在 Span 時
耍廢的 Vector

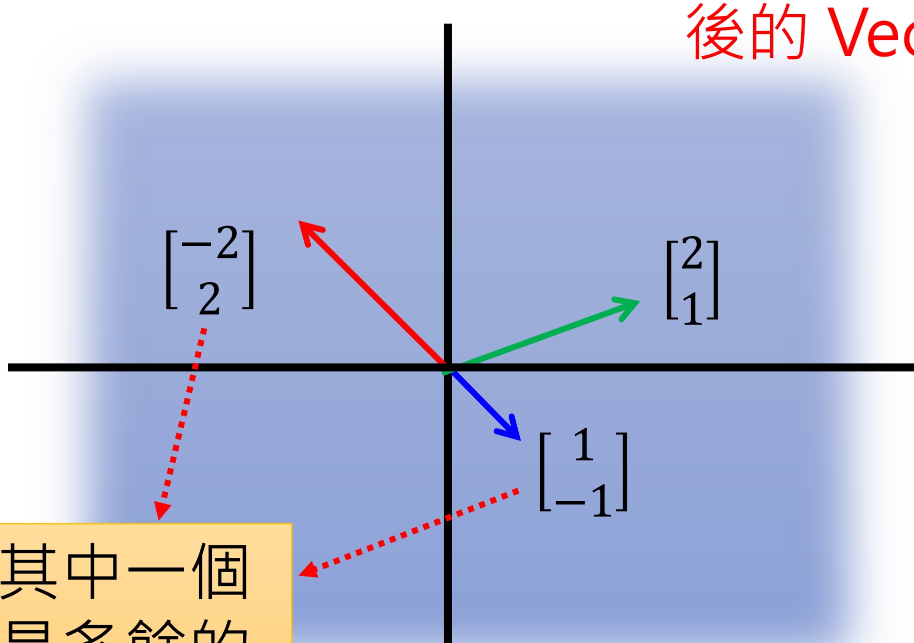
Span

$A\mathbf{x} = \mathbf{b}$

$\begin{bmatrix} 1 & -2 & 2 \\ -1 & 2 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ Every **b** has solution

- Let $S_3 = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$, what is Span S_3?

拿掉其中一個並不會影響 Span 後的 Vector Set



其中一個是多餘的

只要其中一個人在，另一個人就要廢

Every vector in $\mathcal{R}^2$ is their linear combination

Span $S_3 = \mathcal{R}^2$

要廢 Vector 的特徵

Given vector set $S = \{\boldsymbol{u}_1, \boldsymbol{u}_2 \cdots \boldsymbol{u}_k, \boldsymbol{v}\}$

Given vector set $S' = \{\boldsymbol{u}_1, \boldsymbol{u}_2 \cdots \boldsymbol{u}_k\}$

$\boldsymbol{v}$ 在要廢:

$Span\ S = Span\ S'$

⇐ ⇒

$\boldsymbol{v}$ 是 S 其餘成員的 linear combination ($\boldsymbol{v} \in Span\ S'$)

$$\boldsymbol{v} = b_1\boldsymbol{u}_1 + b_2\boldsymbol{u}_2 + \cdots + b_k\boldsymbol{u}_k$$

Target

$\boldsymbol{w} \in Span\ S$

⇓ ⇑

$\boldsymbol{w} \in Span\ S'$

$\boldsymbol{w} \in Span\ S$

$$\boldsymbol{w} = c_1\boldsymbol{u}_1 + c_2\boldsymbol{u}_2 + \cdots + c_k\boldsymbol{u}_k + c\boldsymbol{v} \in Span\ S'$$

$$= (c_1 + cb_1)\boldsymbol{u}_1 + (c_2 + cb_2)\boldsymbol{u}_2 \ldots + (c_k + cb_k)\boldsymbol{u}_k$$

$\boldsymbol{w} \in Span\ S'$

$$\boldsymbol{w} = c_1\boldsymbol{u}_1 + c_2\boldsymbol{u}_2 + \cdots + c_k\boldsymbol{u}_k$$

$c = 0$

$$= c_1\boldsymbol{u}_1 + c_2\boldsymbol{u}_2 + \cdots + c_k\boldsymbol{u}_k + c\boldsymbol{v} \in Span\ S$$

要廢 Vector 的特徵

Given vector set $S = \{\boldsymbol{u}_1, \boldsymbol{u}_2 \cdots \boldsymbol{u}_k, \boldsymbol{v}\}$

Given vector set $S' = \{\boldsymbol{u}_1, \boldsymbol{u}_2 \cdots \boldsymbol{u}_k\}$

$\boldsymbol{v}$ 在要廢:

$Span\ S = Span\ S'$

$\boldsymbol{v}$ 是 S 其餘成員的 linear combination ($\boldsymbol{v} \in Span\ S'$)

$$\boldsymbol{v} = 0\boldsymbol{u}_1 + 0\boldsymbol{u}_2 + \cdots + 0\boldsymbol{u}_k + 1\boldsymbol{v}$$

$\boldsymbol{w} \in Span\ S$

$\boldsymbol{w} \in Span\ S'$

$\boldsymbol{v} \in Span\ S$

$\boldsymbol{v} \in Span\ S'$