Matrix

李宏毅
Hung-yi Lee

Matrix

- A matrix is a set of vectors

$$a_1 = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix} \qquad a_2 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \qquad a_3 = \begin{bmatrix} 5 \\ -1 \\ 1 \end{bmatrix}$$

$$A = [a_1 \quad a_2 \quad a_3] = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 1 & -1 \\ -2 & 1 & 1 \end{bmatrix}$$

Matrix

- If the matrix has m rows and n columns, we say the size of the matrix is m by n, written m x n
  - The matrix is called square if m=n
  - We use $\mathcal{M}_{m\text{x}n}$ to denote the set that contains all matrices whose size is m x n

$$\begin{array}{cc} & \text{3 columns} \\ \text{2 rows} & \begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix} \in \mathcal{M}_{2\text{x}3} \\ & 2 \text{ X } 3\end{array}$$

$$\begin{array}{cc} & \text{2 columns} \\ \text{3 rows} & \begin{bmatrix}1 & 4\\ 2 & 5\\ 3 & 6\end{bmatrix} \in \mathcal{M}_{3\text{x}2} \\ & 3 \text{ X } 2\end{array}$$

先 Row 再 Column

Matrix

- ***Index of component***: the scalar in the i-th row and j-th column is called (i,j)-entry of the matrix

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

先 Row 再 Column

(1,2)-entry

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 1 & -1 \\ -2 & 1 & 1 \end{bmatrix}$$

(3,1)-entry

(3,3)-entry

Matrix

- Two matrices with the same size can add or subtract.
- Matrix can multiply by a scalar

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 6 & 9 \\ 8 & 0 \\ 9 & 2 \end{bmatrix} \qquad 9B$$

$$A + B \qquad\qquad A - B$$

Zero Matrix

- zero matrix: matrix with all zero entries, denoted by O (any size) or $O_{m\times n}$.
  - For example, a 2-by-3 zero matrix can be denoted

$$O_{2\times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A + O = A$$
$$0A = O$$
$$A - A = O$$

- Identity matrix: must be square
  - 對角線是 1, 其它都是 0

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Sometimes I_n is simply written as I (any size).

Properties

- A, B, C are mxn matrices, and s and t are scalars
  - $A + B = B + A$
  - $(A + B) + C = A + (B + C)$
  - $(st)A = s(tA)$
  - $s(A + B) = sA + sB$
  - $(s+t)A = sA + tA$

Transpose

Is “transpose” a linear system?

- If A is an mxn matrix
- A^T (transpose of A) is an nxm matrix whose (i,j)-entry is the (j,i)-entry of A

$$A = \begin{array}{c} (1,2) \\ \begin{bmatrix} 6 & \boxed{9} \\ 8 & 0 \\ 9 & \boxed{2} \end{bmatrix} \\ (3,2) \end{array} \quad \xrightarrow{\text{Transpose}} \quad A^T = \begin{array}{c} \begin{bmatrix} 6 & 8 & 9 \\ \boxed{9} & 0 & \boxed{2} \end{bmatrix} \\ (2,1) \qquad\quad (2,3) \end{array}$$

以左上到右下的對角線為軸進行翻轉

Transpose

$$A = \begin{bmatrix} 5 & 5 \\ 6 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & 7 \\ 8 & 8 \end{bmatrix}$$

- A and B are mxn matrices, and s is a scalar
- $(A^T)^T = A$
- $(sA)^T = sA^T$
- $(A + B)^T = A^T + B^T$

$$2A = \begin{bmatrix} 10 & 10 \\ 12 & 12 \end{bmatrix} \qquad (2A)^T = \begin{bmatrix} 10 & 12 \\ 10 & 12 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 5 & 6 \\ 5 & 6 \end{bmatrix} \qquad 2A^T = \begin{bmatrix} 10 & 12 \\ 10 & 12 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 12 & 12 \\ 14 & 14 \end{bmatrix} \qquad (A + B)^T = \begin{bmatrix} 12 & 14 \\ 12 & 14 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 5 & 6 \\ 5 & 6 \end{bmatrix} \qquad B^T = \begin{bmatrix} 7 & 8 \\ 7 & 8 \end{bmatrix} \qquad A^T + B^T = \begin{bmatrix} 12 & 14 \\ 12 & 14 \end{bmatrix}$$