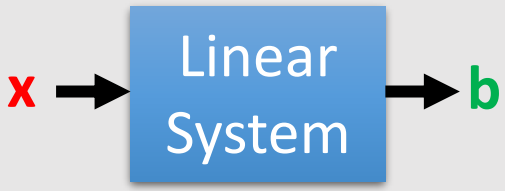
Having Solution or Not

(You already learned in high school)

Review



$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

System of Linear Equations

Matrix-vector product: $A\mathbf{x} = \mathbf{b}$

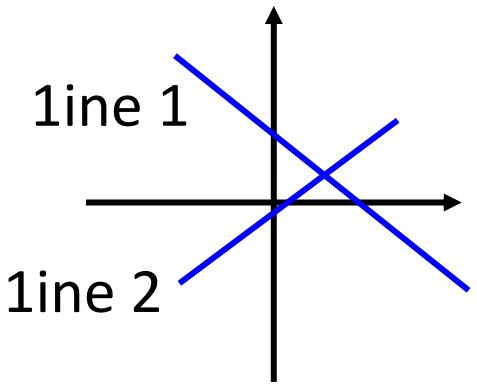
Given A and **b**, let's find **x**

In High School ……

More Variables?

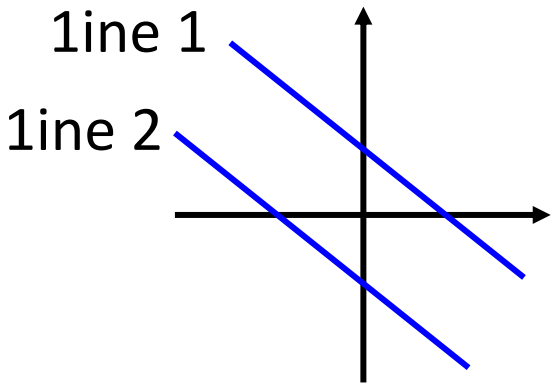
- Considering any system of linear equations with 2 variables and 2 equations

$$a_{11}x_1 + a_{12}x_2 = b_1 \quad \text{…… line 1}$$
$$a_{21}x_1 + a_{22}x_2 = b_2 \quad \text{…… line 2}$$



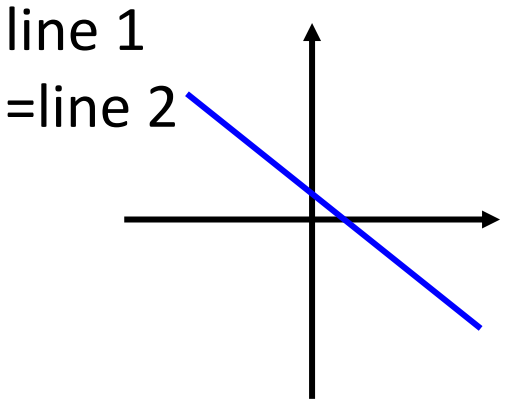
unique solution

consistent



no solution

inconsistent



infinite solutions

consistent

Only Unique and Infinite?

