



Matrix

(You already learned in high school)

Matrix $A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 1 & -1 \\ -2 & 1 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$
 3×1 1×3

- If the matrix has m rows and n columns, we say the size of the matrix is m by n , written $m \times n$
- We use $\mathcal{M}_{m \times n}$ to denote the set that contains all matrices whose size is $m \times n$

3 columns
 2 rows $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \in \mathcal{M}_{2 \times 3}$
 2×3

2 columns
 3 rows $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \in \mathcal{M}_{3 \times 2}$
 3×2

先 Row 再 Column

Matrix

先 Row 再 Column

- **Index of component**: the scalar in the i -th row and j -th column is called (i,j) -entry of the matrix

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 1 & -1 \\ -2 & 1 & 1 \end{bmatrix}$$

(1,2)-entry

(3,1)-entry

(3,3)-entry

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$A = [\mathbf{a}_1 \quad \mathbf{a}_2 \quad \cdots \quad \mathbf{a}_n]$$

vectors

Matrix

- Two matrices with the same size can add or subtract.
- Matrix can multiply by a scalar

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 6 & 9 \\ 8 & 0 \\ 9 & 2 \end{bmatrix} \quad 9B$$

$$A + B$$

$$A - B$$

Properties

- A, B, C are $m \times n$ matrices, and s and t are scalars
 - $A + B = B + A$
 - $(A + B) + C = A + (B + C)$
 - $(st)A = s(tA)$
 - $s(A + B) = sA + sB$
 - $(s+t)A = sA + tA$