Properties of Matrix-Vector Product

Matrix-vector Product

- The size of matrix and vector should be matched.

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 1 & -1 \\ -2 & 1 & 1 \end{bmatrix} \qquad x = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$A' = \begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad A'' = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 0 & -1 \\ 1 & -3 \end{bmatrix}$$

Properties of Matrix-vector Product

- A and B are mxn matrices, **u** and **v** are vectors in $\mathcal{R}^n$, and c is a scalar.
- $A(\boldsymbol{u}+\boldsymbol{v}) = A\boldsymbol{u} + A\boldsymbol{v}$
- $A(c\boldsymbol{u}) = c(A\boldsymbol{u}) = (cA)\boldsymbol{u}$
- $(A+B)\boldsymbol{u} = A\boldsymbol{u} + B\boldsymbol{u}$
- $A\mathbf{0}$ is the mx1 zero vector
- $\boldsymbol{O}\boldsymbol{v}$ is also the mx1 zero vector
- $I_n\boldsymbol{v} = \boldsymbol{v}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$= \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Properties of Matrix-vector Product

需要知道這麼多嗎?

- A and B are mxn matrices. If $A\boldsymbol{w} = B\boldsymbol{w}$ for all $\boldsymbol{w}$ in $\mathcal{R}^n$. Is it true that $A = B$?

$A\boldsymbol{e_j} = \boldsymbol{a_j}$, where e_j is the j-th standard vector in $\mathcal{R}^n$

Column Aspect

$$\underset{j}{\boldsymbol{e_1}} = \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix} \quad A\boldsymbol{e_1} = [\boldsymbol{a_1} \quad \cdots \quad \boldsymbol{a_n}] \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix} = 1 \cdot \boldsymbol{a_1} + 0 \cdot \boldsymbol{a_2} + \cdots + 0 \cdot \boldsymbol{a_n}$$

$$= \boldsymbol{a_1}_{\;j}$$

$$\boxed{A\boldsymbol{e_1} = B\boldsymbol{e_1} \quad A\boldsymbol{e_2} = B\boldsymbol{e_2} \quad \cdots \quad A\boldsymbol{e_n} = B\boldsymbol{e_n}}$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$\boldsymbol{a_1} = \boldsymbol{b_1} \quad \boldsymbol{a_2} = \boldsymbol{b_2} \quad \cdots\cdots \quad \boldsymbol{a_n} = \boldsymbol{b_n}$$

$$\Rightarrow A = B$$