# 5-3

85. Suppose that $A$ and $B$ are similar matrices such that $B = P^{-1}AP$ for some invertible matrix $P$.

(a) Show that $A$ is diagonalizable if and only if $B$ is diagonalizable.

85. Suppose that $A$ and $B$ are similar matrices such that $B = P^{-1}AP$ for some invertible matrix $P$.

(b) How are the eigenvalues of $A$ related to the eigenvalues of $B$?

(c) How are the eigenvectors of $A$ related to the eigenvectors of $B$?

86. A matrix $B$ is called a cube root of a matrix $A$ if $B^3 = A$. Prove that every diagonalizable matrix has a cube root.

88. Let $A$ be a diagonalizable $n \times n$ matrix. Prove that if the characteristic polynomial of $A$ is $f(t) = a_n t^n + a_{n-1} t^{n-1} + \cdots + a_1 t + a_0$, then $f(A) = O$, where $f(A) = a_n A^n + a_{n-1} A^{n-1} + \cdots + a_1 A + a_0 I_n$. This result is called the Cayley-Hamilton theorem.

Hint: If $A = PDP^{-1}$, show that $f(A) = Pf(D)P^{-1}$

88. Let $A$ be a diagonalizable $n \times n$ matrix. Prove that if the characteristic polynomial of $A$ is $f(t) = a_n t^n + a_{n-1} t^{n-1} + \cdots + a_1 t + a_0$, then $f(A) = O$, where $f(A) = a_n A^n + a_{n-1} A^{n-1} + \cdots + a_1 A + a_0 I_n$. This result is called the Cayley-Hamilton theorem.

Hint: If $A = PDP^{-1}$, show that $f(A) = Pf(D)P^{-1}$

89. The trace of a square matrix is the sum of its diagonal entries.

(a) Prove that if $A$ is a diagonalizable matrix, then the trace of $A$ equals the sum of the eigenvalues of $A$.

Hint: For all $n \times n$ matrices $A$ and $B$, show that the trace of $AB$ equals the trace of $BA$.

89. The trace of a square matrix is the sum of its diagonal entries.

(b) Let $\mathrm{A}$ be a diagonalizable $n \times n$ matrix with characteristic polynomial $(-1)^n(t-\lambda_1)(t-\lambda_2)\ldots(t-\lambda_n)$. Prove that the coefficient of $t^{n-1}$ in this polynomial is $(-1)^{n-1}$ times the trace of $A$.

89. The trace of a square matrix is the sum of its diagonal entries.

(c) For $A$ as in (b), what is the constant term of the characteristic polynomial of $A$?

$detD = detA$

55. Let $A$ be a $5 \times 5$ matrix with exactly the eigenvalues $4, 5,$ and $8,$ and corresponding eigenspaces $W_1, W_2$ and $W_3$. For each of the given parts, either write the characteristic polynomial of $A$, or state why there is insufficient information to determine the characteristic polynomial.

(a) $dimW_1 = 2$ and $dimW_3 = 2$

(b) $A$ is diagonalizable and $dimW_2 = 2$

(c) $A$ is diagonalizable, $dimW_1 = 2$ and $dimW_3 = 2$