(High School) Vector

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Vectors

- A vector **v** is a set of numbers

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Column vector

$$\mathbf{v} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

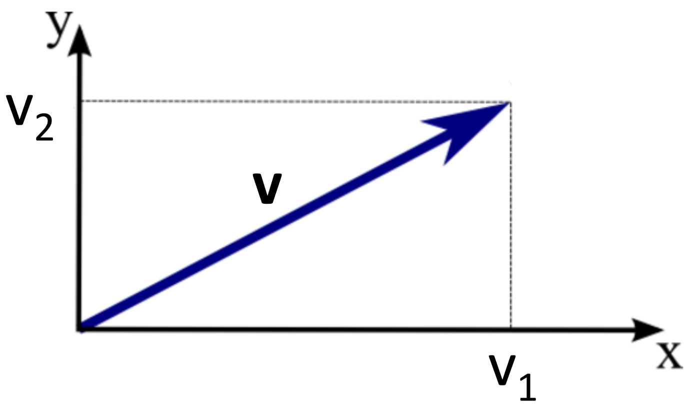
Row vector

In this course, the term **vector** refers to a **column vector** unless being explicitly mentioned otherwise.

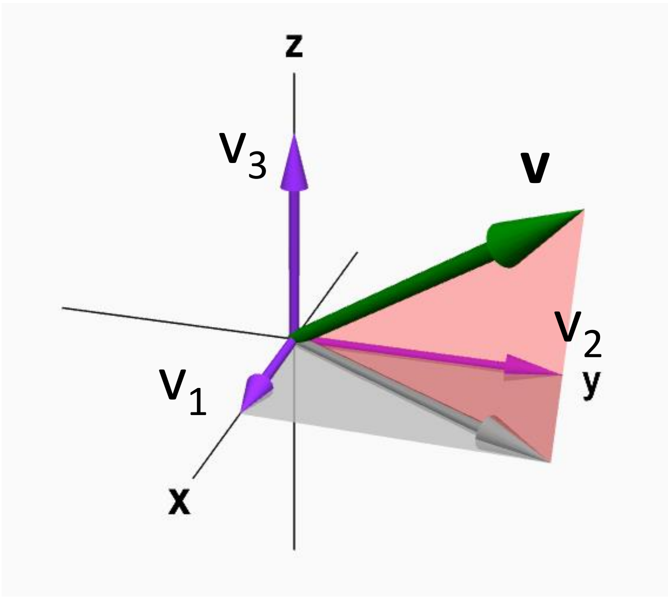
Vectors

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

- components: the entries of a vector.
  - The i-th component of vector **v** refers to v_i
  - $v_1=1$, $v_2=2$, $v_3=3$
- If a vector only has less than four components, you can visualize it.

