

A blue wireframe sphere is centered in the background, composed of numerous small blue dots connected by thin blue lines, creating a mesh-like structure. The sphere is set against a solid black background.

# Properties of Matrix-Vector Product

# Matrix-vector Product

- The size of matrix and vector should be matched.

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 1 & -1 \\ -2 & 1 & 1 \end{bmatrix} \quad \begin{matrix} \leftarrow \text{red X} \\ \swarrow \\ \downarrow \end{matrix} \quad x = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$
$$A' = \begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 1 & 4 \end{bmatrix} \quad A'' = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 0 & -1 \\ 1 & -3 \end{bmatrix}$$

# Properties of Matrix-vector Product

- $A$  and  $B$  are  $m \times n$  matrices,  $\mathbf{u}$  and  $\mathbf{v}$  are vectors in  $\mathcal{R}^n$ , and  $c$  is a scalar.
  - $A(\mathbf{u} + \mathbf{v}) = A\mathbf{u} + A\mathbf{v}$
  - $A(c\mathbf{u}) = c(A\mathbf{u}) = (cA)\mathbf{u}$
  - $(A + B)\mathbf{u} = A\mathbf{u} + B\mathbf{u}$
  - $A\mathbf{0}$  is the  $m \times 1$  zero vector
  - $\mathbf{0}\mathbf{v}$  is also the  $m \times 1$  zero vector
  - $I_n \mathbf{v} = \mathbf{v}$
- $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

# Properties of Matrix-vector Product

需要知道這麼多嗎?

- A and B are  $m \times n$  matrices. If  $A\mathbf{w} = B\mathbf{w}$  for all  $\mathbf{w}$  in  $\mathcal{R}^n$ . Is it true that  $A = B$ ?

$A\mathbf{e}_j = \mathbf{a}_j$ , where  $\mathbf{e}_j$  is the  $j$ -th standard vector in  $\mathcal{R}^n$

$$\mathbf{e}_j = \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix} \quad A\mathbf{e}_j = [\mathbf{a}_1 \quad \cdots \quad \mathbf{a}_n] \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix} = 1 \cdot \mathbf{a}_1 + 0 \cdot \mathbf{a}_2 + \cdots + 0 \cdot \mathbf{a}_n = \mathbf{a}_j$$

*Column Aspect*

$$A\mathbf{e}_1 = B\mathbf{e}_1 \quad A\mathbf{e}_2 = B\mathbf{e}_2 \quad \cdots \quad A\mathbf{e}_n = B\mathbf{e}_n$$

$$\mathbf{a}_1 = \mathbf{b}_1$$

$$\mathbf{a}_2 = \mathbf{b}_2$$

$$\mathbf{a}_n = \mathbf{b}_n$$

$$\Rightarrow A = B$$