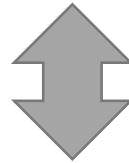


Matrix-Vector Product

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$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$



Matrix-vector product: $A\mathbf{x} = \mathbf{b}$

Row Aspect

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

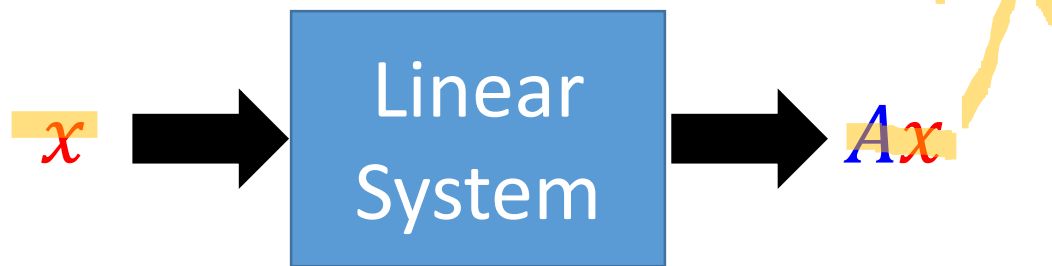
$$Ax = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad Ax = \begin{bmatrix} \\ \end{bmatrix}$$

Matrix-Vector Product

$$\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{array} = \begin{array}{l} b_1 \\ b_2 \\ \vdots \\ b_m \end{array}$$

$Ax = b$



Coefficients are A

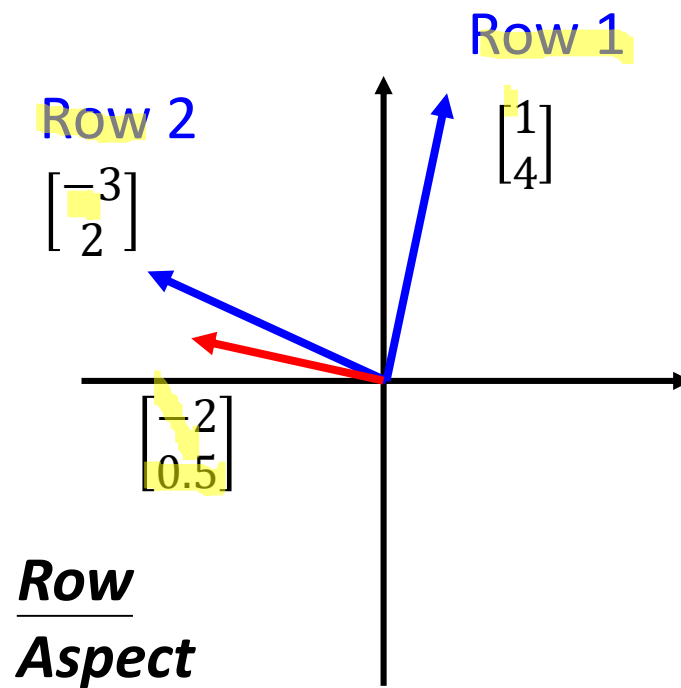
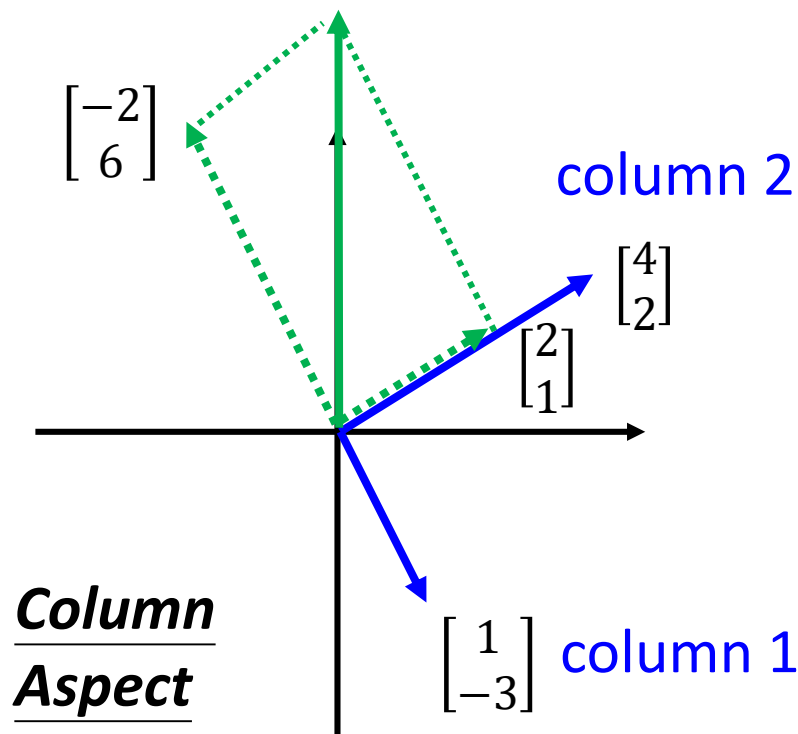
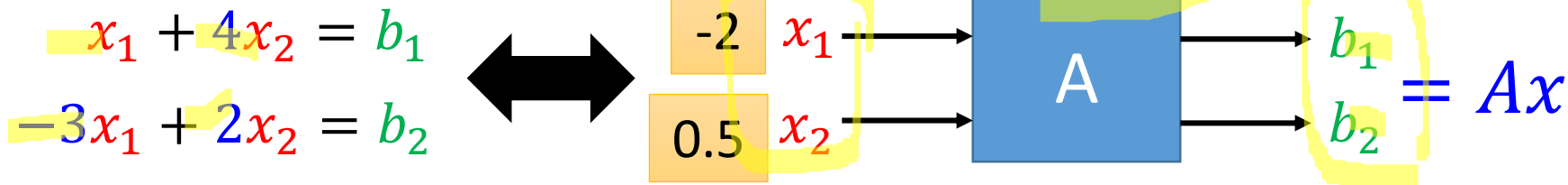
Column Aspect

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$Ax =$$

$$= \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}$$

Example



Matrix-vector Product

- The size of matrix and vector should be matched.

