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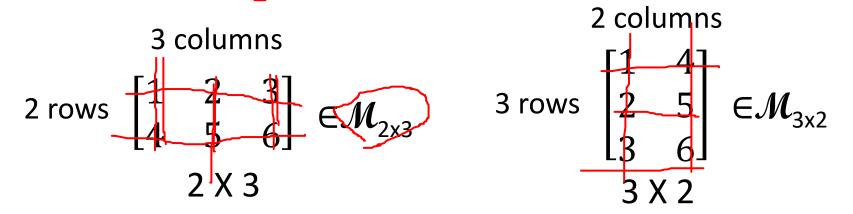
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A matrix is a set of vectors

$$\underline{a_1} = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix} \qquad \underline{a_2} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \qquad \underline{a_3} = \begin{bmatrix} 5 \\ -1 \\ 1 \end{bmatrix}$$

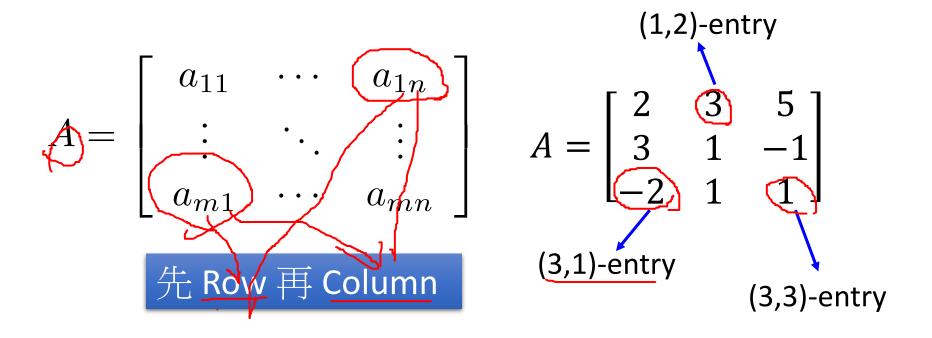
$$A = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 1 & -1 \\ -2 & 1 & 1 \end{bmatrix}$$

- If the matrix has m rows and n columns, we say the size of the matrix is m by n, written m x n
 - The matrix is called square if m=n
 - We use \mathcal{M}_{myn} to denote the set that contains all matrices whose size is m x n



先 Row 再 Column

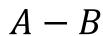
Index of component: the scalar in the i-th row and i-th column is called (i,j)-entry of the matrix



- Two matrices with the same size can add or subtract.
- Matrix can multiply by a scalar

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 6 & 9 \\ 8 & 0 \\ 9 & 2 \end{bmatrix} \qquad 9B$$

$$A + B$$



Zero Matrix

- zero matrix: matrix with all zero entries, denoted by Q (any size) or $O_{m \times n}$.
 - For example, a 2-by-3 zero matrix can be denoted

$$O_{2\times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
 $A + Q = A$
 $0A = Q$
 $A - A = Q$

- Identity matrix: must be square
 - 對角線是 1, 其它都是 0

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

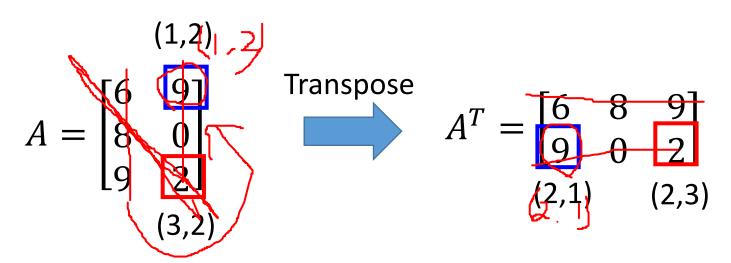
Sometimes I_n is simply written as I (any size).

Properties

- A, B, C are mxn matrices, and s and t are scalars
 - A + B = B + A
 - (A + B) + C = A + (B + C)
 - (st)A = s(tA)
 - s(A + B) = sA + sB
 - (s+t)A = sA + tA

Is "transpose" a linear system?

- If A is an mxn matrix
- A^T (transpose of A) is an <u>nxm</u> matrix whose (i,j)-entry is the (iii)-entry of A



以左上到右下的對角線為軸 進行翻轉

Transpose
$$A = \begin{bmatrix} 5 & 5 \\ 6 & 6 \end{bmatrix}$$
 $B = \begin{bmatrix} 7 & 7 \\ 8 & 8 \end{bmatrix}$

A and B are mxn matrices, and s is a scalar

•
$$(A^T)^T = A$$

•
$$(SA)^T \neq SA^T$$

$$\bullet \ (\underline{A} + \underline{B})^T = (\underline{A}^T) + (\underline{B}^T)^T$$

•
$$(A^T)^T = A$$

• $(3A)^T = SA^T$
• $(A + B)^T = A^T + B^T$
• $(A + B)^T = A^T + B^T$

$$\mathbf{A}^T = \begin{bmatrix} 5 & 6 \\ 5 & 6 \end{bmatrix} \qquad 2A^T = \begin{bmatrix} 10 & 12 \\ 10 & 12 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 12 & 12 \\ 14 & 14 \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} 5 & 6 \\ 5 & 6 \end{bmatrix} \quad B^{T} = \begin{bmatrix} 7 & 8 \\ 7 & 8 \end{bmatrix} \qquad A^{T} + B^{T} = \begin{bmatrix} 12 & 14 \\ 12 & 14 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 12 & 12 \\ 14 & 14 \end{bmatrix} \qquad (A + B)^{T} = \begin{bmatrix} 12 & 14 \\ 12 & 14 \end{bmatrix}$$

$$\begin{bmatrix} A + B \end{bmatrix} = \begin{bmatrix} 12 & 14 \\ 12 & 14 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 12 & 14 \\ 12 & 14 \end{bmatrix}$$