為什麼不能只有兩個解？

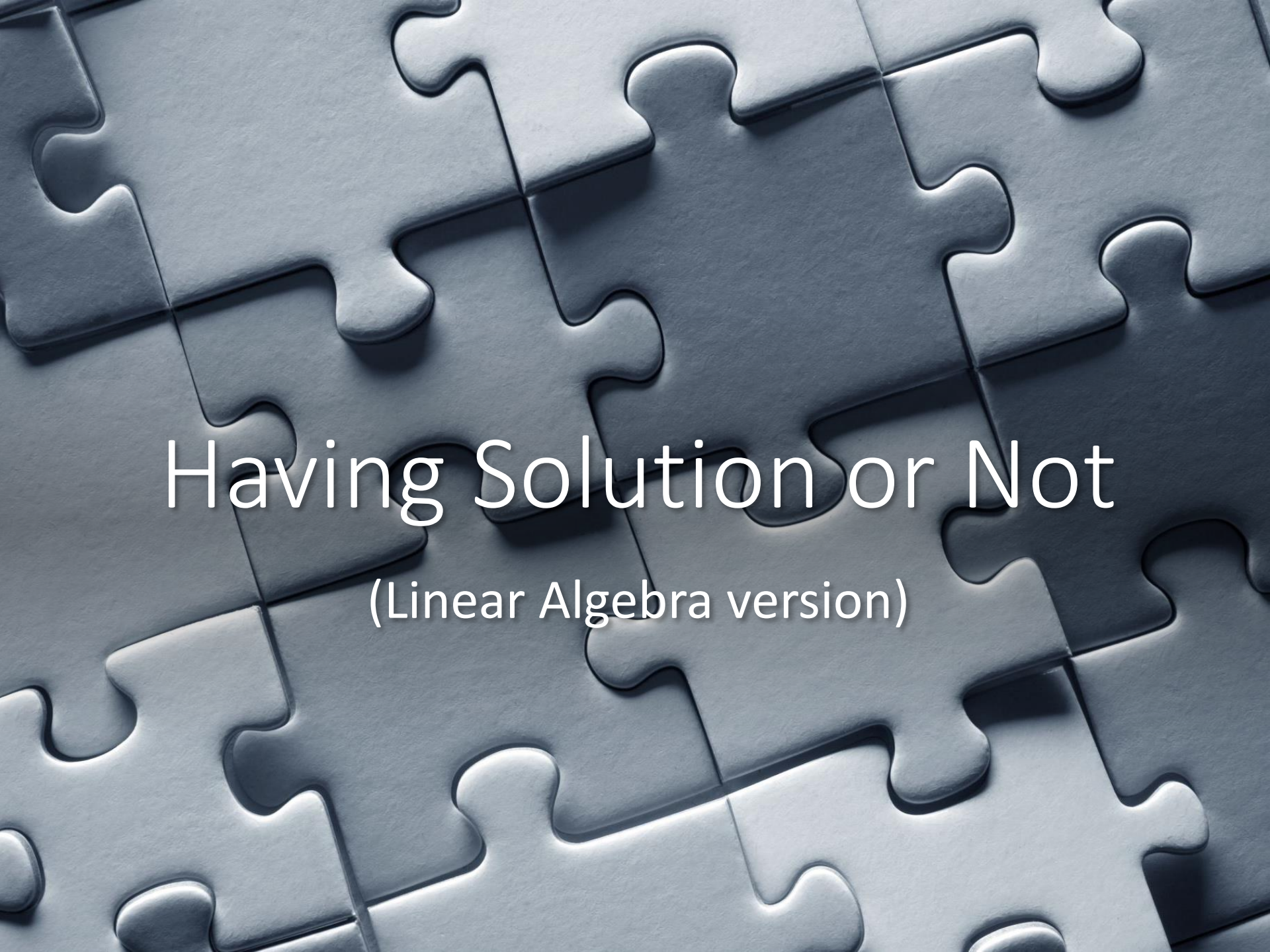
一旦找到兩個解，
就可以找到無窮多解

3rd solution $0.3u + 0.7v$

4th solution $0.4u + 0.6v$

$0.3b + 0.7b = b$

$0.4b + 0.6b = b$



Having Solution or Not

(Linear Algebra version)

Summary

Is ***b*** in the ***span*** of the columns of A?

$A: m \times n \quad \boldsymbol{x} \in R^n \quad \boldsymbol{b} \in R^m$

Is ***b*** a ***linear combination*** of columns of A?

Is ***b*** in the ***span*** of the columns of A?

NO

No solution

YES

The columns of A are ***independent***.

Rank A = n

Nullity A = 0

Unique solution

The columns of A are ***dependent***.

Rank A < n

Nullity A > 0

Infinite solution

Summary

The columns of A are ***independent***.

Rank A = n

Nullity A = 0

$A: m \times n$

$\boldsymbol{x} \in R^n \quad \boldsymbol{b} \in R^m$

NO

Is $\boldsymbol{b}$ a ***linear combination*** of columns of A?

Is $\boldsymbol{b}$ in the ***span*** of the columns of A?

NO

No solution

YES

Infinite solution

YES

Is $\boldsymbol{b}$ a ***linear combination*** of columns of A?

Is $\boldsymbol{b}$ in the ***span*** of the columns of A?

NO

No solution

YES

Unique solution