The diagram shows a matrix A and a vector x with red annotations. A large red 'X' is drawn over the matrix A and the vector x , indicating that their dimensions do not match for multiplication. A red arrow points from the vector x to the first column of matrix A . Below, two matrices A' and A'' are shown. Red lines and arrows indicate the corrections: a vertical red line is drawn under the first column of A' and the first row of A'' . A red arrow points from the first row of x to the first row of A'' , and another red arrow points from the first column of x to the first column of A' .

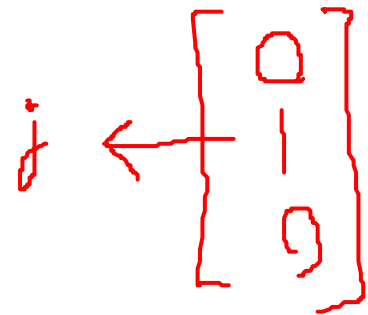
$$A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 1 & -1 \\ -2 & 1 & 1 \end{bmatrix} \quad x = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$
$$A' = \begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 1 & 4 \end{bmatrix} \quad A'' = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 0 & -1 \\ 1 & -3 \end{bmatrix}$$

Properties of Matrix-vector Product

- A and B are $m \times n$ matrices, \mathbf{u} and \mathbf{v} are vectors in \mathcal{R}^n , and c is a scalar.
- $A(\mathbf{u} + \mathbf{v}) = A\mathbf{u} + A\mathbf{v}$
- $A(c\mathbf{u})$ = $c(A\mathbf{u})$ = $(cA)\mathbf{u}$
- $(A + B)\mathbf{u}$ = $A\mathbf{u}$ + $B\mathbf{u}$
- $A\mathbf{0}$ is the $m \times 1$ zero vector
- $\mathbf{0}\mathbf{v}$ is also the $m \times 1$ zero vector
- $I_n \mathbf{v}$ = \mathbf{v}



Properties of Matrix-vector Product

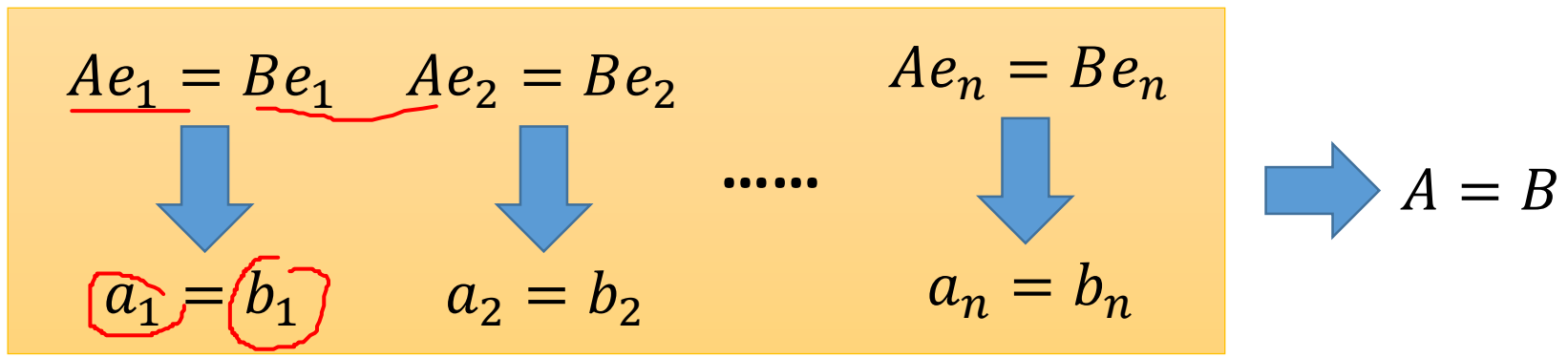


- A and B are $m \times n$ matrices. If $Aw = Bw$ for all w in \mathcal{R}^n . Is it true that $A = B$?

$Ae_j = a_j$ for $j = 1, 2, \dots, n$, where e_j is the j -th standard vector in \mathcal{R}^n

$e_1 = \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix}$ $Ae_1 = \begin{bmatrix} a_1 & \dots & a_n \end{bmatrix} \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix} = 1 \cdot a_1 + 0 \cdot a_2 + \dots + 0 \cdot a_n = a_1$

Column Aspect



Concluding Remarks

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$Ax = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}$$

Row Aspect

$$= x_1 \begin{bmatrix} a_{11} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ \vdots \\ a_{m2} \end{bmatrix} + \cdots + x_n \begin{bmatrix} a_{1n} \\ \vdots \\ a_{mn} \end{bmatrix}$$

Column Aspect