5-2

74. If $f(t)$ is the characteristic polynomial of a square matrix A, what is $f(0)$?

$f(t) = \det(A - tI)$

$f(0) = \det(A - 0I) = \det A$

75. Suppose that the characteristic polynomial of an $n \times n$ matrix A is

$$f(A) = a_n t^n + a_{n-1} t^{n-1} + \cdots + a_1 t + a_0 \qquad \det A = a_0$$

Determine the characteristic polynomial of $-A$.

$$f(A) = \det(A - tI)$$

$$f'(A) = \det(-A - tI) = (-1)^n \det(A + tI)$$

$$= (-1)^n \det(A - (-A)I) = (-1)^n f(-A)$$

$$f(-A) = (-1)^n a_n (-A)^n + (-1)^n a_{n-1} (-A)^{n-1} + \cdots + (-1)^n a_1 (-A) + (-1)^n a_0$$

76. What is the coefficient of t^n in the characteristic polynomial of an $n \times n$ matrix?

$(-1)^n$

$$\det\begin{pmatrix} a_{11}-\lambda & & & \\ & a_{22}-\lambda & & \\ & & a_{33}-\lambda & \\ & & & \ddots \\ & & & & a_{nn}-\lambda \end{pmatrix} = (a_{11}-\lambda)(a_{22}-\lambda)\cdots(a_{nn}-\lambda) + \cdots$$

$$= (-\lambda)^n + \cdots$$

77. Suppose that A is a 4×4 matrix with no nonreal eigenvalues and exactly ~~three~~ real eigenvalues, 5 and -9. Let W_1 and W_2 be the eigenspaces corresponding to 5 and -9, respectively. Write all the possible characteristic polynomials of A that are consistent with the following information:

(b) $dim W_2 = 1$

78. Suppose that A is a 5×5 matrix with no nonreal eigenvalues and exactly three real eigenvalues, 4, 6, and 7. Let W_1, W_2, and W_3 be the eigenspaces corresponding to 4, 6, and 7, respectively. Write all the possible characteristic polynomials of A that are consistent with the following information:

(d) $dimW_2 = 2$ and $dimW_3 = 2$

$A: n \times n$

$\lambda_1 \quad \lambda_2 \quad \lambda_3$ no more λ

$(-1)^n$

$c(A-\lambda_1)^{m_1}(A-\lambda_2)^{m_2}(A-\lambda_3)^{m_3}$

81. (a) Determine a basis for each eigenspace of $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$

(b) Determine a basis for each eigenspace of $-3A$

(c) Determine a basis for each eigenspace of $5A$

81. (d) Establish a relationship between the eigenvectors of any square matrix B and those of cB for any scalar $c \neq 0$.

(e) Establish a relationship between the eigenvalues of any square matrix B and those of cB for any scalar $c \neq 0$.

83. (a) Determine the characteristic polynomial of A^T, where $A = \begin{bmatrix} 5 & -2 \\ 1 & 8 \end{bmatrix}$

(b) Establish a relationship between the characteristic polynomial of any square matrix B and that of B^T.

(c) What does (b) imply about the relationship between the eigenvalues of a square matrix B and those of B^T?

(d) Is there a relationship between the eigenvectors of a square matrix B and those of B^T?

84. Let A and B be $n \times n$ matrices such that $B = P^{-1}AP$, and let λ be an eigenvalue of A (and hence of B). Prove the following results.

$$I = P^{-1}P$$

84. Let A and B be $n \times n$ matrices such that $B = P^{-1}AP$, and let λ be an eigenvalue of A (and hence of B). Prove the following results.

(a) A vector $\boldsymbol{v}$ in R^n is in the eigenspace of A corresponding to λ if and only if $P^{-1}\boldsymbol{v}$ is in the eigenspace of B corresponding to λ.

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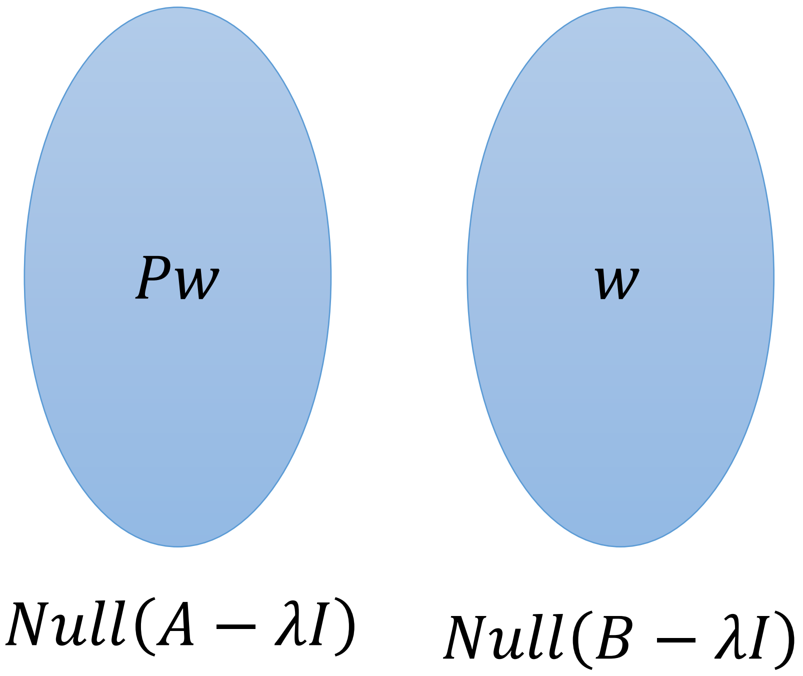
(b) If $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_k\}$ is a basis for the eigenspace of A corresponding to λ, then $\{P^{-1}\boldsymbol{v}_1, P^{-1}\boldsymbol{v}_2, \dots, P^{-1}\boldsymbol{v}_k\}$ is a basis for the eigenspace of B corresponding to λ.



84. Let A and B be $n \times n$ matrices such that $B = P^{-1}AP$, and let λ be an eigenvalue of A (and hence of B). Prove the following results.

(a) A vector $\boldsymbol{v}$ in R^n is in the eigenspace of A corresponding to λ if and only if $P^{-1}\boldsymbol{v}$ is in the eigenspace of B corresponding to λ.

(b) If $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k\}$ is a basis for the eigenspace of A corresponding to λ, then $\{P^{-1}\boldsymbol{v}_1, P^{-1}\boldsymbol{v}_2, \ldots, P^{-1}\boldsymbol{v}_k\}$ is a basis for the eigenspace of B corresponding to λ.

(c) The eigenspaces of A and B that correspond to the same eigenvalue have the same dimension.