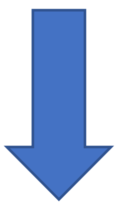


http://mathinsight.org/vectors_cartesian_coordinates_2d_3d#vector3D

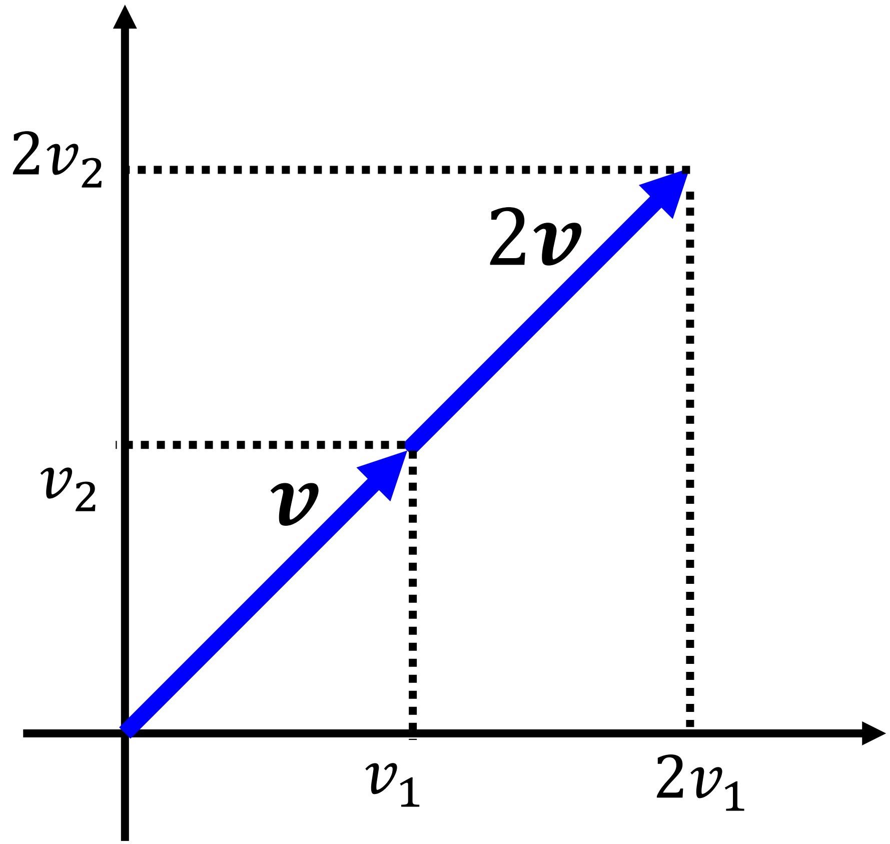


Scalar Multiplication

$$\boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

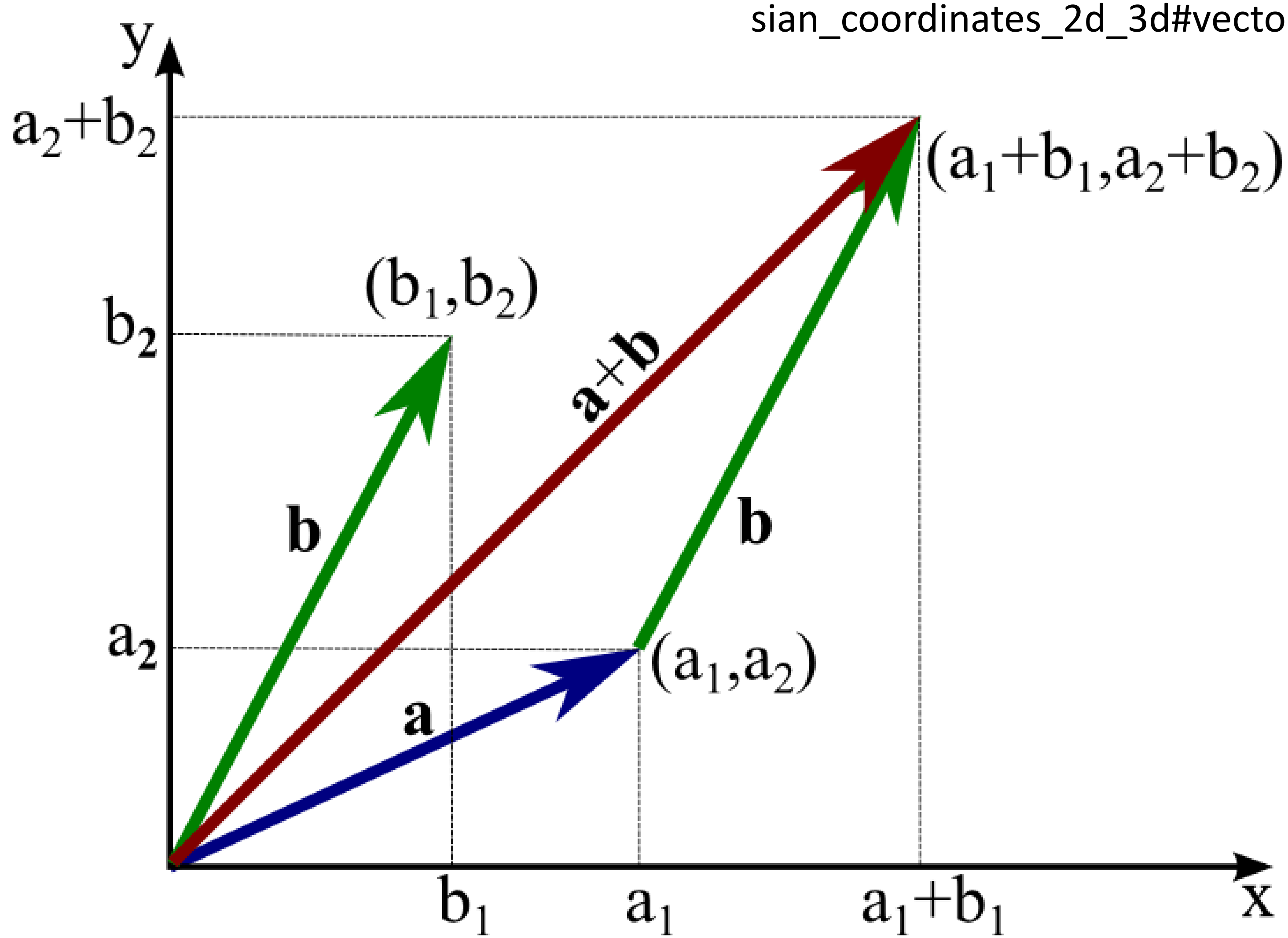


$c\boldsymbol{v}$



