} 0 & 3 \\ 0 & 5 \end{bmatrix}$$

$Rank\ A = ?$

$Nullity\ A = ?$

$$A = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$Rank\ A = ?$

$Nullity\ A = ?$

Summary

$$A\mathbf{x} = \mathbf{b}$$

$$A: m \times n \quad \boldsymbol{x} \in R^n \quad \boldsymbol{b} \in R^m$$