Vector Addition

http://mathinsight.org/vectors_cartesian_coordinates_2d_3d#vector3D

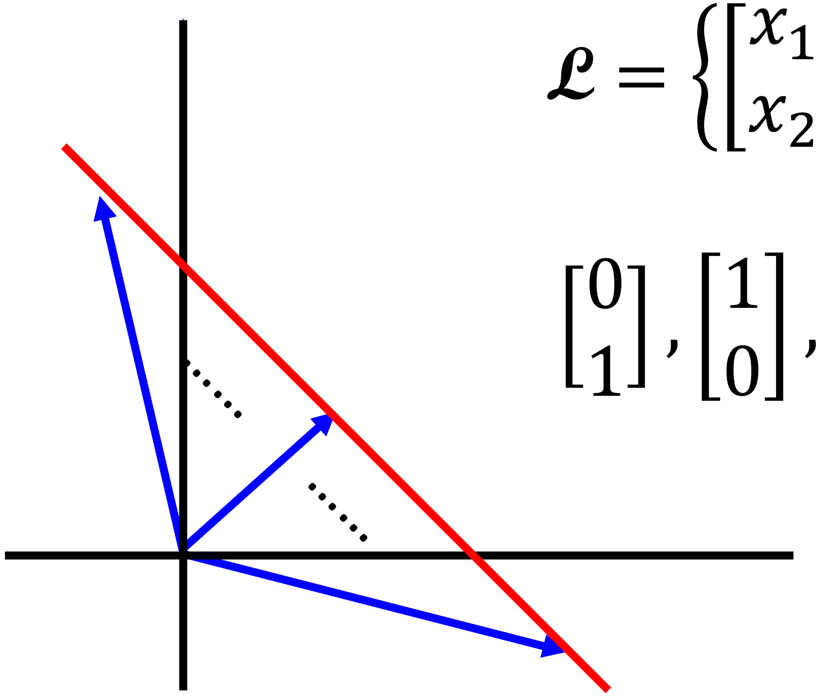


Vector Set

$$\left\{\begin{bmatrix}1\\2\\3\end{bmatrix},\begin{bmatrix}4\\5\\6\end{bmatrix},\begin{bmatrix}6\\8\\9\end{bmatrix},\begin{bmatrix}9\\0\\2\end{bmatrix}\right\}$$

A vector set with 4 elements

- A vector set can contain infinite elements

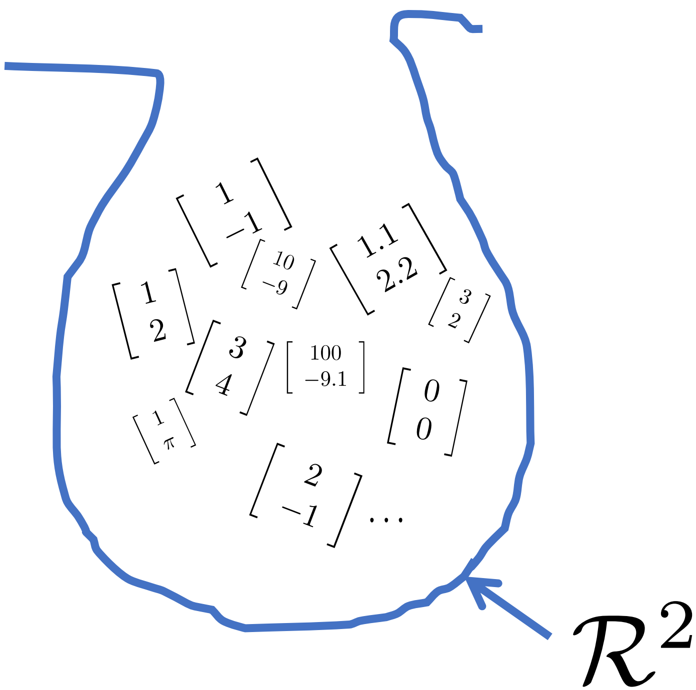


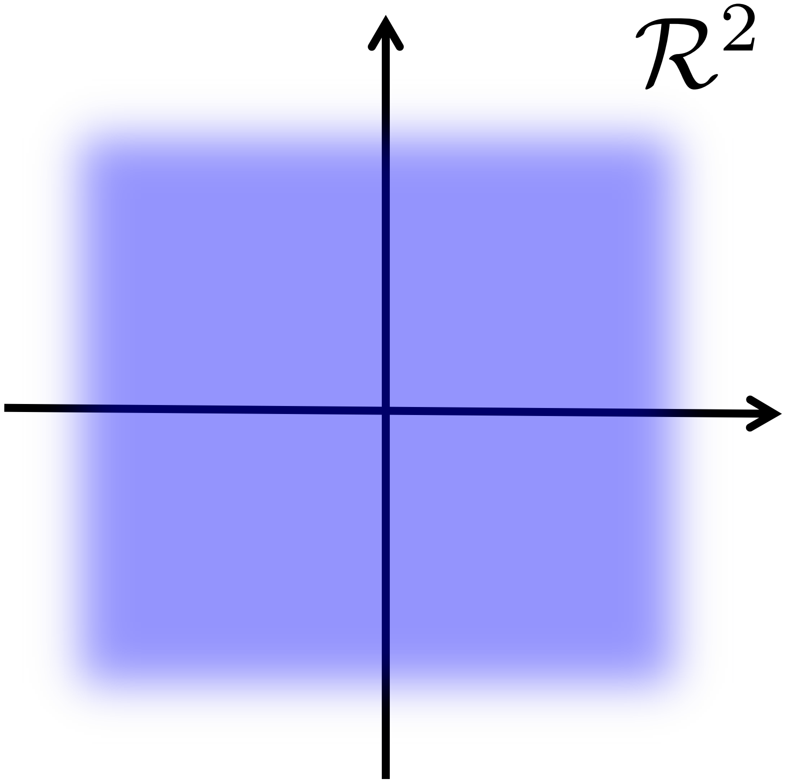
$$\mathcal{L}=\left\{\begin{bmatrix}x_1\\x_2\end{bmatrix}: x_1+x_2=1\right\}$$

$$\begin{bmatrix}0\\1\end{bmatrix},\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0.3\\0.7\end{bmatrix},\begin{bmatrix}0.7\\0.3\end{bmatrix}\cdots\cdots$$

Vector Set

- $\mathcal{R}^n$:We denote the set of all **vectors** with *n* entries by $\mathcal{R}^n$.





Properties of Vector

The objects have the following 8 properties are "vectors".

- For any vectors **u**, **v** and **w** in $\mathcal{R}^n$, and any scalars a and b
  - **u** + **v** = **v** + **u**
  - (**u** + **v**) + **w** = **u** + (**v** + **w**)
  - There is an element $\mathbf{0}$ in $\mathcal{R}^n$ such that $\mathbf{0}$ + **u** = **u**
  - There is an element **u'** in $\mathcal{R}^n$ such that **u'** + **u** = 0
  - 1**u** = **u**
  - (ab)**u** = a(b**u**)
  - a(**u**+**v**) = a**u** + a**v**
  - (a+b)**u** = a**u** + b**u**

$$\mathbf{0} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$ zero vector

u' is the additive inverse of **u**

More Properties of Vector

0 + **u** = **u**
u’ + **u** = **0**

- For any vectors **u**, **v** and **w** in $\mathscr{R}^n$, and any scalar a
  - If **u** + **v** = **w** + **v**, then **u** = **w**
  - If **u** + **v** = **u** + **w**, then **v** = **w**
  - The zero vector **0** is unique. It is the only vector in $\mathscr{R}^n$ that satisfies **0** + **u** = **u**
  - Each vector in $\mathscr{R}^n$ has exactly one **u**’
  - 0**u** = **0**
  - a**0** = **0**
  - **u**’ = -1(**u**) = -**u**
  - (-a)**u** = a(-**u**) = -(a**u**)